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# **ADVANCED SUPPORT FOR MULTILEVEL HETEROGENEOUS EMBEDDED HIGH PERFORMANCE COMPUTING**

**Texas Tech University**

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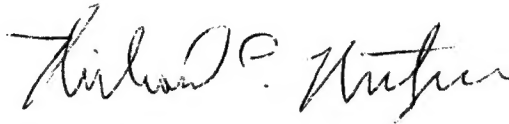
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## ABSTRACT

Embedded systems often must adhere to strict size, weight and power (SWAP) constraints and yet provide tremendous computational throughput. Increasing the difficulty of this challenge, there is a trend to utilize commercial-off-the-shelf (COTS) components in the design of such systems to reduce both total cost and time to market. Two embedded high-performance radar applications are investigated in this effort: synthetic aperture radar (SAR) and space-time adaptive processing (STAP). Advanced techniques for optimally configuring and utilizing the components of a commercially available multicomputer platform are described for these two applications. Although a particular platform is targeted in this study - Mercury Computer Systems' RACE multicomputer - the techniques described in this report are generic and could be applied to a range of different computational platforms.

For the SAR application, a system performance model, in the context of SWAP, is developed based on mathematical programming. An optimization technique using a combination of constrained nonlinear and integer programming is developed to determine system configurations that minimize SWAP.

A major challenge of implementing parallel STAP algorithms on multiprocessor systems is determining the best method for distributing the 3-D data cube across processors of the multiprocessor system (i.e., the mapping strategy) and the scheduling of communication within each phase of computation. It is important to understand how mapping and scheduling strategies affect overall performance. A network simulator is developed for this purpose and is used to evaluate the performance of various mapping and scheduling strategies.



## PREFACE

In essence, this report is the combination of work described in the Master's theses of Mr. Jeffrey T. Muehring [25] and Mr. Jack M. West [24]. These two graduates of Texas Tech University performed research under the direction of their major professor, Dr. John K. Antonio, who is also the Principal Investigator (PI) for this effort, supported by Rome Laboratory under Grant No. F30602-96-1-0098. This research effort began in April 1996 and carried on (after a no-cost extension) through March 1998.

In July 1997, the PI was awarded another contract through the Defense Advanced Research Projects Agency (DARPA), entitled "Configuring Embeddable Adaptive Computing Systems for Multiple Application Domains with Minimal Size, Weight, and Power," Contract No. F30602-97-2-0297. The research undertaken in the DARPA effort, especially near the beginning of that effort, overlapped with the concluding work being performed under the Rome Laboratory effort. In fact, the early successes of the Rome effort were reported in the proposal that was ultimately funded by DARPA. Due to the overlap in topics researched and funding provided by the two organizations (Rome Laboratory and DARPA), it is difficult to define exactly where the results supported by Rome Laboratory stop and those supported by DARPA begin. In fact, the research results reported here that were obtained during the period from July 1997 through March 1998 were due to the joint support from both Rome Laboratory and DARPA. For this reason, some of the material reported here will also appear in a future report for the DARPA effort.

The report is divided into two parts. The first part describes research entitled "Optimal Configuration of a Parallel Embedded System for Synthetic Aperture Radar Processing" [25] and the second part is entitled "Simulation of Communication Time for Space-Time Adaptive Processing on a Parallel Embedded System" [24].

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**PART 1:**  
**OPTIMAL CONFIGURATION OF A PARALLEL EMBEDDED**  
**SYSTEM FOR SYNTHETIC APERTURE RADAR PROCESSING [25]**

## CHAPTER I

### INTRODUCTION TO PART 1

Even as increasingly more computing power is available on ever decreasing areas of silicon, the processing requirements of modern applications often exceed the capabilities of individual processors. That is, regardless of the speed and memory of a system, there always will exist some application that pushes the envelope of imaginable computation. It is highly probable that this maxim will remain valid for all generations of computers to come. Out of this truth was born parallel processing.

When current technology cannot provide a single chip with adequate performance, it seems reasonable to assume that multiple chips might work in tandem to provide for the shortcomings of the single chip. However, apart from the fact that a vast number of computational tasks are not easily parallelizable, the physical requirements of multiple processors can pose critical difficulties in terms of size, weight, and power (SWAP). Such constraints especially hold true for embedded systems.

Synthetic aperture radar (SAR) data processing often belongs to this genre of problems that require both high-performance computing and adherence to tight SWAP constraints. Intensive computing results from the massive amount of information that is required to process a SAR image and SWAP constraints are due to the nature of the host vehicles of such systems — often unmanned aerial vehicles (UAVs) or spaceborne orbiting satellites. Assuming the requirement of multiple processors and exploiting the well-defined parallelization of SAR processing, it is beneficial to determine the exact configuration of hardware and software that will optimize limited resources (i.e., SWAP). This work proposes two optimization models based on mathematical programming. The models are

applied to a Mercury Computer Systems' RACE heterogeneous multicomputer [7], assumed to be onboard a tightly SWAP-constrained UAV, on which a SAR stripmap image processing algorithm is mapped across multiple computing elements.

This work begins with an overview of the background material. Chapter II briefly covers the principles of radar and synthetic aperture radar and the formulas that are most relevant to the processing of the data. Chapter III provides an overview of the Mercury RACE multicomputer and applies the processing techniques discussed in Chapter II to the Mercury RACE system. Chapter IV formulates the optimization problem in the context of mathematical programming and establishes a basis for applying it to the configuration of a Mercury RACE system. Chapter V introduces an ideal shared-memory model (ISMM) and investigates a representative sample of solutions using this model. Chapter VI introduces a more sophisticated and realistic approach, the CN-constrained model (CNCM). Comparison to the ISMM is conducted and the utility of the ISMM as an approximator to the CNCM is investigated. Chapter VII explores the use of random configurations to both verify the solutions obtained from the models discussed and also possibly provide an alternative method of performing optimization. Chapter VIII concludes the work with a summary of the investigation and results.

## CHAPTER II

### PRINCIPLES OF SYNTHETIC APERTURE RADAR

Synthetic aperture radar (also known as synthetic array radar) is implemented in numerous systems for military, commercial, and scientific purposes. SAR's widespread use is due to its ability to produce photo-quality images with the use of radio waves. Uses include ground surveillance, terrain mapping, weather mapping, ocean current and ice floe tracking, and detection of earthquake faults. Because radio waves are relatively unaffected by poor weather and/or lighting, radar's performance remains constant in most conditions. In contrast to most optical techniques, as a ranging instrument radar can deliver true three-dimensional images. As discussed below, SAR distinguishes itself from conventional radar by its drastically reduced size requirements of the physical antenna in exchange for a substantial amount of postprocessing. A brief overview of basic radar and more specific SAR principles as is relevant to this research is given below. For a thorough treatment of basic radar, the reader is referred to books such as [6, 21, 22]. Synthetic aperture radar is covered in works such as [3, 5, 9, 12].

#### 2.1 Conventional Radar

The fundamental principle of radar involves the detection of objects by the transmission and return of electromagnetic waves. When pulses are emitted from the radar transmitter, portions of the signals are returned (with significant attenuation in power) after colliding with objects in their path. Since electromagnetic waves travel at the speed of light, the range  $R$  of an object can be



easily calculated by

$$R = \frac{cT_e}{2},$$

where  $c$  is the speed of light and  $T_e$  is the elapsed time from the transmission to the reception of the signal.

If the transmitter consisted simply of a point with no direction of the signal, the range information returned by an object would yield only the radius of the spherical surface on which the object resides, with the transmitter located at the center. However, transmitters typically direct the signal beam so as to sweep out a solid angle of the sphere. In the case of an airborne radar directed toward the ground, such as employed for terrain mapping or ground surveillance, the solid angle effectively becomes an elliptical area on the ground illuminated by electromagnetic waves, known as the radar's footprint (Fig. 2.1). This two-dimensional area is referred to in terms of range and azimuth, where the range dimension extends orthogonally from the aircraft and the azimuth dimension runs parallel to the aircraft's line of flight. The range swath  $R_s$  is the length of the footprint in the range dimension, and the width of the footprint in the azimuth dimension is the beamwidth at a given range. Although the beamwidth increases with range, typically it is treated as a constant, assuming an insignificant variance in the beamwidth from the bottom to the top of the range swath, at least at ranges of interest.

The radar resolution is the minimum distance between two distinguishable points on the ground. Resolutions for azimuth and range are individually calculated. However, physical parameters of the system are typically determined such that the resolutions in both dimensions are equal. Other factors, as discussed below, determine the actual resolution for a given system. Distinction is made between a *simple radar*, which employs a minimum of signal processing, a *conven-*

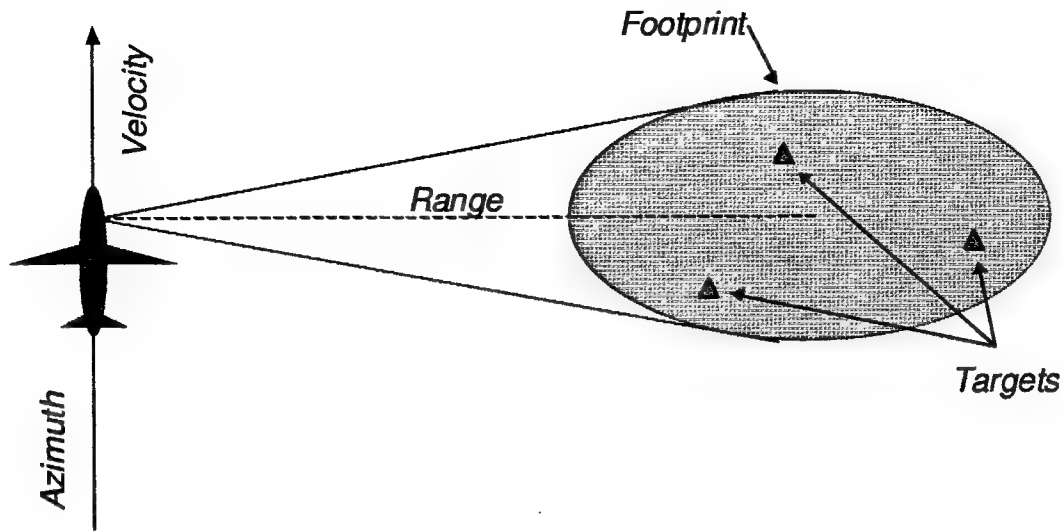


Fig. 2.1: Footprint of aerial radar.

*tional radar*, which is mounted on a stationary platform, and finally a *synthetic aperture radar*.

Range resolution  $\delta_R$  of a simple radar is affected by the transmission pulse. Directly proportional to the duration of the pulse  $\tau_p$ ,  $\delta_R$  is defined by the following equation:

$$\delta_R = \frac{c\tau_p}{2}. \quad (2.1)$$

Therefore for fine resolution,  $\tau_p$  must be small. However, a significant signal to noise ratio (SNR) in the returned signal must be maintained, requiring a high total power in the transmitted signal. A small  $\tau_p$  and a set total power entails a very high burst of energy for fine resolutions, which is impractical for most systems.

To overcome this difficulty, a carrier frequency that varies with time is often applied to the pulse, known as analog linear frequency modulation. Physically,

this pulse is represented by Fig. 2.2. Mathematically, however, it should be noted that each pulse is visualized as a signal with both positive and negative frequency components, centered at time  $t = 0$  (Fig. 2.3). The resultant pulse is known as a chirp, and the rate with which the frequency varies is the chirp rate. With signal processing techniques, this method allows definition of the compressed pulse width  $\tau_c$  in time as

$$\tau_c = \frac{1}{B},$$

where the bandwidth  $B$  of the pulse is the frequency differential between the lowest and highest frequencies of the carrier signal. A new equation for  $\delta_R$  follows:

$$\begin{aligned} \delta_R &= \frac{c\tau_c}{2} \\ &= \frac{c}{2B}. \end{aligned} \quad (2.2)$$

The above equation for range resolution is greatly improved over the previous one employing  $\tau_p$  because of the high bandwidths feasible in typical systems. The carrier frequency is often in the gigahertz range, although the frequency range (i.e.,  $B$ ) is typically in megahertz.

With a conventional radar, azimuth (also known as cross-range) resolution  $\delta_{\text{real}}$  is defined simply by the beamwidth. Most systems, however, cannot produce a sufficiently narrow beam at ranges of interest to provide acceptable resolution. Beamwidth depends upon the real antenna aperture  $A_{\text{real}}$  (length of the antenna) and is approximated as follows:

$$\delta_{\text{real}} \approx \frac{R\lambda}{A_{\text{real}}}, \quad (2.3)$$

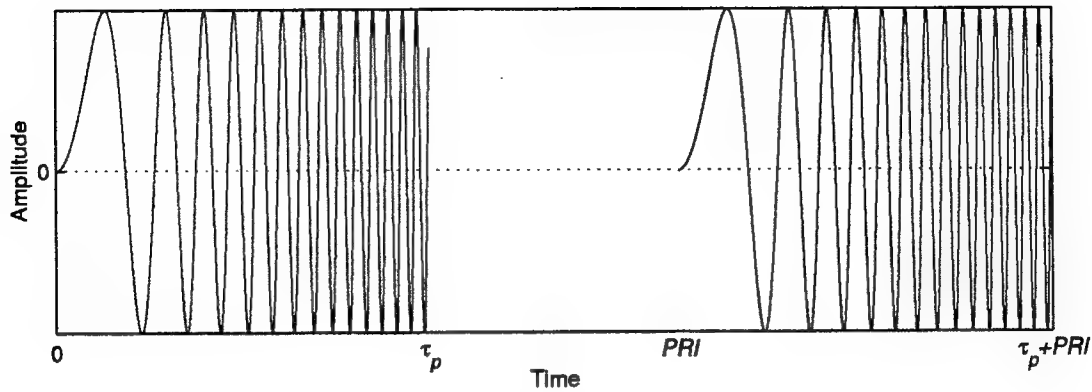


Fig. 2.2: Reference chirp starting at  $t = 0$ .

where  $\lambda$  is the wavelength.

Fine azimuth resolution at a set range therefore can be obtained with a small wavelength or a large effective aperture. Different wavelengths are preferential for different types of targets, so this variable may also be set to some degree. The effective aperture is extremely limited by the allowable size and weight of an antenna mountable on an aircraft, especially a UAV.

An intuitive method of increasing the effective aperture would be to line up several smaller antennas and average the returns. Such an array of antennas might be feasible for a ground radar system, but this provides no help in the case of airborne radar. This idea of an antenna array, however, leads to the discussion of SAR, which simulates an antenna array with only one antenna.

## 2.2 Synthetic Aperture Radar

SAR provides a method of obtaining high (fine) cross-range resolution with a small effective aperture. Assuming a fixed-angle radar and straight line of flight, the method (called stripmapping) exploits the inherent motion of an aircraft by utilizing many successive large-beamwidth footprints, each slightly offset in

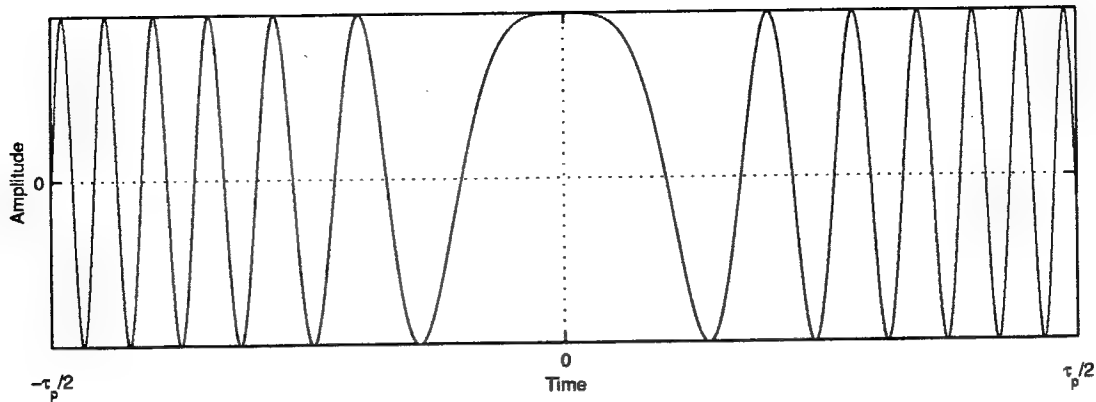


Fig. 2.3: Reference chirp centered at  $t = 0$ .

time by the interval between transmitted pulses, or pulse repetition interval  $PRI$  (Fig. 2.4). After processing, the resultant radar image has a resolution that is otherwise obtainable only with a beamwidth many times narrower than the real one (Fig. 2.5). The real antenna aperture then can be replaced by the synthetic aperture  $A_{syn}$ . Counterintuitively,  $A_{syn}$  can be as great as the width of the real radar footprint  $\delta_{real}$ , creating an inverse relationship between real antenna length  $A_{real}$  and synthetic aperture length  $A_{syn}$ . This phenomenon is shown by substituting the equation for the real beamwidth  $\delta_{real}$  (Eqn. 2.3) for  $A_{syn}$  in the corresponding equation for  $\delta_{syn}$ :

$$\begin{aligned}\delta_{syn} &= \frac{R\lambda}{A_{syn}} \\ &= \frac{R\lambda}{\frac{R\lambda}{A_{real}}} \\ &= A_{real}.\end{aligned}$$

Because of increased phase sensitivity that occurs from processing the return signals of the synthetic aperture, an additional  $\frac{1}{2}$  can be factored into the equation

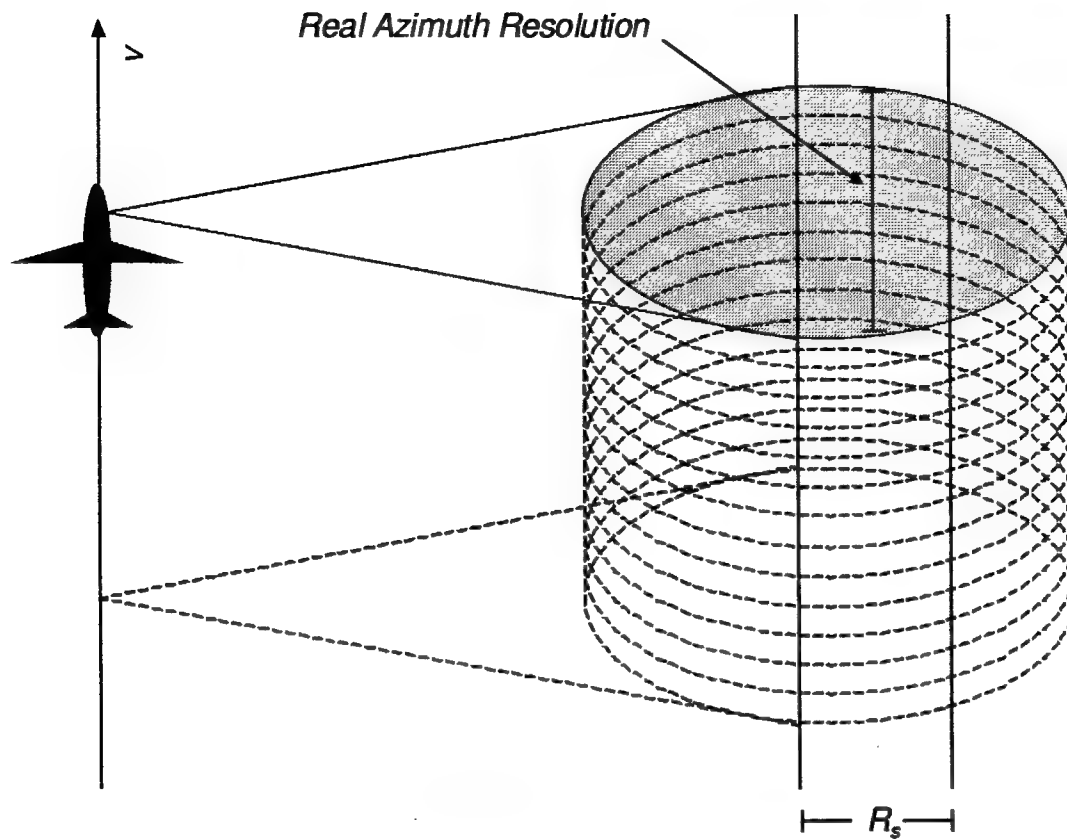


Fig. 2.4: Overlapping realbeam footprints.

for  $\delta_{\text{real}}$  [9], resulting in

$$\delta_{\text{syn}} = \frac{A_{\text{real}}}{2}. \quad (2.4)$$

Notice that the above equation for  $\delta_{\text{syn}}$  involves only the antenna length  $A_{\text{real}}$  and is independent of range.

Processing of the returned signals in both the range and azimuth dimensions involves the use of matched filtering, a signal processing technique in which the returned signal is convolved with a reference signal to reduce noise and/or

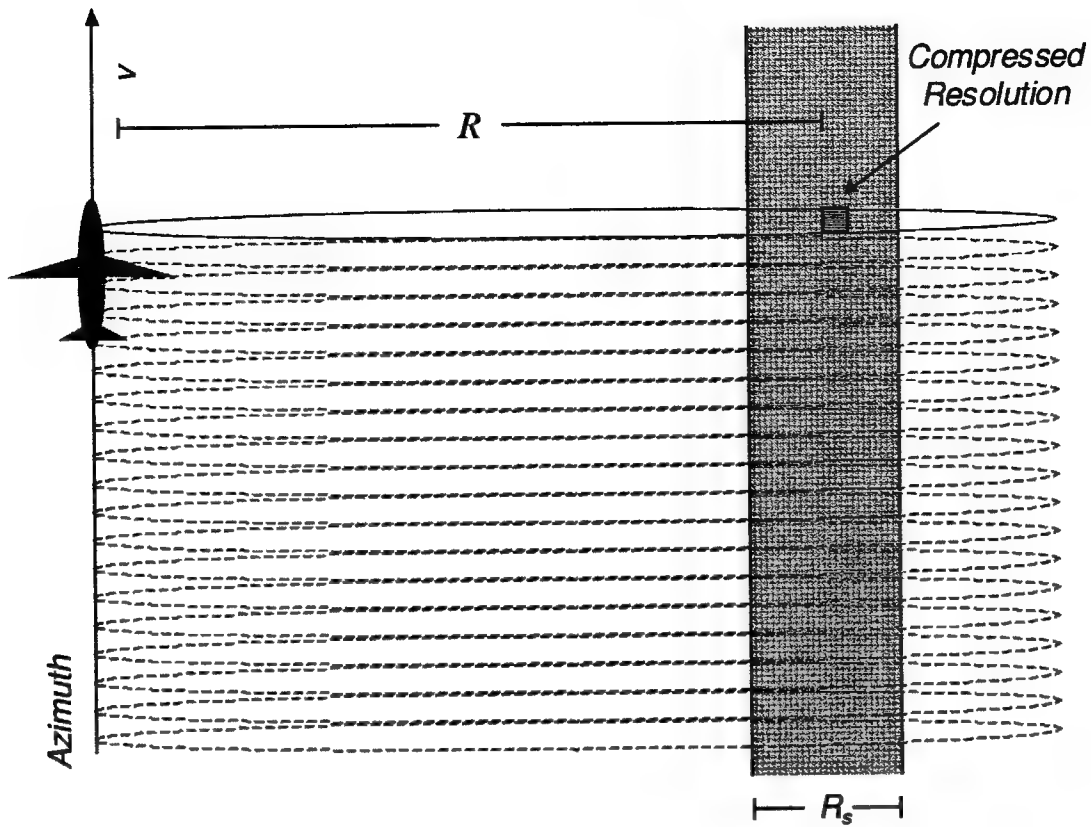


Fig. 2.5: Resultant narrow synthetic beams.

increase resolution. The convolution integral is defined as

$$g(t) = \int_{-\infty}^{\infty} h(\tau) f(t - \tau) d\tau, \quad (2.5)$$

where  $g(t)$  is the output,  $h(t)$  is the impulse response of the filter, and  $f(t)$  is the input signal. For a matched filter, the reference function  $h(t)$ , also known as the convolution kernel, is essentially a mathematically manipulated version of the original signal. The resultant waveform from the convolution contains a "spike," or mainlobe, representing the return for the compressed range pulse or compressed azimuth beamwidth (Fig. 2.2).

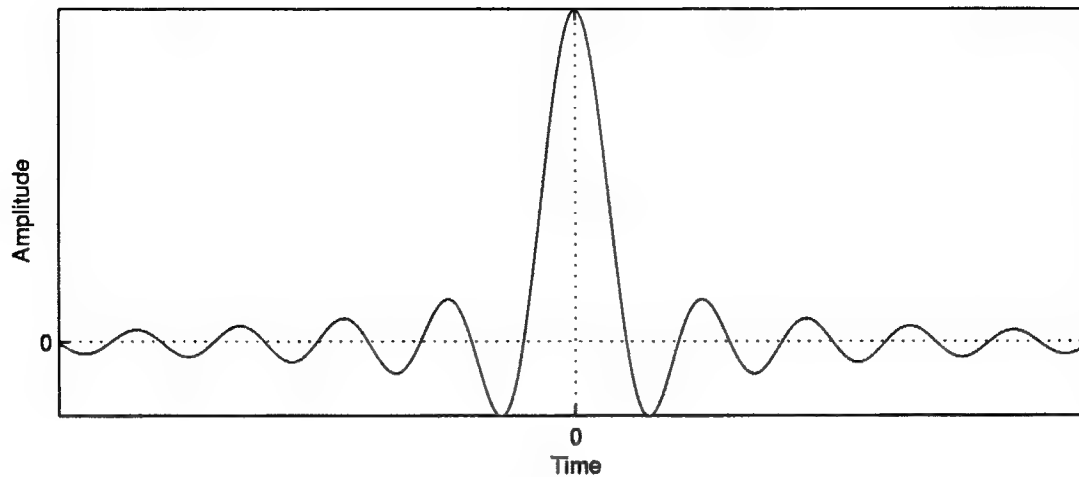


Fig. 2.6: Example waveform for the convolution of a return signal with reference signal.

In azimuth processing, the matched filter effects the focusing of multiple real beamwidths to the compressed beamwidth by emulating the geometry of a radar receiver dish. Receiver dishes are classically parabolic in shape because of the property of parabolas in which lines parallel to the axis and incident on the parabola all converge at the focus. Furthermore, the lengths of all such line segments originating from a common range are equal. At ranges of interest, the return signal is approximated by a plane wave composed of parallel lines. SAR emulates a parabola, even though the synthetic array is a straight line, by mathematically setting the time delays in the return signals that would have occurred if the array was parabolic in shape. This compensation is accomplished by the convolution of the return signal and the reference function. Without this adjustment, an antenna array is termed “unfocused.”

The convolution integral in Eqn. 2.5 is most efficiently implemented with the use of Fast Fourier Transforms (FFTs), where the Fourier Transform  $F(\omega)$  of a



function  $f(t)$  is given by

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt. \quad (2.6)$$

The inverse Fourier Transform is similarly expressed as

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{j\omega t} d\omega.$$

The FFT describes a family of efficient algorithms for computing the discrete form of Eqn. 2.6. This use of FFTs to perform convolution is known as fast convolution. Fast convolution exploits the Fourier transform property in which the product of two functions in the frequency domain is equivalent to their convolution in the time domain. The Fourier transform of Eqn. 2.5 is

$$G(\omega) = H(\omega)F(\omega), \quad (2.7)$$

where  $H(\omega)$  and  $F(\omega)$  represent the reference and received signals, respectively. Computationally, Eqn. 2.7 can be faster to perform on a digital computer than Eqn. 2.5. It should be noted that at the digital processing level, the operations are discrete, with the input function a sampled representation of the real returned signal. The discrete forms of Eqns. 2.5 and 2.7 are, respectively,

$$g[t] = \sum_{\tau=-\infty}^{\infty} h[\tau]f[t-\tau],$$

and

$$G[\omega] = H[\omega]F[\omega].$$

Similarly, the reference function is represented by a discrete number of sample

points equal to a value known as the time-bandwidth product.

In the case of range processing, the time-bandwidth product depends strictly upon the original signal, where the time component is  $\tau_p$  and bandwidth is  $B$ . This value also represents the compression ratio achieved by processing, as determined in Eqn. 2.2. Because the reference function in a matched filter convolution is called the kernel, this parameter will be referenced in this work as the range kernel size  $K_r$ , representing the number of discrete points in the kernel. A convenient form of this value is derived by taking the ratio of uncompressed range resolution (Eqn. 2.1) to the compressed range resolution, resulting in

$$K_r = \frac{c\tau_p}{2\delta_R}. \quad (2.8)$$

A brief discussion of Doppler effects precedes derivation of the azimuth reference function and time-bandwidth product. Doppler frequency shift describes the effect of relative motion between two objects on the reception of a signal by one object when transmitted or reflected by the other object. The shift is inversely related to the rate of change of distance between the two objects. Given a fixed radar platform velocity, the distance from the aircraft to an object constantly changes within the synthetic aperture and thus so does the Doppler frequency. The frequency of the returned signal then will not be identical to that of the transmitted signal.

Taking time  $t = 0$  to be the point at which the radar is directly perpendicular to the target in question, i.e., when the center of the real beam (and synthetic beam) footprint is upon the target, the formula for Doppler frequency  $f_d$  is approximated as follows:

$$f_d \approx \frac{2v^2t}{R\lambda} \quad (2.9)$$

where  $v$  is the velocity of the aircraft. When a target first appears in the real beam footprint,  $f_d$  is at its maximum because the rate of change in the azimuth dimension toward the target is at a maximum. When the object is in the center of the aperture,  $f_d$  is zero. Then, as the object leaves the footprint,  $f_d$  assumes its greatest negative value. Doppler frequency information, as well as signal intensity, from the received signal is stored for azimuth resolution processing.

The azimuth time-bandwidth product depends upon the range of Doppler frequency in the returned signal rather than on the bandwidth of the transmitted signal as in range compression. The frequency range is defined by Eqn. 2.9. The time parameter is the duration of the synthetic aperture rather than that of the real aperture. As for range, the value also defines the azimuth compression ratio, describing the ratio of the real beamwidth to the synthetic beamwidth. Using Eqns. 2.3 and 2.4, azimuth kernel size  $K_a$  can be represented as follows:

$$\begin{aligned} K_a &= \frac{\delta_{\text{real}}}{\delta_{\text{syn}}} \\ &= \frac{2R\lambda}{A_{\text{real}}^2} \\ &= \frac{R\lambda}{2\delta_{\text{syn}}} \end{aligned}$$

This chapter has presented an overview of the fundamental mathematics of SAR. The next chapter incorporates the equations derived in this chapter into the framework of a SAR processing system based on the Mercury RACE multicomputer.

## CHAPTER III

### THE MERCURY RACE SYSTEM

Following is an overview of the Mercury RACE multicomputer. This first section has been taken in part from J. West's introduction to the Mercury RACE system [24]. West is currently researching the Mercury RACE at the network level for the target application of space-time adaptive processing.

In recent years, Mercury Computer Systems, Inc. has emerged as one of the leaders in the development and manufacturing of high-performance embedded heterogeneous message-passing systems designed to address complex real-time applications requiring tremendous computational throughput. The computational requirement is often in the order of billions of floating point operations per second (Gflops). Mercury's RACE multicomputer provides a foundation for parallel systems and offers a set of building blocks that provide upward scalability. A high-level diagram of a typical RACE multicomputer is illustrated in Fig. 3.1. The system's primary components include DSPs, reduced-instruction-set-computing (RISC) processors, I/O ports, and a network interface all connected via the RACEway interconnection network.

The fundamental computing unit in the RACE system is the compute node (CN). A CN houses one or more compute elements (CEs) of the same type, either central processing units (CPUs) or digital signal processors (DSPs). Daughter-cards accommodate one or two CNs of the same type. Two or more daughter-cards reside on one mainboard. Communication between CNs, daughtercards, and mainboards takes place through the RACEway interconnect. Communication between processors residing on the same CN, however, is routed through the logic within the CN. This hardware logic also facilitates access to a shared memory block by all processors on the same CN, as well as accomodating remote

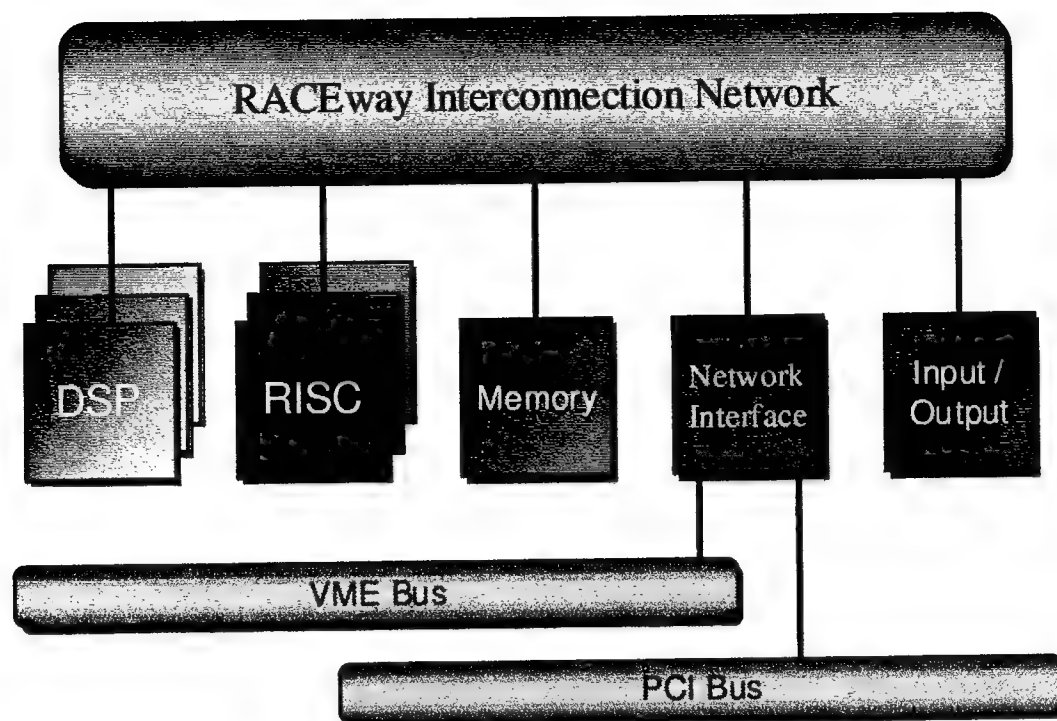


Fig. 3.1: The RACE multicomputer.

memory accesses from other CNs.

The RACEway interconnection network is the framework used to provide high-performance communications among the interconnected processors and devices. Each node in the multicomputer interfaces the network through the RACE network chip. The network chip (see Fig. 3.2) is a crossbar with six bidirectional channels consisting of 32 parallel data lines and eight control leads. Each crossbar transfers data synchronously at a clock rate of 40 megahertz (MHz). Each channel is bidirectional but is only driven in one direction at a time at 160 megabytes per second (MB/s) [15]. Among the six ports comprising a RACE crossbar, each switch can either interconnect any three port pairs, providing an aggregate bandwidth of 480 MB/s, or can cause data to be broadcasted to all or

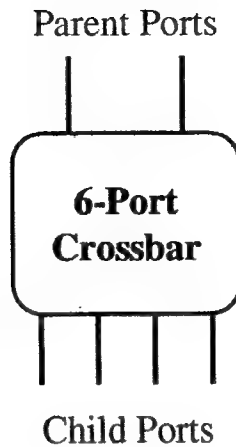


Fig. 3.2: The RACEway six-port network chip (derived from [15]).

a subset of the remaining five ports [7].

The versatility of the RACE network chip allows the RACE multicomputer to be configured into a number of different network topologies. Possible network topologies include two-dimensional (2-D) and three-dimensional (3-D) meshes, 2-D and 3-D rings, grids, and Clos networks. However, the most common configuration is a fat-tree architecture (see Fig. 3.3). For a fat-tree configuration, the crossbar switches are connected in a parent-child arrangement. Each crossbar has two parent ports and four child ports (see Fig. 3.2). The crossbars of the RACE multicomputer are connected to form the branches of the fat tree. The compute nodes represent the leaves of the tree.

To route a message from one processor to another, the message goes up the tree, selecting one of the two parents as it goes, until it reaches a network chip that is a common ancestor of both the source and destination node [15]. After reaching the common ancestor network switch, the message travels down the fat tree to the destination compute node. Fig. 3.4 illustrates a message transfer between two CNs.

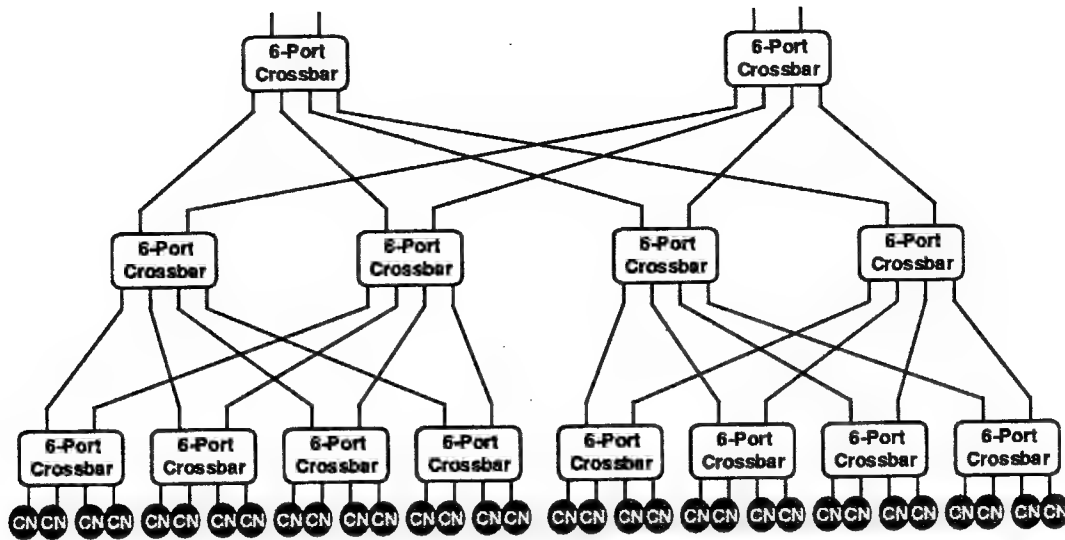


Fig. 3.3: The RACE multicomputer fat-tree interconnection network.

In conventional tree architectures, there is only one path between any pair of processors. One major problem associated with such conventional networks is that they suffer communication bottlenecks at higher levels in the tree. For example, when several compute nodes in the left subtree communicate with compute nodes in the right subtree, the root node must handle all the messages [14]. This problem can be partially alleviated by increasing the number of effective parallel paths between compute nodes. This type of modified tree architecture is referred to as a fat tree. Mercury's RACE system is based on the fat-tree topology.

The RACE system is a circuit-switched network. In a circuit-switched network, a compute node establishes a path through the network. Once the compute node has been granted a path to the destination node, the path is occupied for the duration of the message transmission.

To send a message through Mercury's fat-tree network, the first step is to

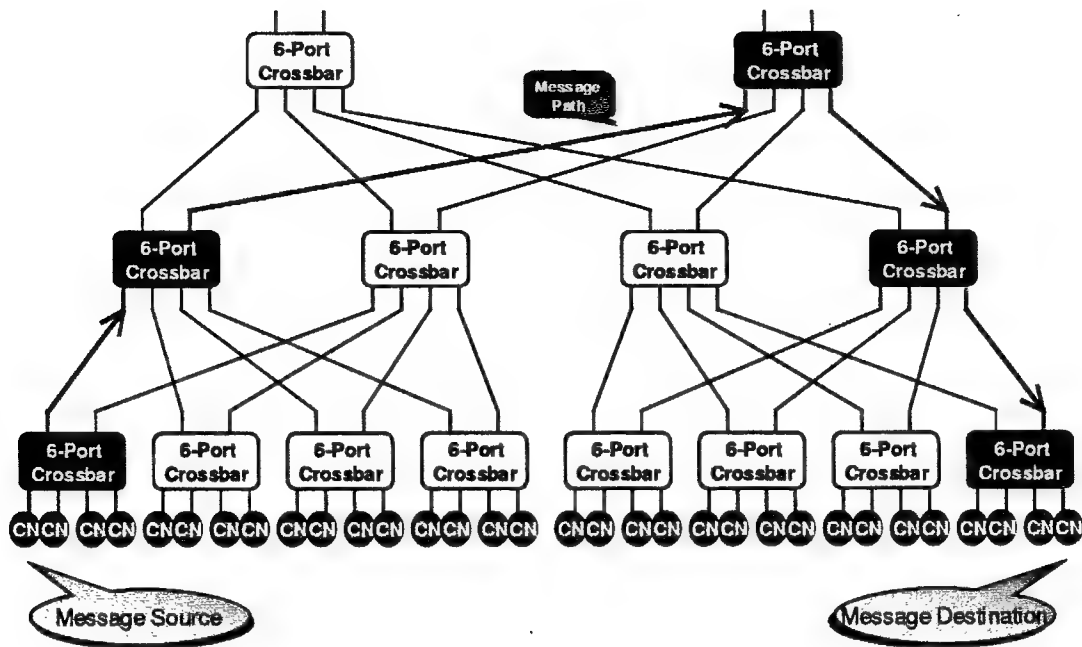


Fig. 3.4: Message transfer between two CNs.

establish a path. To establish a path, a message header specifying a path is sent through the network along a given channel. The status of a channel is categorized as either free or occupied. The header makes as much progress as possible through the network until blocked. After a message header has been blocked, it waits until a free channel becomes available. When a free channel matching the path specification (of the message header) becomes available, the channel is flagged as occupied, and the message header advances along that path. After establishing a path to the destination node, the message header sends an acknowledgment to the source along the allocated path. Upon receiving acknowledgment of a granted network path, the source node sends its message down the path in a pipelined fashion [15]. During the transmission of the last byte of data, the status of each occupied channel is set to free.

As stated above, the Mercury interconnection network under consideration



is a fat-tree architecture comprised of multiple parallel paths. An interesting feature of the Mercury system is that it provides auto route path selection at the crossbar level, which means the multiple paths in the RACEway network may be automatically and dynamically selected by the RACE network crossbars. For instance, if one path is currently occupied with a data transfer and another path matching the path specification is free, the free path is automatically selected by the crossbar logic [19]. Auto-route path selection frees the programmer from the details of path routing. In addition, processes that require high amounts of interprocessor communication, such as a distributed matrix transposition, benefit from adaptive routing [7].

In networks that take advantage of adaptive routing, some type of priority scheme is typically used to avoid deadlocks and guarantee that an application will meet tight real-time constraints. To facilitate the implementation of a priority scheme, each message header includes a priority number, ranging from zero to three. To understand the role of priorities, suppose a high priority message arrives at a crossbar, and all the outgoing channels matching the message's path specification are occupied by other messages. If a lower priority message occupies one of the channels that the higher priority message needs, the lower priority message is required to release the channel in the Mercury system [15]. The lower priority message is suspended by sending a "kill" signal backwards along the path to the source node. Data that was already in the path propagates down the pipeline to the destination node with the current byte releasing the channels as if it were the last byte of data in the message. After the path becomes free, the higher priority message may gain access to the channel. The lower priority message resumes when a free channel becomes available. The processor-network hardware contains built-in facilities that handle the suspension and reestablishment of a killed message.

For messages contending for the same channel with the same priority, the incoming port number is the tie-breaking mechanism. Furthermore, messages coming from parents have a higher priority than messages from children, and messages coming from a higher numbered parent (or child) port number have a higher priority than messages originating from lower numbered ports. However, this is only a tie-breaking mechanism for messages arriving or blocked at the same crossbar, and it does not result in suspension of any message that has already been routed to the next switch [15].

With the network configured as a fat tree, the RACEway interconnection fabric provides very good scaling properties. In a  $p$ -processor system, the height  $h$  of the fat tree is  $h = \lceil \log_4 p \rceil$ . Thus, the network diameter  $D$  or maximum number of links traversed is  $D = 2h - 1$ . The bisection bandwidth  $B$  of a system, which is defined as the minimum number of edges (or channels) that have to be removed along a cut that partitions the network into two equal halves, assuming  $p = 4^k$  processors, where  $k$  is an integer, is  $B = 160\sqrt{p}$  MB/s. (Each channel in the RACEway system has a bandwidth of 160 MB/s [15].

The RACEway system may be configured as a heterogeneous multicomputer composed of two or more different types of processors. The potential heterogeneity of the RACE multicomputer includes various possible configurations of i860, PowerPC, and Super Harvard Architecture Computer (SHARC) DSP processors. The SHARC DSP is ideally suited for embedded vector signal processing operations such as FFTs where physical size and power are at a premium or other similar algorithms that have a high ratio of data-to-computation. Furthermore, Analog Devices' 21060 SHARC processor provides more than twice the physical processor density of RISC-based CNs. In contrast, the PowerPC and i860, both RISC processors, are appropriate for executing scalar-type applications, with a low ratio of data to computation, generated by arbitrary compiled code.

Because this work focuses on optimization of the FFT-intensive operations involved in SAR processing, it is assumed that the system studied uniformly employs SHARC CNs. However, there exist different types of CNs even with the same type CE. At the time of this writing, the two standard SHARC-based daughtercards are the S2T16B and the S1D64B. The S2T16B implements two CNs for a total of six CEs and 32 MB of DRAM. The S1D64B implements only two CEs but 64 MB of DRAM, all contained within one CN. The power consumption of the S2T16B and the S1D64B are 12.2 and 9.6 watts, respectively. Clearly, both daughtercards have different characteristics, each with a different CE-to-memory ratio and power consumption penalty.

### 3.1 Mapping of SAR Processing onto the RACE System

The basic computational framework and mapping of CEs assumed here is the same as that described in [8]. The descriptions given in this section and the next represent an overview; for more details refer to [8].

CEs are divided into range and azimuth CEs. Every CE is dedicated exclusively to the processing of data either in the range or azimuth direction. Although it would be possible to investigate the utilization of individual CEs for the simultaneous processing of both range and azimuth data, only one fractional CE each for range and azimuth is potentially wasted. Consideration of the processing overhead associated with multitasking and the memory overhead of multiple programs quickly diminishes any benefit that might be obtained from such a configuration. Furthermore, [8] recommends availability of both memory and CEs above the calculated requirement to provide for flexibility and any contingencies. Any such excess resources are usually in excess of that associated with a single CE.

After radar returns have been sampled and converted to digital signals, sam-

ples are typically read into memory at a rate of 5–50 Msamples/s [8]. By visualizing memory as a two-dimensional grid, a row of memory contains the returns from a single radar pulse, whereas a column contains returns of different pulses from the same range. Memory is therefore sequentially filled a row at a time. When a sufficient number of rows have been filled, this data is processed by a range CE. These blocks of data are sent to the range CEs in a round-robin fashion. After a number of range CEs have processed data, the conglomerate block of data is “corner-turned,” or matrix-transposed, and then sent to the azimuth CEs. Note that the number of range and azimuth CEs need not be the same. The matrix transposition of the data dictates that the azimuth CEs receive the range-processed rows as columns and the unprocessed columns of the azimuth direction as rows. Fig. 3.5 illustrates the communication in a matrix transposition. Note that although each range processor is responsible for several signal returns (set of pulses), each range processor only needs to hold one entire return in memory for computation before sending the result to the azimuth processors.

### 3.2 Computational Framework

As discussed earlier, SAR processing primarily involves convolution of the data with reference functions. For the sake of simplicity and without loss of significant performance (because of the relatively small requirement of range processing as compared to azimuth), it is assumed that the entire vector of range samples for a given pulse return is processed as a single section of data. The azimuth CEs perform similar operations on the data as the range CEs (i.e., fast convolution) but with one important difference: the length of the data stream in the azimuth direction is indefinite, whereas in the range direction it is of a fixed length. Therefore the data cannot be convolved as a single entity in the azimuth dimension.

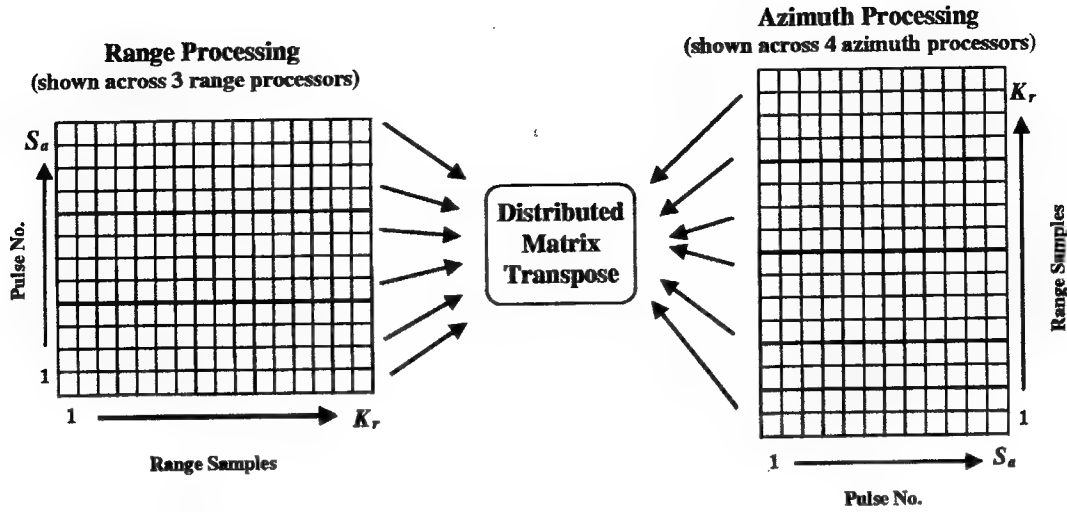


Fig. 3.5: Parallelization of the matrix transpose operation.

Sectioned fast convolution [18] provides a method for processing data streams of indefinite length. For such a data stream, the data is divided into sections of arbitrary length. A section is then convolved with the prestored kernel as in the case of a regular fast convolution. (Note that this prestored kernel saves the time of taking the FFT of the transmitted signal each time, which ideally should be the same for each pulse. Furthermore, functions such as windowing and other filtering techniques can be included in this kernel and precalculated.) Overlapping the sections by an amount equal to the kernel size and performing fast convolutions on each overlapped section yields the same result as if the entire data stream were convolved at once. However, there is a price to be paid in computational efficiency for using this method. A portion (of length equal to the kernel size) of each convolution resultant must be discarded. Therefore computational efficiency decreases as the ratio of the section of new data to the kernel size decreases. Fig. 3.6 illustrates the principle of sectioned convolution.

Besides memory, another limiting factor to the size of the new data to be



**Large Overlap/Section ratio  $\Rightarrow$  Small azimuth memory, large number azimuth processors**  
**Small Overlap/Section ratio  $\Rightarrow$  Large azimuth memory, small number azimuth processors**

Fig. 3.6: Sectioned convolution.

convolved is the  $O(N \lg N)$  time complexity of the standard FFT algorithm. An important objective is to balance computational efficiency with memory requirements. For instance, selecting a section size that maximizes computational efficiency alone, without regard for concomitant memory requirements, may be unfavorable due to high power consumption by the memory. Accounting for this tradeoff is an important aspect of the model presented in this work.

A fast convolution consists of an  $N$ -point FFT,  $N$  complex multiply operations, and an  $N$ -point inverse-FFT, where  $N$  is the number of data points to be processed, including any overlap. The complexity of this computational load is therefore  $L = O(N \lg N + N)$ . The exact number of floating point operations generally depends on CE- and implementation-specific details. If SHARC CEs are assumed, the exact number of floating point operations is given by [8]:

$$L = 10N \lg N + 6N.$$

The computational load per sample is obtained by dividing  $L$  by the number of new data points processed, which reflects the efficiency of the calculation. For

range processing this load per sample  $\phi_r$  due to the fast convolution is given by

$$\phi_r = \frac{10F_r \lg F_r + 6F_r}{S_r},$$

where  $F_r$  is the FFT size for the range and  $S_r$  is the number of points in the range to be processed. These two values can differ because of the stipulation in the FFT algorithm that requires the FFT size to be a power of two (i.e.,  $F_r = 2^k$ ). Although this implies some inefficiency, it is usually still faster than using a direct convolution algorithm based on the exact sequence length.

The number of range points  $S_r$  is equal to the range swath  $R_s$  divided by the desired resolution  $\delta$  (assuming  $\delta_{\text{syn}} = \delta_R = \delta$ ). That is,

$$S_r = \frac{R_s}{\delta}. \quad (3.1)$$

Using this expression, the equation for  $\phi_r$  becomes

$$\phi_r = \frac{\delta F_r (6 + 10 \lg F_r)}{R_s}.$$

Similarly, the azimuth processing load per sample due to the fast convolution is given by

$$\phi_a = \frac{F_a (6 + 10 \lg F_a)}{S_a},$$

where  $F_a$  is the azimuth FFT size and  $S_a$  is the azimuth section length.

To determine the number of CEs required for both range and azimuth processing, the total computational load must be derived. The fast convolution comprises the majority of the load. However, several other operations are also involved, including fix-to-float conversion, complex signal formation, motion compensation, magnituding, and the matrix transpose already mentioned [8]. It is

important to realize that different operations can take different amounts of time, even if they are considered to be a "single floating point operation." Therefore, calculating the total computational load requirement per data sample involves dividing the number of real operations per sample of each type by their respective tested throughputs for a given type of CE. This value multiplied by the sample rate yields the total number of CEs required.

Range and azimuth processing have unique load requirements in addition to the fast convolution load and are noted by the constants  $\alpha_r$  and  $\alpha_a$ , respectively. The required number of range CEs is then defined by

$$P_r = Q(\alpha_r + \frac{\phi_r}{\gamma}), \quad (3.2)$$

where  $Q$  is the sample rate and  $\gamma$  is the throughput in Mflops for a fast convolution based on the assumed CE type used. Similarly, the number of azimuth CEs required is given by

$$P_a = Q(\alpha_a + \frac{\phi_a}{\gamma}). \quad (3.3)$$

It can be shown that the sample rate is determined by the following equation [8]:

$$Q = \frac{vR_s}{\delta^2}.$$

If this expression is substituted for  $Q$  and the expressions for  $\phi_r$  and  $\phi_a$  are also applied, then Eqns. 3.2 and 3.3 become

$$\begin{aligned} P_r &= \frac{v(6\delta F_r + \alpha_r \gamma R_s + 10\delta F_r \lg F_r)}{\gamma \delta^2} \\ P_a &= \frac{vR_s(\alpha_a + \frac{F_a(6+10 \lg F_a)}{\gamma S_a})}{\delta^2}. \end{aligned} \quad (3.4)$$



The total memory required for range processing is a product of the number of range CEs  $P_r$  and the number of range samples  $S_r$ . This value represents the number of complex range samples that are stored in memory at a given instant, each complex sample consisting of 16 bytes. Therefore the total range memory required is

$$M_r = 16P_r S_r,$$

or equivalently,

$$M_r = \frac{16R_s v (6\delta F_r + \alpha_r \gamma R_s + 10\delta F_r \lg F_r)}{\gamma \delta^3}. \quad (3.5)$$

Azimuth memory requirements dominate total system memory, necessitating a double-buffer (for the matrix transpose operation) and an output image buffer, both of size  $S_r(S_a + K_a)$ . The double-buffer stores complex values; the output image buffer stores reals. Storing the data in the double buffer as complex fixed-point values instead of complex reals reduces the double-buffer storage by 50%. Computation, however, is performed using reals to prevent a substantial compromise in precision. The total azimuth memory requirement in bytes is expressed as

$$M_a = 10S_r(S_a + K_a). \quad (3.6)$$

The value of  $K_a$  can be expressed in terms of basic parameters of the radar. Let  $\lambda$  be the wavelength of the radar. The value for  $K_a$  is derived in [8] to be:

$$K_a = \frac{\lambda R}{2\delta^2}. \quad (3.7)$$

Substituting this expression and  $S_r = R_s/\delta$  into Eqn. 3.6 yields

$$M_a = \frac{5R_s(\lambda R + 2\delta^2 S_a)}{\delta^3}. \quad (3.8)$$

## CHAPTER IV

### THE OPTIMIZATION PROBLEM

The final equations derived above for  $P_r$ ,  $P_a$ ,  $M_r$ , and  $M_a$ , given by Eqns. 3.2, 3.4, 3.5 and 3.6, depend on several different types of basic system parameters. These basic parameters can be divided into four major categories:

- radar parameters:  $R$  (range),  $R_s$  (range swath), and  $\lambda$  (wavelength);
- application parameters:  $\delta$  (desired resolution) and  $v$  (platform velocity);
- processor parameters:  $\alpha_r$ ,  $\alpha_a$ , and  $\gamma$ ; and
- software parameter:  $S_a$ .

From Eqns. 3.2, 3.4, 3.5 and 3.6, it appears that there is also a dependence on the parameters  $F_r$  (range FFT size) and  $F_a$  (azimuth FFT size). However, recall that  $F_r$  and  $F_a$  are functions of  $S_r$  and  $S_a + K_a$ , respectively, and  $S_r$  and  $K_a$  can both be expressed in terms of basic radar and application parameters (see Eqns. 3.1 and 3.7).

Let the total processor requirement  $P_r + P_a$  and the total memory requirement  $M_r + M_a$  be denoted as  $P$  and  $M$ . Any variable will be represented as a function when it is a function of another variable that is to be implicitly or explicitly optimized in the problem. In the example of  $P$  and  $M$ , they will normally be represented as  $P(S_a)$  and  $M(S_a)$  as a reminder that they are dependent on the optimization variable  $S_a$ .

Throughout this work, it is assumed that all radar and processor parameters are fixed. Depending on the optimization objective, one or more of the application or software parameters will be optimized, with the remaining parameters fixed or implicitly computed. However, in each case the optimal  $S_a$

must be determined. The problem then focuses on finding an optimal value for  $S_a$  and the effect of  $S_a$  on the resulting computing platform configuration and the optimization objective.

#### 4.1 Mathematical Programming

Optimization of parameters can be achieved through a method known as mathematical programming. The expression that represents the variable or variables to be minimized is called the objective function, consistently designated in this work as  $Z$ . Additional inequality and equality relations comprise the set of constraints for the programming problem.

Many algorithms exist for different types of mathematical programming problems. The algorithm employed depends on the nature of the optimization. If possible, it is favorable to formulate the problem such that the solution space can be constructed by a set of linear equations. Such a problem falls into the category of linear programming. One common linear programming algorithm is the Simplex Method. The Simplex Method merely examines vertices of the two-dimensional solution space constructed by the constraints until the minimum value is found, the linear nature of the solution space assuring that a minimal solution will fall on a vertex.

Often, however, nonlinear equations enter into the problem, as is the case in this work. Nonlinear programming requires more complex algorithms. MATLAB's `constr` function in the Optimization Toolbox is employed in this work to solve the nonlinear programming problems.

The `constr` function implements a Sequential Quadratic Programming (SQP) algorithm, which constitutes a family of the most efficient constrained optimization algorithms currently known. See [2] for specific information on the MATLAB implementation of the `constr` function and [10] for an overview of the SQP

algorithm.

Like most optimization algorithms, the SQP algorithm requires that the solution space is convex to guarantee an optimal solution. That is, no local minima or maxima exist in the solution space to deceive the algorithm to settle for a suboptimal solution. In general, without an exhaustive search, which is not practical in the continuous domain, no algorithm can guarantee an optimal solution from a nonconvex solution space. Optimization algorithms operate on the basis of finite changes in the objective function value due to small changes in the optimization variables. When no more improvement can be obtained, within a set tolerance, from various excitations of the optimization variables, the algorithm is done. If the optimization algorithm ventures into a local minimum or maximum, the conditions for algorithm completion are met even if the value of the solution is very bad relative to other points in the solution space.

Proving the convexity of a multidimensional problem can be very time consuming, without the guarantee of an eventual solution. Even if a problem can be proved to be nonconvex, optimal solutions still often are desired, without a way of restructuring the problem to be convex. One method of countering such problems in a nonconvex space is to either backtrack from a solution and go in a different direction from the one previously taken for a certain number of steps, even if the immediate results from such an action return an inferior objective function value. If the algorithm consistently returns to the original solution, it may be assumed with increasing confidence that the given solution is not a local minimum or maximum. This procedure resembles a Tabu search or simulated annealing algorithm, of which portions may be applied, with either deterministic or probabilistic methods of determining the next direction to try [4, 11, 17].

Another method of approaching nonconvex or possibly nonconvex problems is to solve the same problem many times, with each solution given a different

initial guess. To provide the highest level of confidence that a resultant solution is good (optimal or near optimal), the range of initial guesses should be as disparate as is practically feasible. These initial guesses, consisting of values for one or more optimization variables, can be deterministically or nondeterministically generated.

Unless explicitly stated, it is assumed that the problems optimized in the succeeding chapters are convex. Although convexity has not been formally proved for any case, it is considered adequate that widely ranging initial guesses affect only the speed of convergence of the optimization algorithm, not the final solution. Furthermore, obvious logic dictates that when plotted against linearly spaced independent variable values, the optimal solution surfaces would represent either nondecreasing or nonincreasing functions of those variables in each dimension. As this research illustrates, this trend holds true for all the problems investigated except where explicitly noted. See Section 5.3 for an example of a possibly nonconvex problem formulation.

Integer programming constitutes another type of mathematical programming. Integer programming imposes the stipulation that one or more of the variables to be optimized must be an integer, and can be applied to either linear or nonlinear programming. In general, efficient algorithms do not exist for integer programming, and solving such optimization problems can be computationally intensive (see [13] for a summary on integer programming techniques). Although different approaches exist in integer programming, essentially a problem must be solved using linear or nonlinear programming, the noninteger result rounded up or down, and the solution then recomputed based on the new values. Because it is difficult to predict how the rounding of one variable will affect the other variables, testing of many permutations of the discrete variables may be required. In many cases, the optimal solution may not involve the floor or ceil-

ing of the calculated optimal value for a given variable, especially in nonlinear problems with many variables.

## 4.2 Optimization Objectives

Several different optimization objectives are investigated in this work. An optimization objective can be categorized in two different dimensions. The first dimension concerns the SWAP constraint of interest. In UAV systems, all SWAP constraints are critical. However, in terms of computer processing, the most variable and controllable parameter is power. The weight of a computer system largely results from the chassis, which for UAVs is commonly custom designed. The size of the embedded system is a combination of volume of individual components and the geometry by which they are arranged and physically configured. Straightforwardness and generalized application in the calculation of power requirements lend power consumption to be the fundamental case of study in this research.

The second dimension of the objective function involves the parameter, or performance measure, to be optimized. The most obvious parameter to be optimized is power consumption. The minimization of power is indeed the primary objective function under consideration for this work. However, within the framework of computable power consumption, the maximization of velocity and the minimization (making as fine as possible) of resolution are explored as objective functions, assuming a preset maximum power consumption level for the onboard computing system. Further constraints of weight and/or size could be added to the problem, but these are not investigated in this work because of the reasons given above.

### 4.3 Hardware Configurability

Basic guidelines about the hardware components available for system design determine which variables can be optimized. One such set of guidelines includes the option of custom designing each component. Such endeavors should result in very efficient and highly specialized components. Another guideline that is often applied is that widely available preexisting components of less specialized purpose must be employed in the design of the system. This latter assumption is the basis of this research. Both of these approaches are examined below.

#### 4.3.1 Optimal Configuration Using Custom-Designed Boards

The custom design of a card entails the capability to produce a board with the desired ratio of memory to processing power. The objective function  $Z$  of such an approach is given by the following:

$$\kappa P + \beta M, \quad (4.1)$$

where  $\kappa$  and  $\beta$  are constants that represent power requirements on a per processor (or per Mflop) and per byte of memory basis, respectively. Although the above optimization problem appears to be a simple linear programming problem (assuming the existence of constraints), even for this most basic formulation it must be noted that both  $P$  and  $M$  are functions of several variables. In particular, they are functions of the software parameter  $S_a$ . If the expressions for  $P$  and  $M$  are substituted into Eqn. 4.1, the following equivalent expression results:

$$\begin{aligned} & \kappa \left[ \frac{v(6\delta F_r + \alpha_r \gamma R_s + 10\delta F_r \lg F_r)}{\gamma \delta^2} + \frac{v R_s (\alpha_a + \frac{F_a(6+10 \lg F_a)}{\gamma S_a})}{\delta^2} \right] \\ & + \beta \left[ \frac{5 R_s (\lambda R + 2\delta^2 S_a)}{\delta^3} + \frac{16 R_s v(6\delta F_r + \alpha_r \gamma R_s + 10\delta F_r \lg F_r)}{\gamma \delta^3} \right]. \end{aligned} \quad (4.2)$$



Even if all variables except for  $S_a$  are fixed, Eqn. 4.2 is obviously nonlinear and requires a nonlinear programming algorithm such as **constr**. Determining a value of  $S_a$  that minimizes this function defines an optimal configuration, with a corresponding optimal number of  $P$  processors and  $M$  bytes of memory. Modeling total consumed power as described above is unrealistic in many cases because of the overhead in time and money involved in production of customized boards. In addition, the resultant system would be fairly inflexible because it would have been optimized for a specific velocity and resolution. If either of these two application parameters changed at a later point in time or even if a radar parameter changed, the system would become inefficient at best and possibly obsolete. Any change thus would necessitate the design of a new board.

#### 4.3.2 Optimal Configurations Using COTS

The disadvantages discussed above of custom designed boards often lead to the employment of commercially available boards, or commercial off-the-shelf (COTS) products, that contain differing numbers of processors and amounts of memory. The COTS components under consideration here are Mercury's S2T16B and the S1D64B daughtercards. Although systems composed of COTS components are theoretically never as efficient as custom designed boards because of their more generalized purpose, COTS systems often can be built to provide adequate performance for a fraction of the cost and in much less time. Furthermore, if any parameters change, the system can be quickly adapted to accommodate the changes merely by adding or removing cards and appropriately modifying the software. This work is premised on the assumption that COTS hardware must be used, specifically the Mercury RACE system and associated individual components.

With this assumption, the total power requirements of a system can be com-

puted directly from the number of daughtercards employed. When appropriate, the power consumption will be denoted by the symbol  $\Pi$ , usually with an appropriate subscript depending on the exact context. In all systems this research considers, power consumption is assumed to be equal to the product of the number of daughtercards of a given type employed and the power consumption for a single daughtercard of that type. The total power consumption of a system is the sum of all such products for each of the types of daughtercards.

#### 4.4 Architectural Models

This research investigates in depth two models of processor-memory architectures. The first and simplest architecture assumes that all memory and processors that the daughtercards in the system contribute can be pooled and treated as a single entity. For reasons discussed below, this architectural assumption is termed as the ideal shared-memory model (ISMM). The second model treats each CN as a separate entity, and thus is designated as the CN-constrained model (CNCM).

##### 4.4.1 Ideal Shared-Memory Model

The simplest and most intuitive approach to determine how many daughtercards are required to provide the necessary total memory  $M$  and processors  $P$  for correct system function involves taking the maximum of  $M$  and  $P$  divided by the amount of memory or number of processors, respectively, contributed by a single daughtercard (discussed in Chapter V). In the case of the S2T16B, 32 MB of memory and six processors are contributed, or in the case of the S1D64B, 64 MB and two processors. Because the tasks involved in SAR processing are readily parallelizable, the assumption of pooled processors does not present a problem. However, the concept of pooled memory across daughtercards (or more specifi-

cally, CNs), can be dangerously naive, as discussed in the next subsection.

The above assumption provides a simplified model of a real system. Although the usefulness of such a model might be questioned, this research demonstrates that such a model can be used as a lower bound heuristic that yields adequately consistent results for the expected power consumption. Such a heuristic is useful when considering that the solution to the more realistic model discussed below involves a large amount of integer programming, resulting in a more complex problem formulation that is much more computationally intensive in terms of time. Furthermore, in the case that a different system is investigated for which memory can be effectively pooled, this model is valid. This model is investigated in Chapter V.

#### 4.4.2 CN-Constrained Model

A more realistic model for the Mercury RACE system necessitates transition from the aggregate of daughtercards as the quantity of concern to the CN as the fundamental hardware denominational unit in terms of memory and processors. In the ISMM, an optimal configuration of components may allow all processors on one daughtercard to be employed in computation, but each of which require memory of the daughtercard on which they reside and all the memory on several other daughtercards. In such a case, the active constraint on minimum power is memory, leaving the processors on most daughtercards idle. Although such a configuration is theoretically possible, such never occurs in practice because of the prohibitive costs in communication time involved with remote memory accesses from a processor on one CN to the DRAM of another CN. Remote memory accesses occur only during the distributed matrix transposition, which is relatively time consuming considering the amount of data handled. Note that for even two CNs on the same daughtercard to communicate, they must send

requests through the crossbar, just as if they were on distinct daughtercards. Because of this fact, the CN is the fundamental unit of concern in this more realistic model.

To avoid remote memory accesses, a formulation must ensure that all processors employed on a single CN have adequate local memory to complete their tasks. Only configurations that abide by such constraints are considered in the solution. In addition, range and azimuth processors are treated as distinct since they operate based on distinct programs. That is, a processor is dedicated to either azimuth or range processing. However, there is no need for entire dedicated range and azimuth CNs, since each processor on the CN still can operate from a different program. As discussed in depth in Chapter VI, integer programming is required to ensure integral solutions of range and azimuth processor assignments to CNs. Just for the two daughtercards under consideration, it will be shown that the number of processor assignment combinations that may need to be evaluated is in excess of 50, which translates to a 5000% increase in computation. Needless to say, in the case that there exist more than two types of daughtercards from which to select, quick solutions are not forthcoming due to the computational intensity.

#### 4.5 Hardware Availability Constraints

The most general type of configuration assumes that there are multiple daughtercards from which to select. Furthermore, the optimization routine is given freedom to populate a system completely with only one type of card or to determine some mixture of the two card types. This case will be designated the mixed card type configuration.

If a constraint is added to the formulation such that the number of daughtercards of one type or the other must equal zero, this special case will be called

the single card type configuration. Such a formulation represents a scenario in which a system will not tolerate multiple types of daughtercards or the scenario in which hardware has already been acquired and the hardware happens to con-

sist of all one type daughtercard. Furthermore, solutions to the single card type formulation provide insight into the characteristics and best usages of a card type.

#### 4.6 Points of Reference: Nominal Configurations

It has been suggested that using a section size  $S_a$  equal to the azimuth kernel size  $K_a$  is a good heuristic for adequate system performance yet with moderate conservation of memory, which is usually the scarce resource [7]. Such a heuristic ensures the intuitively comfortable result of processing at least as much new data as old data with each convolution. However, as the research exemplifies, the optimal section size often deviates from this heuristic choice.

To illustrate the advantage of optimizing the software parameter  $S_a$ , comparisons are made of the optimal solutions to what will be termed nominal solutions, meaning that the above heuristic was employed in determining the section size instead of the optimization routine. However, even in such nominal configurations, there are often still other variables that may be optimized. If a system was designed without any forethought to optimization, basing configuration only on heuristics or the most obvious approach, it is true that no optimization of any sort might occur. In this case, even most of the nominal configurations presented in this work will outperform arbitrarily designed systems. However, giving the benefit of the doubt to system designers, and for the purpose of analyzing the utilization of optimized section sizes, all other variables besides  $S_a$  are optimized even in the nominal configurations.

#### 4.7 Summary

The next two chapters present the results of computed solutions for the ISMM and the CNCM, respectively. Because of the computational intensity involved

with the CNCM and because of the greater insight into parameter interrelationships provided by the ISMM, more examples have been investigated in the ISMM. For the CNCM, more space is dedicated to examination of the formulation and its complexity, with fewer examples than the ISMM with the assumption that the ISMM provides a good heuristic for solutions. However, it must be noted that the CNCM provides, in addition to more realistic power consumption values, more detailed information for the actual implementation of a configuration. That is, whereas the ISMM only provides information on the number of daughtercards necessary for the entire system, the optimal azimuth section size, and the resultant power consumption, the CNCM also provides precise information on which processors are dedicated to range and azimuth and how much memory of the total local memory is allocated to each. This latter information is sufficient to design a system, whereas the designer still has several problems left, the solutions to which may be physically infeasible, to resolve with the ISMM results.

In addition to the two distinct models, within each model examples are generated by taking combinations of different optimization objectives and hardware availability scenarios. To illustrate the utilization of optimization, comparison is also made between optimal and nominal configurations.

All numerical results produced in this work are based on radar and processor parameters fixed at the following values:  $R = 100000$  m,  $R_s = 20000$  m,  $\lambda = 0.03$  m,  $\alpha_r = 0.3528$  Mflops,  $\alpha_a = 0.9068$  Mflops, and  $\gamma = 94$  Mflops. These values are derived from [8], representative of a real SAR system and experimental SHARC throughputs.

Most examples investigated in this work are represented by three-dimensional graphs with the  $x$  and  $y$  axes formed by the range of values for two independent variables and the  $z$  axis the optimal solution to the given optimization problem, respective of the two independent variables. In the case of power minimization, the two independent variables are resolution and velocity, consistently ranging from 0.5 m to 2.0 m and 50 m/s to 400 m/s, respectively. In the velocity maximization problem, power and resolution are the independent variables, with the same resolution range as noted above and a power range of 30 w to 100 w. Similarly, in the resolution minimization problem, power and velocity comprise the independent variable set with the same range of values for both variable as just noted above for the two other optimization problems. In each case, solution values on the  $z$  axis correspond to linear samplings on the  $x$  and  $y$  axes such that 25 points of equal intervals on each axis are generated. As a result, each graph represents  $25^2 = 625$  separate optimizations.



## CHAPTER V

### IDEAL SHARED-MEMORY MODEL

The first COTS model to be investigated considers the two Mercury RACE daughtercards but assumes no penalty for communication between CNs physically located on different cards. After calculating the system requirements, the optimal number of cards is found by dividing the system requirements by the processing and memory capacity of the cards. Such an approach ignores communication costs for remote memory accesses and treats the aggregate throughput of all the processors as a single entity. Although vendors often advertise total throughput and memory to characterize a system, real performance rarely can be modeled accurately based on such a preposition.

Research shows that the only significant flaw in this assumption in regards to SAR processing on the Mercury RACE is the communication costs for remote memory accesses. It is theoretically feasible that a problem would call for an optimal FFT size too large for a single processor to handle and thus would require a distributed FFT algorithm, which would change the entire model. However, test cases with real data show that this case does not occur within the scenarios studied in this research.

Therefore, although this approach is unrealistic for determining the real requirements of a Mercury RACE or similar system, this model reflects the performance of an ideal shared-memory system (henceforth, this model is referred to as the ISMM). Furthermore, the fundamental nature of the problem and optimal solutions are illustrated more clearly with such a model. The addition of real constraints to ensure solutions of considerable fidelity tends to obscure lower level dynamics of the problem.

## 5.1 Minimization of Power

Minimization of power is the simplest of the optimization problems because of the variables that can be fixed before the optimization algorithm is invoked. All the range variables  $K_r$ ,  $S_r$ ,  $F_r$ ,  $P_r$ , and  $M_r$  are functions of only radar, processor, and application parameters. Among the azimuth variables, only  $K_a$  can be fixed, a function of radar parameters and resolution. The other azimuth variables are dependent on the the section size  $S_a$ , which is an optimization variable, and therefore cannot be calculated statically outside the optimization algorithm.

Because the range variable values will remain the same for each of the power minimization problems, it is useful to examine them only once initially.  $F_r$  is a function of  $K_r$  and  $S_r$ , only assuming values equal to integer powers of two.  $K_r$  and  $S_r$  are linear functions of fixed radar parameters and resolution (Eqns. 2.8 and 3.1) and therefore will be planes that decrease as resolution becomes coarser. The graph of  $F_r$  is shown in Fig. 5.1. The figure illustrates  $F_r$  as a step function, assuming only the following values: 16384, 32768, and 65536. Note that  $F_r$  is independent of velocity.

The number of processors and megabytes of memory necessary for range processing is represented in Figs. 5.2 and 5.3. Both these parameters can be considered as constants added to the azimuth processor and memory requirements. The average number of range processors required is 16.8, with a minimum of 0.85 and a maximum of 121.7. The plot of the required range memory displays a similar trend with an average of 7.2, a minimum of 0.14, and a maximum of 77.9 MB. It is observed that both these graphs exhibit rapid growth, increasing as resolution becomes finer. This observation is confirmed by Eqns. 3.2 and 3.5, which show  $\delta^2$  in the expression for  $P_r$  and  $\delta^3$  in the expression for  $M_r$ .

Because the azimuth variables are the only variables that can be optimized in the power minimization problem, unless otherwise specified, variables such as

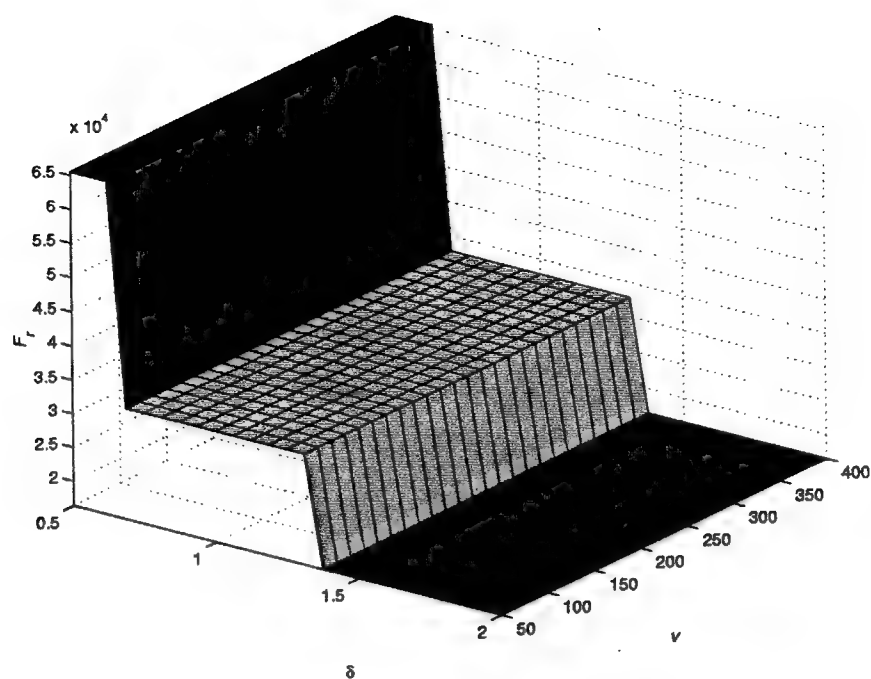


Fig. 5.1: Range FFT size for power minimization.

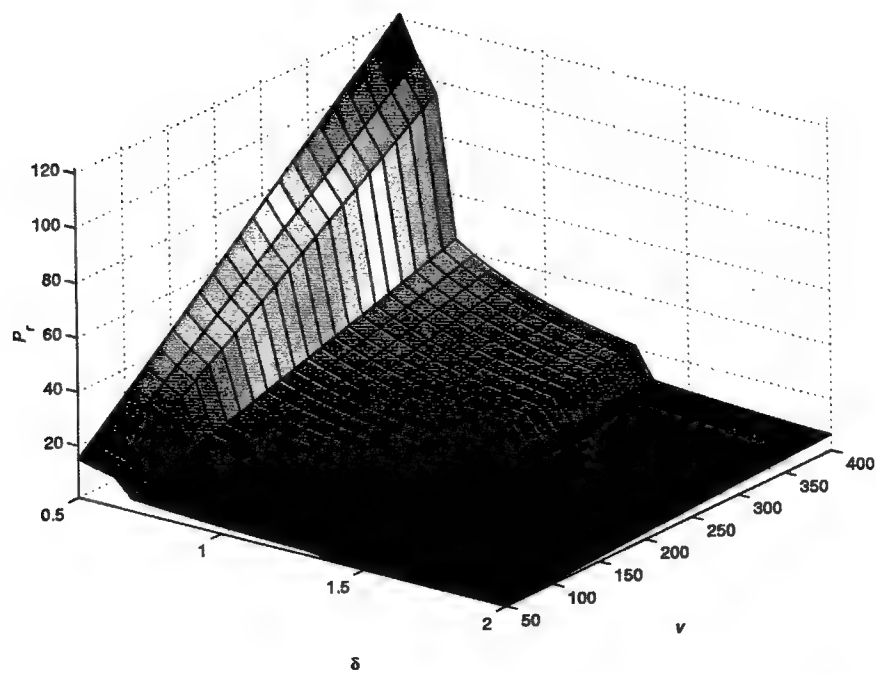


Fig. 5.2: Range processor requirements (in number of processors) for power minimization.

FFT kernel size, section size, and FFT size will refer to the azimuth parameters as opposed to the range parameters. These variables are discussed and graphed below.

### 5.1.1 Optimal Mixed Card Type Configuration

Let  $C_1$  and  $C_2$  denote the number of S2T16B and S1D64B cards utilized, respectively. Thus the function for total consumed power (in watts) is defined as

$$Z = 12.2C_1 + 9.6C_2, \quad (5.1)$$

reflecting the total power requirements of the two boards. Next, two required constraint equations naturally follow based on the values of  $P(S_a)$  and  $M(S_a)$ :

$$6C_1 + 2C_2 \geq P(S_a) \quad (5.2)$$

$$32C_1 + 64C_2 \geq M(S_a). \quad (5.3)$$

These two constraint equations ensure that the total number of processors in the configuration is no less than the total number of required processors and the total amount of memory in the configuration is no less than the total amount of memory required. In this framework, values for the parameters  $C_1$  and  $C_2$  must be optimized in addition to the value of the parameter  $S_a$ . Although the parameter  $S_a$  does not explicitly appear in the objective function that is to be minimized (i.e.,  $Z$ ), its effect is implicit through the constraint equations. That is, the optimal values for  $C_1$  and  $C_2$  are contingent on some calculated value of  $S_a$ .

The only discontinuous portion in the formulation is due to the definition of  $F_a$ , which is a discontinuous function of  $S_a$ . (Recall that  $F_a$  is defined as the

smallest integer power of two that is greater than  $S_a + K_a$ .) This discontinuous function prevents the direct application of nonlinear programming. However, by selecting  $F_a$  as an integer power of two, and adding a constraint to ensure that  $K_a + S_a$  is no greater than this selected value, the discontinuity can be removed. Thus, in addition to the constraints given by Eqns. 5.2 and 5.3, the following constraint equation is added:

$$K_a + S_a \leq F_a, \quad (5.4)$$

where the value of  $F_a = 2^k$  is fixed (the value of  $K_a$  is known based on the values of the specified basic parameters). The discrete nature of  $F_a$  constitutes an integer programming problem and precludes the direct application of nonlinear programming techniques with  $F_a$  as an optimization variable. Thus, to ensure optimality it may be necessary to solve several constrained optimizations based on different feasible values for  $F_a$ . In practice, however, only a few values for  $F_a$  need to be tried: from the smallest feasible value ( $2^k$  where  $k = \lceil \lg K_a \rceil$ ) up to the point at which the optimal value of  $S_a$  is such that  $K_a + S_a < F_a$  (i.e., the constraint becomes inactive). The inactiveness of the constraint suggests that every value of  $F_a$  greater than that last tried would produce an increasingly worse solution. This phenomenon is true because if  $S_a$  does not inflate to its maximum value given the constraint (i.e.,  $F_a - K_a$ ), memory is in shortage and will become more in shortage with every increment in the FFT size.

With the addition of constraints that ensure that optimization variables are greater than zero ( $S_a \geq 1, C_1 \geq 0, C_2 \geq 0$ ), Eqns. 5.1–5.4 constitute the first constrained nonlinear and integer optimization problem solved in this work. Representative samples of the MATLAB code used to solve all the optimization problems and produce the data are included in the Appendix.

Fig. 5.4 represents the total power consumption of the ISMM for a range of

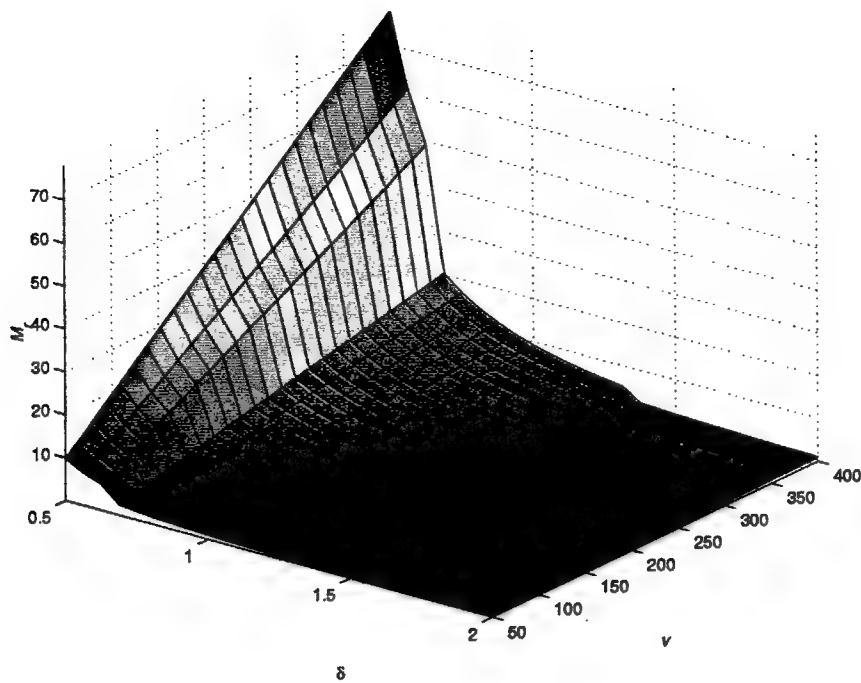


Fig. 5.3: Range memory requirements (in MB) for power minimization.

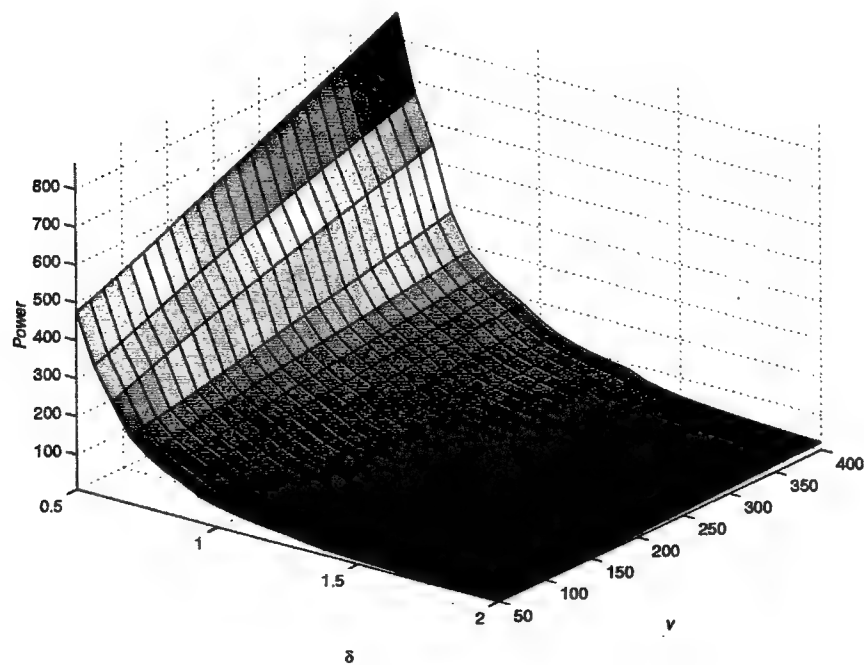


Fig. 5.4: Optimal power consumption.

resolution and velocity pairs. As would be expected, more power is required for higher velocities and finer resolutions. However, it is noted that resolution has a more dramatic effect on power consumption than does velocity. The graph is smooth except for several almost imperceptible ridges at resolution values of approximately 0.65 m, 0.91 m, 1.28 m, and 1.78 m. When the power graph is compared to the graph for  $F_a$  (Fig. 5.5), the cause of the anomalies becomes apparent. The ridges result from the discontinuous nature of  $F_a$ , as described above. For this set of resolution and velocity values, optimal  $F_a$  values range from 512 to 8192 points, corresponding to coarse and fine resolutions, respectively. This finding supports the observation that resolution requirements dominate system performance, and fine resolution demands high memory usage, which in turn drives power consumption high, even in the case of very low velocity. In this scenario, at fine resolutions, relatively inefficient data processing is being performed because memory is in shortage, entailing a surplus of processors.

Although most of the attention paid to explaining Fig. 5.4 will be in terms of the role of azimuth processing requirements, to understand all the intricacies of the graph the role of range processing also must be taken into consideration. As already noted, Figs. 5.3 and 5.2 illustrate the requirements of range processing. Similarly, Figs. 5.6 and 5.7 show the graphs of the azimuth memory and processor requirements. Note that for the power minimization model, although the range requirements remain constant for different configurations, the azimuth requirements change according to the optimally computed  $S_a$ . Analysis of the ratio of azimuth requirements to range requirements therefore is useful. Figs. 5.8 and 5.9 represent these ratios for memory and processors. It is obvious that the disparity between azimuth and range memory is much greater than that of processor requirements. The ratio of azimuth to range processor requirements varies from 0.9:1 to 7.2:1, the lower ratio entailing a larger range processor requirement

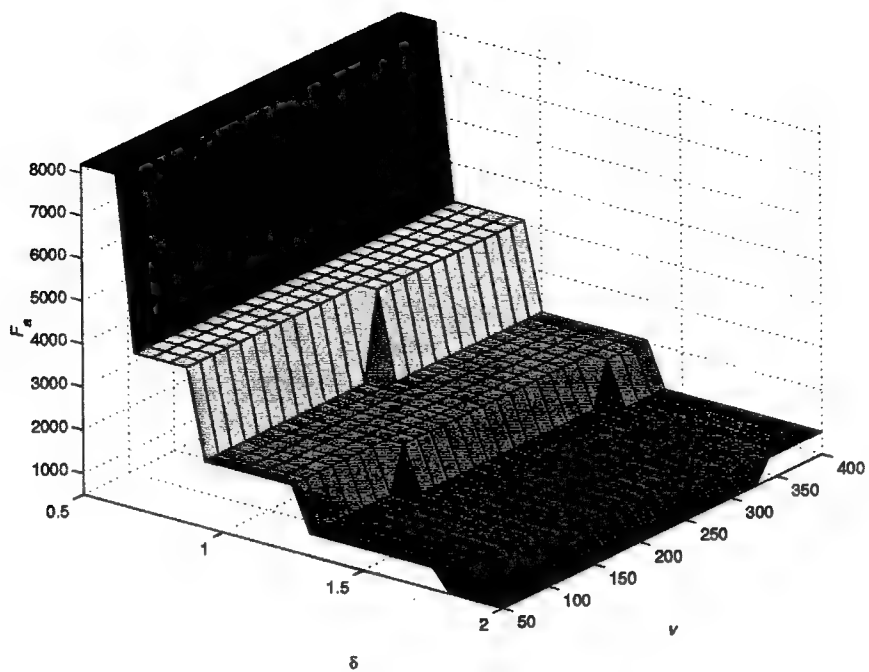


Fig. 5.5: Optimal azimuth FFT size for power minimization.

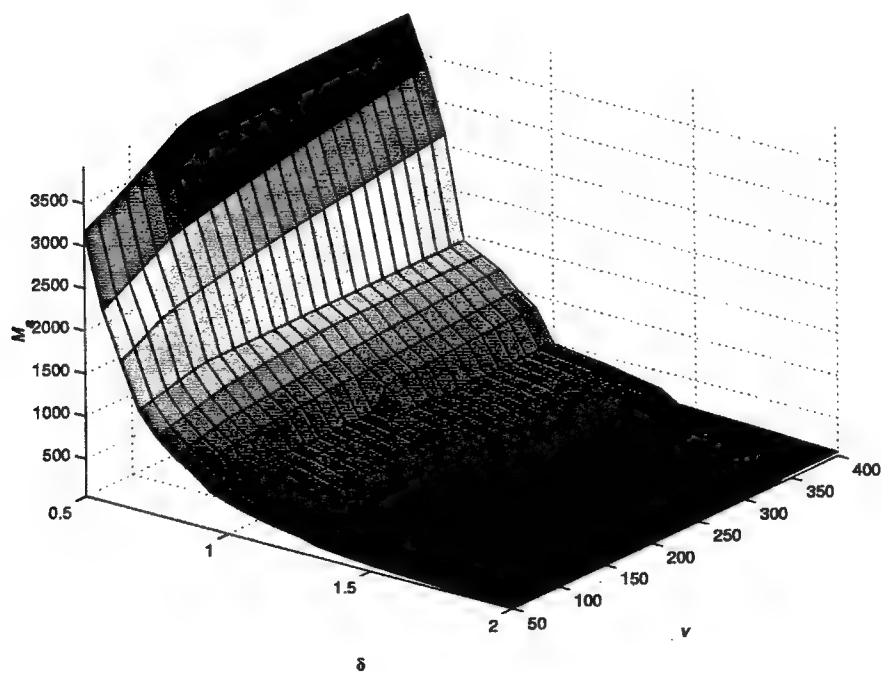


Fig. 5.6: Optimal azimuth memory requirements for power minimization.



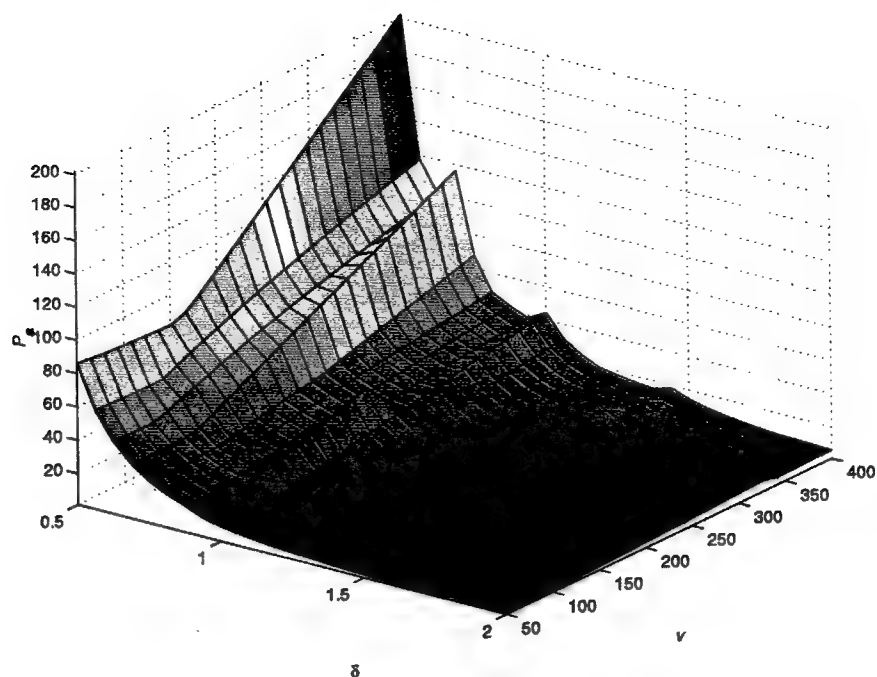


Fig. 5.7: Optimal azimuth processor requirements for power minimization.

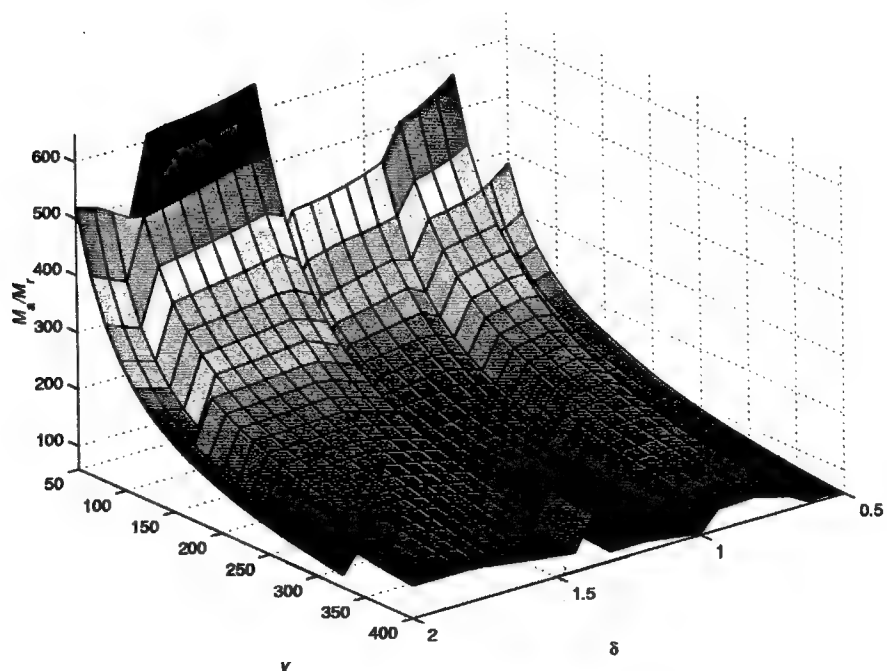


Fig. 5.8: Optimal ratio of azimuth to range memory requirements for power minimization.

than that for azimuth. However, the ratio of azimuth to range memory requirements varies from 59:1 to 648:1, a minimum disparity of almost sixty times the amount of required memory for azimuth than for range processing, even at the few points where more range processors are required than azimuth processors.

A general statement can therefore be made that azimuth requirements always dominate a power minimization configuration (for the given range of resolution, velocity, and radar parameters). Furthermore, every visible ripple in the power consumption graph of Fig. 5.4 can be accredited to discontinuities in azimuth requirements because the discontinuities in range requirements correspond spatially to discontinuities in azimuth requirements. It might seem that if the power graph is to be analyzed primarily in terms of azimuth requirements, then the power consumption by range requirements should be subtracted from the total power requirements before analysis. This method is implausible, however, because the power consumption can not be measured strictly by the product of the requirements and some constant representing the power per megabyte or power per processor, as was theorized in the custom-VLSI model of Section 4.3.1. Adherence to both the processor and memory constraints of Eqns. 5.2 and 5.3 leads to taking the maximum of the daughtercards required by both constraints to determine total power consumption. Consequently, power consumption by only range or azimuth processing has no meaning because optimization of the azimuth section size automatically seeks to utilize all available resources, which are dependent on the range requirements. Therefore, throughout the rest of the power minimization model, knowledge of the range requirements and the impact they have on total power requirements should be considered, but discussion of variables will be limited to the azimuth requirements because they are the values of optimization.

As is expected from Eqn. 3.7, the graph of  $K_a$  (Fig. 5.10) is completely

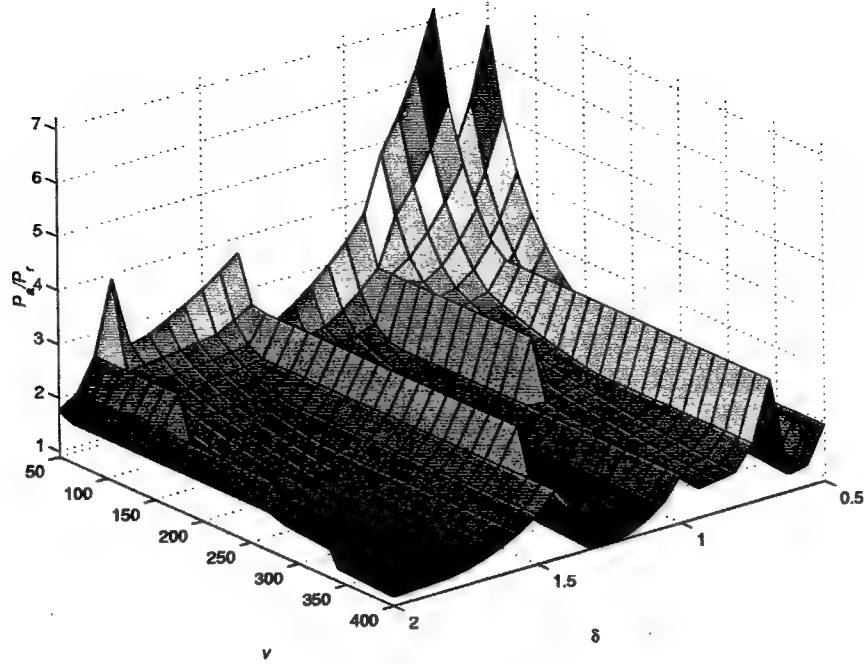


Fig. 5.9: Optimal ratio of azimuth to range processor requirements for power minimization.

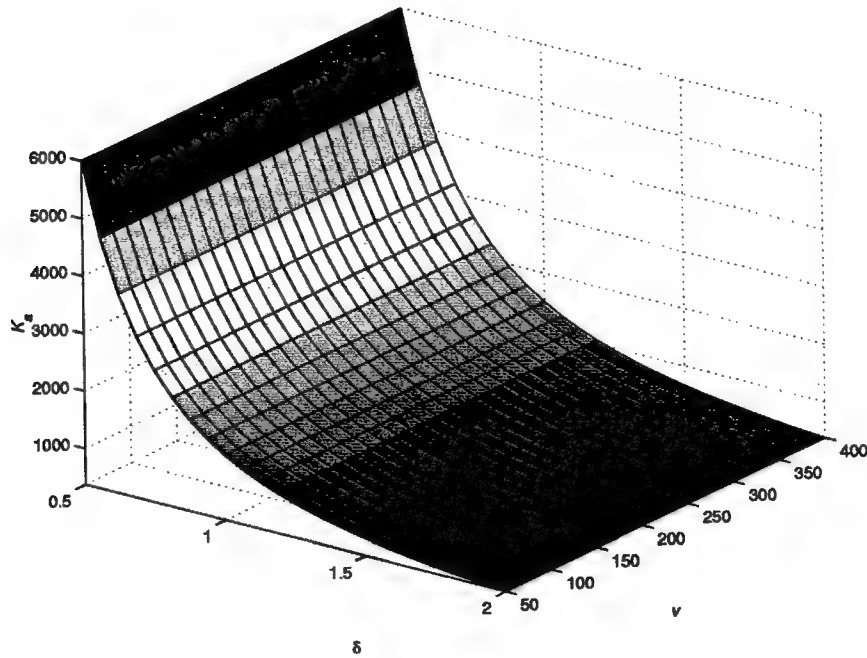


Fig. 5.10: Optimal azimuth FFT kernel size for power minimization.

smooth, increasing as resolution becomes finer, and independent of velocity. The graph of  $S_a$  (Fig. 5.11), however, is more interesting.  $S_a$  is nondecreasing in the velocity dimension, but undulates in the resolution dimension. This rippling effect results from the graph of  $F_a$ . The tiers of the  $F_a$  graph determine the discontinuities of the  $S_a$  graph. As resolution becomes finer,  $S_a$  also decreases to compensate for the additional memory that is required by the resolution. When the processing becomes too inefficient, the next value of  $F_a$  becomes optimal and evokes a corresponding increase in  $S_a$ . Resolution demands again necessitate reductions in  $S_a$  to save memory and utilize processors until the next value of  $F_a$  becomes optimal. However, notice that in Fig. 5.12, which represents the ratio of  $S_a$  to  $F_a$ , overall  $S_a$  gradually decreases as a proportion of  $F_a$  as resolution becomes finer. As a result of the gradual decrease in this ratio, there is a corresponding decrease in the computed optimal employment of the processor-rich (S2T16B) boards to the memory-rich boards (S1D64B). This trend is illustrated in Figs. 5.13 and 5.14.

Surprisingly, note that velocity seems to have a more dramatic effect on the card type utilization than does resolution. However, recall that the two card types vary by a factor of two in memory capacity but vary by a factor of three in processors. The undulations in both graphs again result from the discontinuities in the graph of  $F_a$ , but the effect of the  $F_a$  discontinuities is very transient, resulting in spikes that quickly return to the general shape of the graph.

### 5.1.2 Optimal Single Card Type Configuration

In the case that only one or the other daughtercard type is available for system configuration, the optimization problem is easily adapted to accommodate this

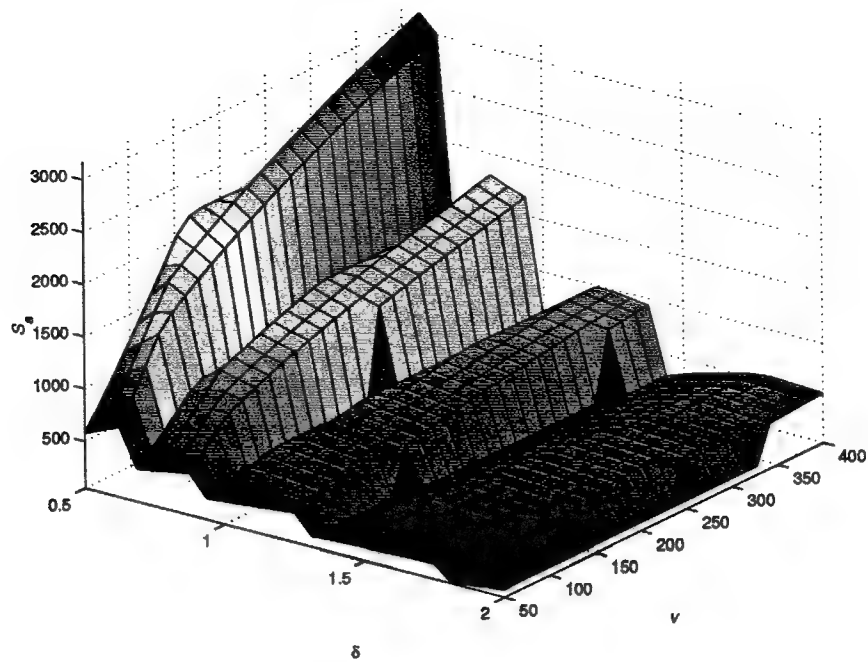


Fig. 5.11: Optimal azimuth section size for power minimization.

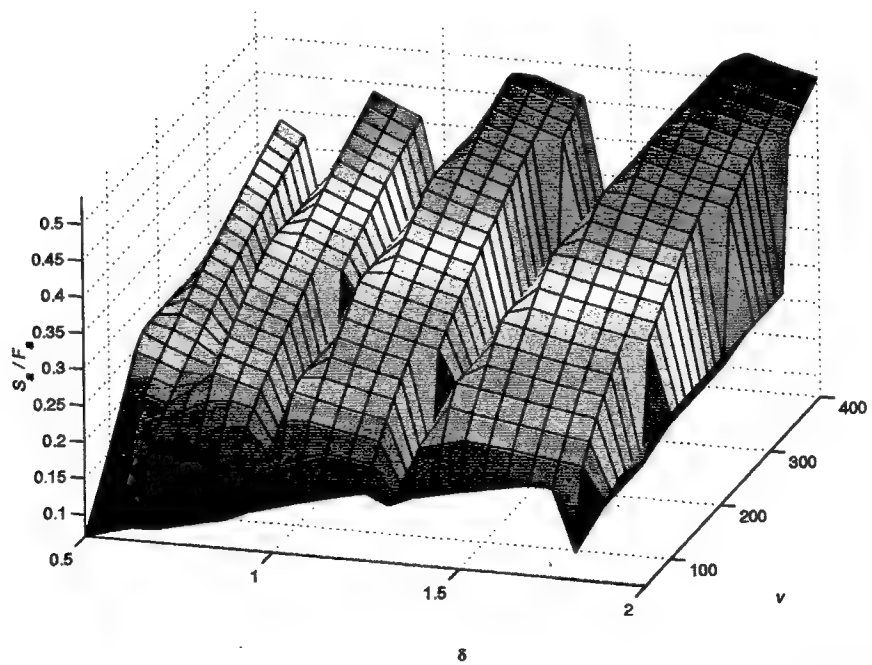


Fig. 5.12: Optimal ratio of azimuth section size to FFT size for power minimization.

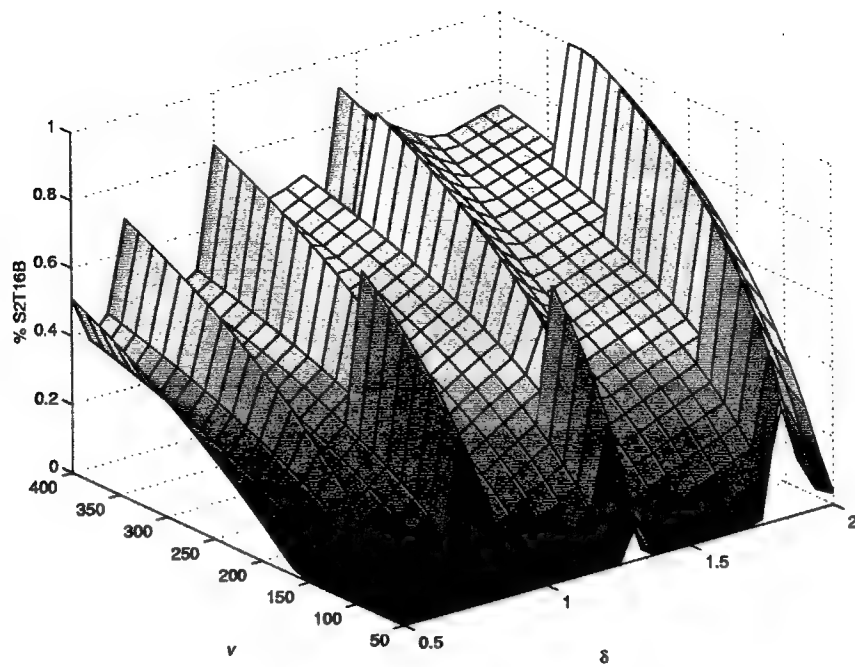


Fig. 5.13: Optimal percentage of power usage by the S2T16B for power minimization.

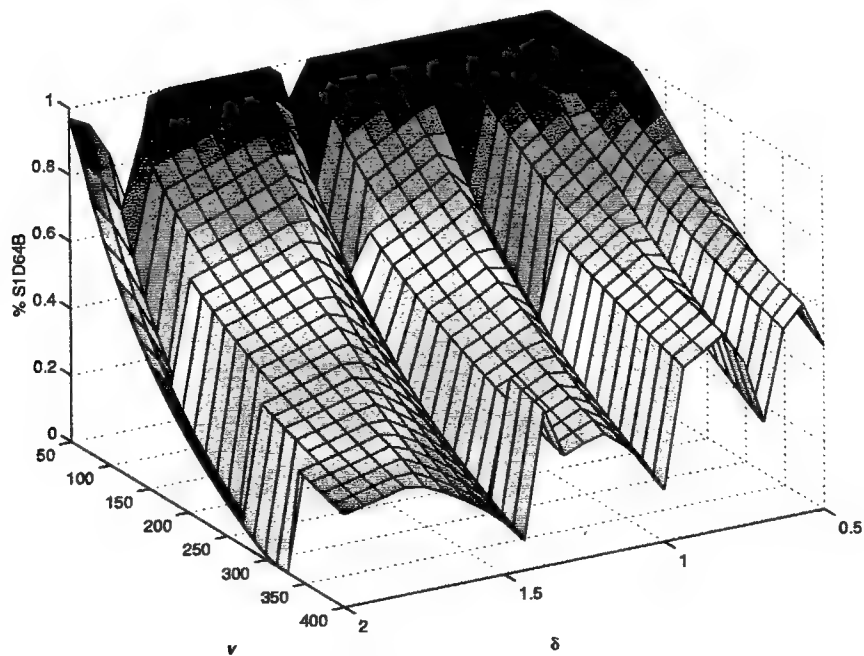


Fig. 5.14: Optimal percentage of power usage by the S1D64B.

tighter constraint. The generalized objective function becomes

$$Z = \Pi_d(C_T), \quad (5.5)$$

where  $\Pi_d$  denotes the power consumption per daughtercard as a function of the card type and  $C_T$  is the card type. Similarly, the constraint equations become:

$$CP_d(C_T) \geq P(S_a) \quad (5.6)$$

$$CM_d(C_T) \geq M(S_a), \quad (5.7)$$

where  $C$  is the number of cards employed, and  $P_d$  and  $M_d$  are the number of processors and amount of memory available as functions of the daughtercard type. All other constraints remain the same. Solving this problem for both card types produces the power consumption graphs of Figs. 5.15 and 5.16. Fig. 5.15 for the S2T16B is a much smoother graph than that of the S1D64B in Fig. 5.16.

There is a noncoincidental resemblance between Fig. 5.15 and the perfectly smooth curled plane of  $K_a$  (note that the graph of  $K_a$  is the same for every configuration involving optimal power with resolution and velocity fixed). As  $K_a$  increases, so does the card requirement. However, Fig. 5.16 depicts a less smooth function. Obviously, the S1D64B configuration depends on more than just  $K_a$ . This difference is explained by an examination of the resource utilizations of both configurations. In both cases, there is a 100% processor utilization. The S2T16B similarly has an average 99.7% memory utilization. In contrast, average memory utilization in the S1D64B configuration was only 90.5%, seemingly low for an optimal solution. The low memory usage in the latter case is a consequence of the more extreme ratio of memory to processors in the S1D64B, having one-third as many processors but twice as much memory as the S2T16B. Thus, in regards to resource utilization, the processor-rich S2T16B is better suited for the range

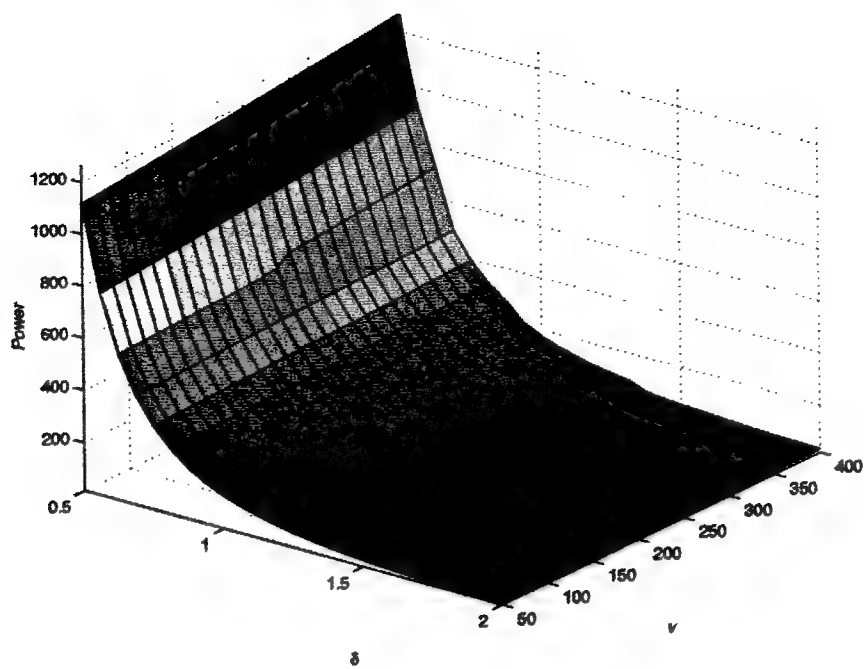


Fig. 5.15: Optimal power consumption in S2T16B-only configuration.

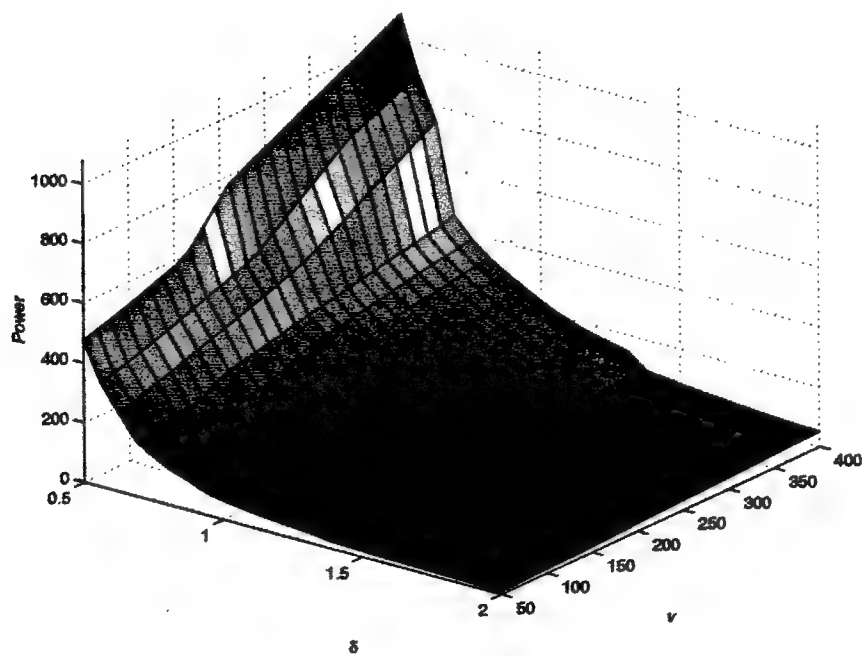


Fig. 5.16: Optimal power consumption in S1D64B-only configuration.



of resolution and velocity pairs in this investigation.

Resource utilization, however, was not the goal of the optimization problem. It seems reasonable to assume that efficient resource utilization would entail low power consumption, but that is not necessarily the case. As is observed from examining Figs. 5.15 and 5.16, peak power consumptions were 1265 and 1079 w for the S2T16B and S1D64B configurations, respectively. Similarly, average power consumption was 213.2 and 164.3 w. The S1D64B, with its poorer memory utilization, consumed an average 29.8 w less than the S2T16B configuration.

Such statistics can be misleading, however, if overgeneralized. If it is necessary to employ only one type of card in a system, the S1D64B is not necessarily a better choice. As seen by Figs. 5.17 and 5.18, there is a clear demarcation of the areas where each card type is most appropriate. Fig. 5.17 shows the percent gain in power consumption of employing the S2T16B over the S1D64B. The plane running through the graph marks zero percent gain. Everywhere above the plane therefore signifies that the S1D64B card is more efficient, noting a gain in power over the S1D64B. Similarly, areas of the graph below the plane denote better performance by the S2T16B. Fig. 5.18 represents the surface formed in Fig. 5.17 as a binary function, with blue denoting gains in power and red losses (improvements) in power. The S1D64B provides up to a 135% decrease in power and at worst consumes up to 39% more power. However, the S1D64B is better-suited for 59% of the cases considered.

Clearly, the required resolution and velocity determine which card is most appropriate in a single-card type system. The two extremes of the card type power consumptions occur at the extremes of the resolution and velocity graph. The S1D64B's advantage is most apparent at the highest performance scenario—where velocity is at a peak (400 m/s) and resolution is finest (0.5 m). Conversely, the S2T16B outperforms the S1D64B most drastically in the low performance

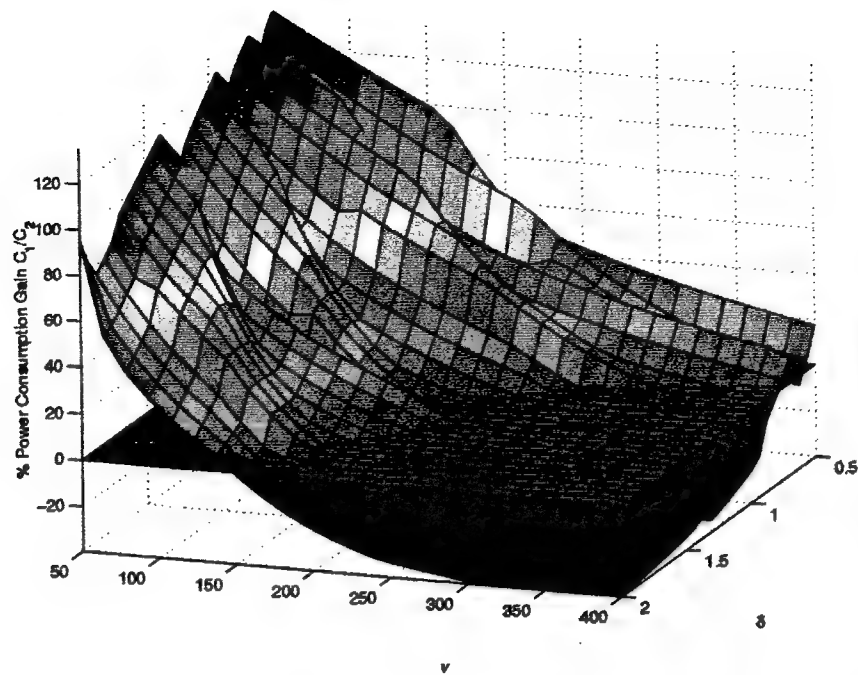


Fig. 5.17: Percentage power gain and loss of S2T16B-only over S1D64B-only configurations. Positive values therefore indicate that the S1D64B or the S2T16B is better-suited, respectively.

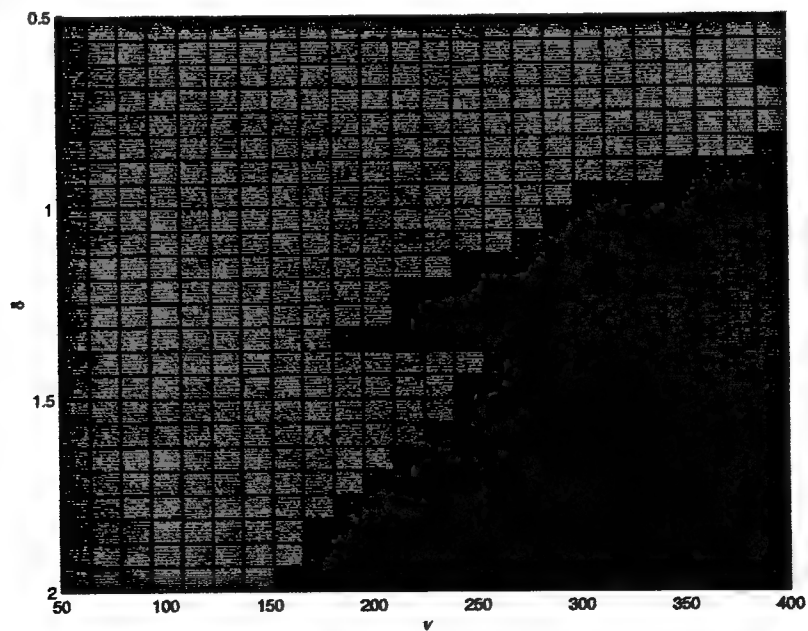


Fig. 5.18: Red and blue areas represent lowest power consumption either by S1D64B or S2T16B configurations, respectively.

scenario—where velocity is lowest (50 m/s) and resolution is coarsest (2 m).

### 5.1.3 Nominal Mixed Card Type Configurations

It has been suggested that using an azimuth section size ( $S_a$ ) equal to the kernel size ( $K_a$ ) is a good heuristic for adequate performance with moderate conservation of memory, which is usually the scarce resource [7]. The optimization problem is simplified by removing  $S_a$  from the optimization variables and setting it equal to  $K_a$ . The third constraint (Eqn. 5.4) is also removed, resulting in only one meaningful value for  $F_a$  ( $F_a = 2^{\lceil \lg(2K_a) \rceil}$ ). Fig. 5.21 graphs the power consumption of a system using the section size heuristic yet still optimizing the number of cards of each type. For the range of values tested in this investigation, it was found that for 91.5% of the cases, the optimal kernel to section size ratio is larger than the 1:1 ratio associated with the heuristic. Fig. 5.20 shows the ratio of the kernel size to the optimal section size. The average ratio in this scenario is 2.24 with a minimum of 0.72 and maximum of 10.42. Consequently, there is a substantial increase in power requirements of the nominal configuration. Adaptation of the number of cards of each type by the optimization routine keeps the increase from attaining the ten fold that might otherwise occur if the same card type ratio was maintained from the optimal to the nominal configuration. Nevertheless, Fig. 5.21 shows a power increase from 0–82.3% with an average of 19.4%. The 0% increase occurs where the optimal section size happens to be equal to the kernel size and the 82.4% increase intuitively occurs in the area where the optimal  $K_a : S_a$  ratio is highest, where velocity is at a minimum and resolution finest.

The entire region where velocity is low exhibits extreme improvements for the optimal section size. At first glance, this seems to be a surprising result because resolution and memory requirements usually dominate power requirements. This

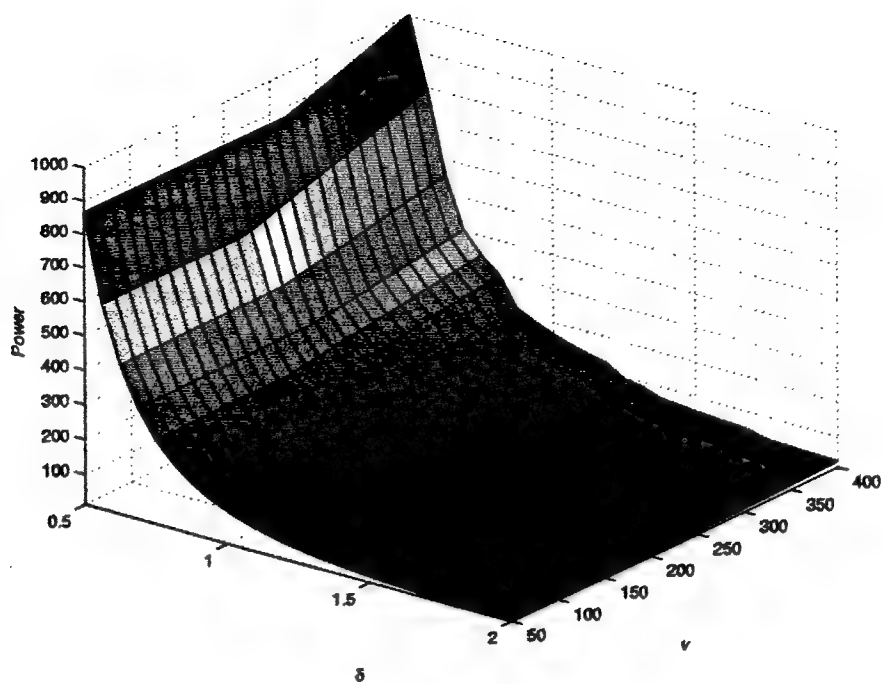


Fig. 5.19: Optimal card type configuration and nominal section size for power minimization.

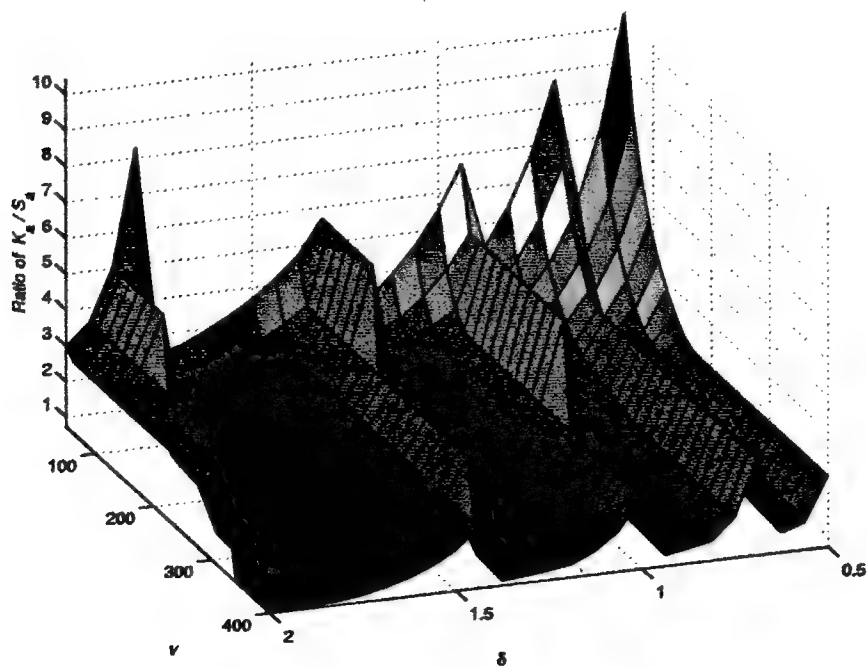


Fig. 5.20: Ratio of azimuth kernel size to optimal section size for power minimization.

rule remains true in this case also but indirectly. At lower velocities, processing power becomes less crucial. A nominal section size results in a surplus of memory. The optimal section size lowers the section size, resulting in less efficient processing but more efficient memory usage. The undulations in the surface of Fig. 5.21 correspond to the rippling nature of  $S_a$  (Fig. 5.11).

#### 5.1.4 Nominal Single Card Type Configurations

To complete the comparison of optimal and nominal section sizes in both single and mixed card type configurations, nominal single card type configurations are now investigated. As expected, this configuration requires the highest power. Figs. 5.22 and 5.23 represent the power consumption graphs of the two single card type configurations. The S2T16B graph now follows  $K_a$  (Fig. 5.10) even more closely than its optimal section size counterpart. This resemblance is due to the total lack of configuration optimization. In the nominal mixed card type configuration,  $C_1$  and  $C_2$  are still optimization variables. In the single card type configuration,  $C$  is calculated by the following formula:

$$C = \max \left\{ \frac{P(K_a)}{P_d(C_T)}, \frac{M(K_a)}{M_d(C_T)} \right\}. \quad (5.8)$$

Therefore memory is always the active constraint for the memory-poor S2T16B configuration. That is, the right side of Eqn. 5.8 is always dominant. However, Fig. 5.23 still exhibits sharp points. The memory-rich S1D64B configuration is still susceptible to both memory and processor constraints, being processor bound in 73.1% of the cases and memory bound the other 26.9% of the cases at low velocities.

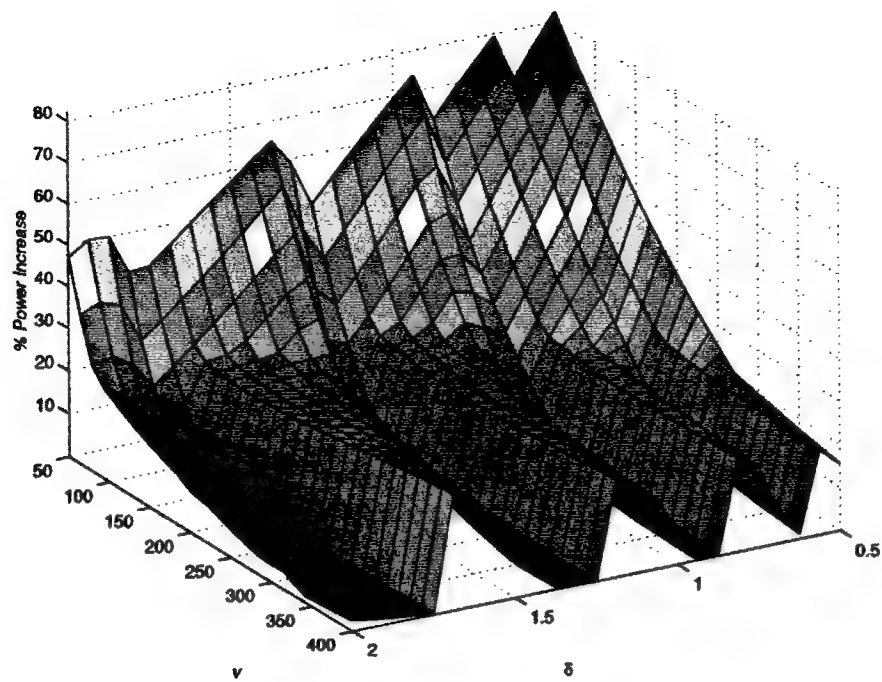


Fig. 5.21: Percentage increase in power of nominal section size over optimal section size configuration.

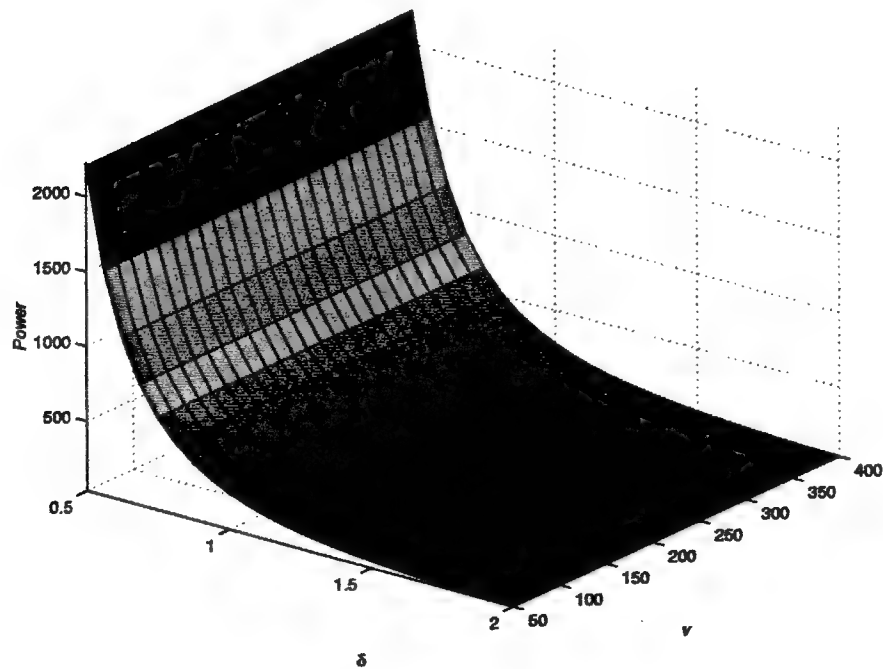


Fig. 5.22: Power consumption of nominal single card type configuration using the S2T16B.

### 5.1.5 Summary of Power Minimization Models

Fig. 5.24 compares the six possible configurations discussed so far. The velocity is fixed at 298 m/s. Fig. 5.25 similarly compares the six configurations but with the resolution fixed at 0.875 m. As expected, the optimal mixed configuration requires the least power for all values of resolution and velocity. Table 5.1 summarizes the comparison across all values.

## 5.2 Maximization of Velocity

All models presented thus far have considered the minimization of power consumption as the objective function. In this section the maximization of velocity  $v$  for a given system is investigated. It is assumed that the resolution  $\delta$  and either available power or the number of daughtercards of each type are the independent variables.

### 5.2.1 Set Power with Variable Number of Cards

Fixing power still leaves the number of each card type to be maximized. The formulation for velocity maximization is very similar to that of power minimization. The objective function is simply the maximization of the following:

$$Z = v. \quad (5.9)$$

The constraint equations are also similar to the power minimization model except for the addition of a power constraint that is almost identical to the objective function in the power minimization model. This additional constraint is as follows:

$$\Pi \geq 12.2C_1 + 9.6C_2, \quad (5.10)$$

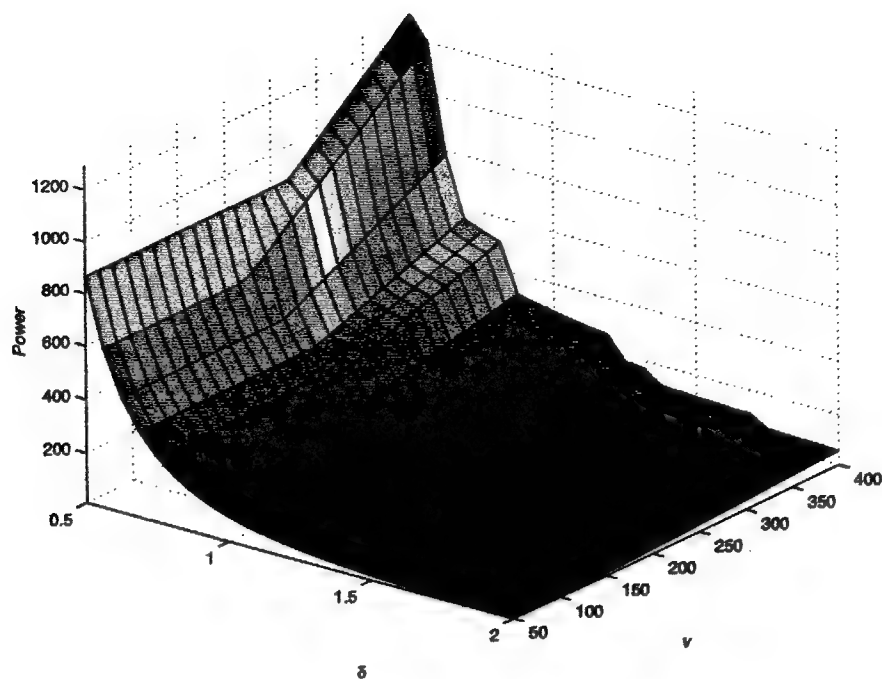


Fig. 5.23: Power consumption of nominal single card type configuration using the S1D64B.

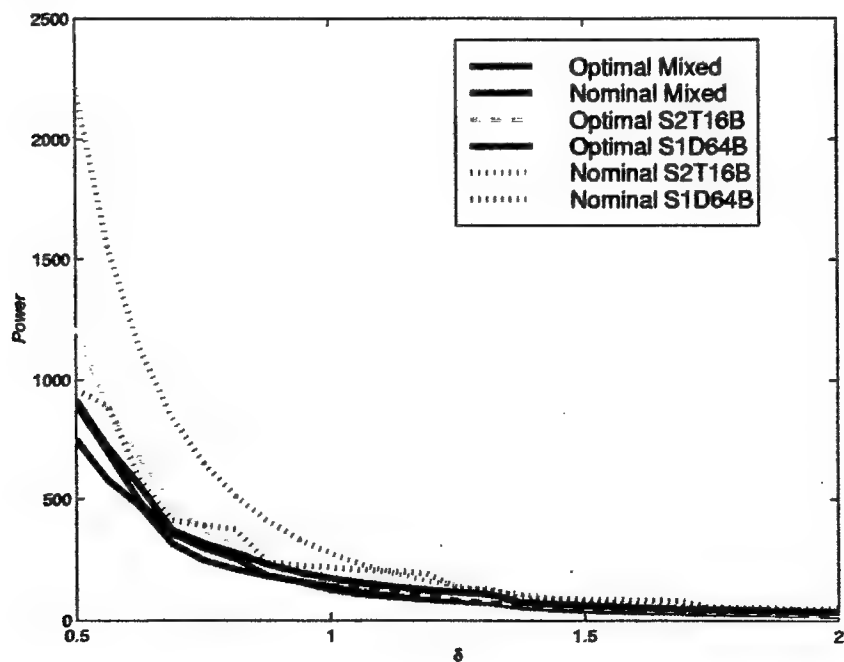


Fig. 5.24: Comparison of power consumption of six configurations with velocity fixed at 298 m/s.



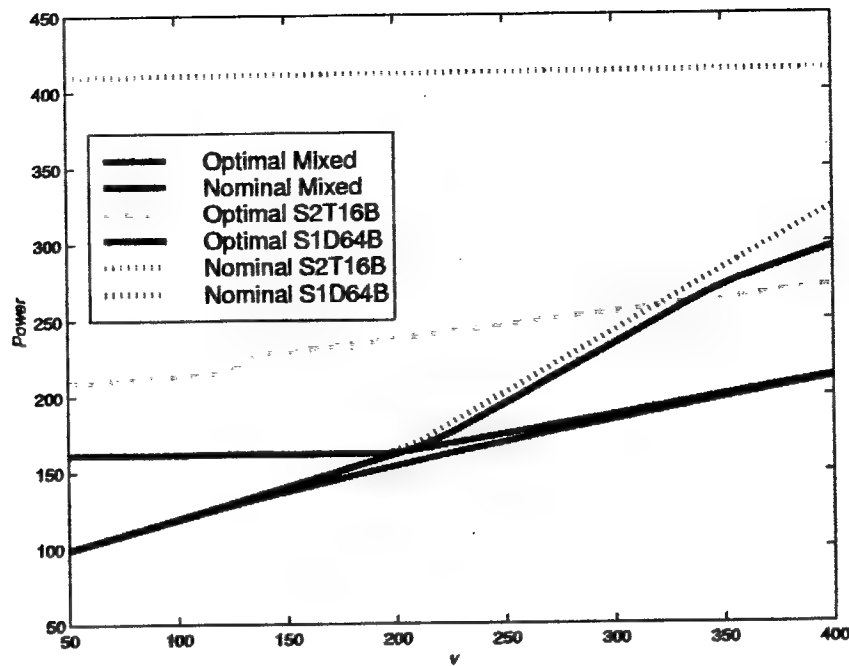


Fig. 5.25: Comparison of power consumption of six configurations with resolution fixed at 0.875 m.

Table 5.1: Comparison of configurations showing the minimum, maximum, and average power and the percent increase of each statistic over the power consumption of the optimal mixed configuration.

Configuration	Min.	% Inc.	Max.	% Inc.	Avg.	% Inc.
Optimal Mixed	9.192	—	867.6	—	135.5	—
Nominal Mixed	13.52	47.05	1002	15.49	166.8	23.13
Optimal S2T16B	17.96	95.37	1265	45.82	213.2	57.40
Optimal S1D64B	9.203	.1134	1079	24.41	164.3	21.28
Nominal S2T16B	34.36	273.8	2221	156.0	379.1	179.9
Nominal S1D64B	13.52	47.05	1289	48.61	206.3	52.30

where  $\Pi$  represents the power allocated for the system. The optimization problem is slightly more complex than in the case of minimizing power because  $P_r$ ,  $P_a$ , and  $M_r$  are all functions of  $v$ . Therefore both  $S_a$  and  $v$  are implicit in the constraint equations. Following the convention set forth, Eqns. 5.2 and 5.3 become

$$6C_1 + 2C_2 \geq P(S_a, v) \quad (5.11)$$

$$32C_1 + 64C_2 \geq M(S_a, v). \quad (5.12)$$

In addition, the following lower bound is added:

$$v \geq 0. \quad (5.13)$$

Eqn. 5.10 could be expressed as an equality constraint because fractional numbers of cards are not disallowed in this formulation. The inequality expression is left, however, because the optimization algorithm always finds a solution utilizing all available power. Furthermore, the inequality constraint is more correct in the generalized form of the problem if  $C_1$  and  $C_2$  are forced to be integers.

#### 5.2.1.1 Optimal Mixed Card Type Configuration

The graph of the maximum attainable velocity given a set power and resolution is shown in Fig. 5.26. The expressed maximum velocity at high power and fine resolution is probably impracticably high for a real airborne UAV, but such speeds might be realistic for a spaceborne satellite, although other parameters in the radar system would probably change and necessarily the range  $R$ .

Fig. 5.27 illustrates the different geometry of the solution space in reference to the power minimization problem. The three plateaus are at FFT sizes of 1024, 2048, and 4096. About one third of the graph is missing (32.0%) because

there was no feasible solution to the given power-resolution pair. The boundary of infeasibility in this scenario runs roughly on the line where resolution equals 1.0. Recalling that azimuth memory is defined by an expression with  $\delta^3$  in the denominator (Eqn. 3.8), the 1.0 resolution boundary is logical. Increasing the power range would provide at least some feasible solutions for all resolutions. However, the maximum attainable velocity at coarse resolutions would become unreasonably high for the given scenario because the velocity already exceeded 1800 m/s (over Mach 5) in Fig. 5.26. Note that the graph of  $K_a$  is the same as for the power minimization problem (Fig. 5.10). Fig. 5.28 shows the optimal section size. Each point of inflection corresponds to a jump in the FFT size.

The plot of the optimal S2T16B usage for maximum velocity is shown in Fig. 5.29. The plot for the S1D64B can be easily visualized by turning the graph upside down, or taking one minus the graph for the S2T16B. The high plateau in Fig. 5.29 corresponds to the low plateau in the graph of the FFT size (Fig. 5.27). When the optimal FFT size was low, implying a great quantity of processing, the processor-rich S2T16B became the exclusively ideal choice. Outside of this region, a mixture of the two cards was optimal, with the S2T16B usage generally increasing both as resolution became coarser and available power increased. For this range of values, the S2T16B consumed an average 65.7% of the power.

#### 5.2.1.2 Optimal Single Card Type Configuration

Observing that the S2T16B seems to be favored in the velocity maximization problem, the optimal single card type configurations are now investigated. Figs. 5.30 and 5.31 show the maximum velocities attainable using only the S2T16B or the S1D64B daughtercards. As expected, for feasible scenarios the S2T16B accommodates an average maximum velocity of 721 m/s compared to 286 m/s for the S1D64B. Thus, the S2T16B shows a 150% improvement over

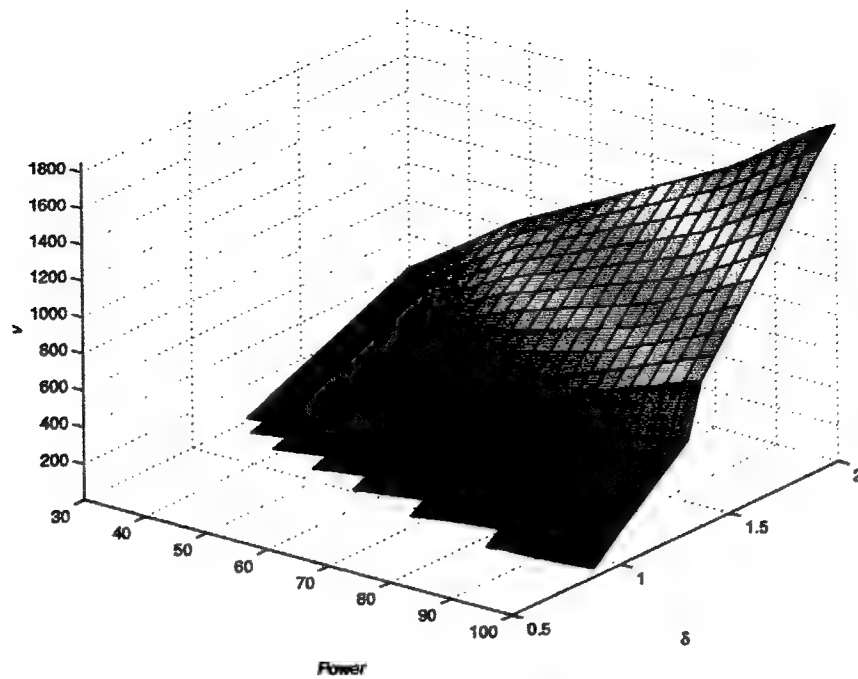


Fig. 5.26: Maximum velocity attainable at fixed power and resolution.

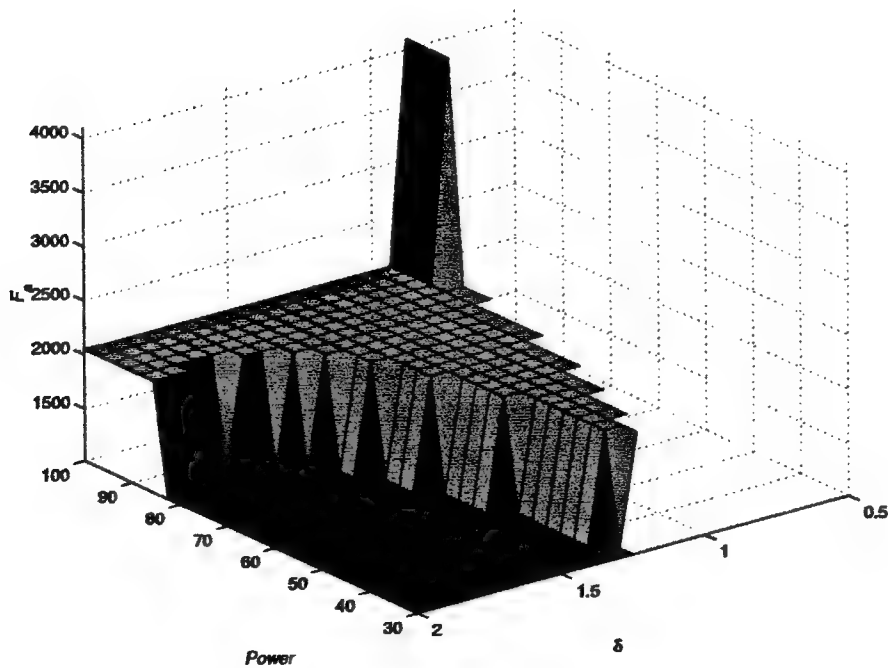


Fig. 5.27: FFT size of maximum velocity solutions.

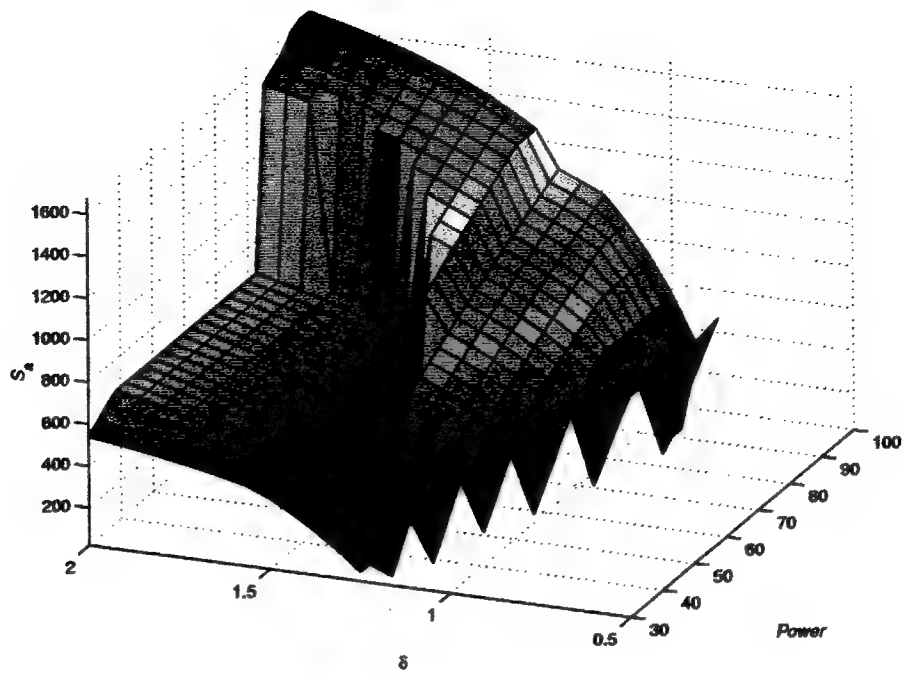


Fig. 5.28: Optimal section size for maximum velocity.

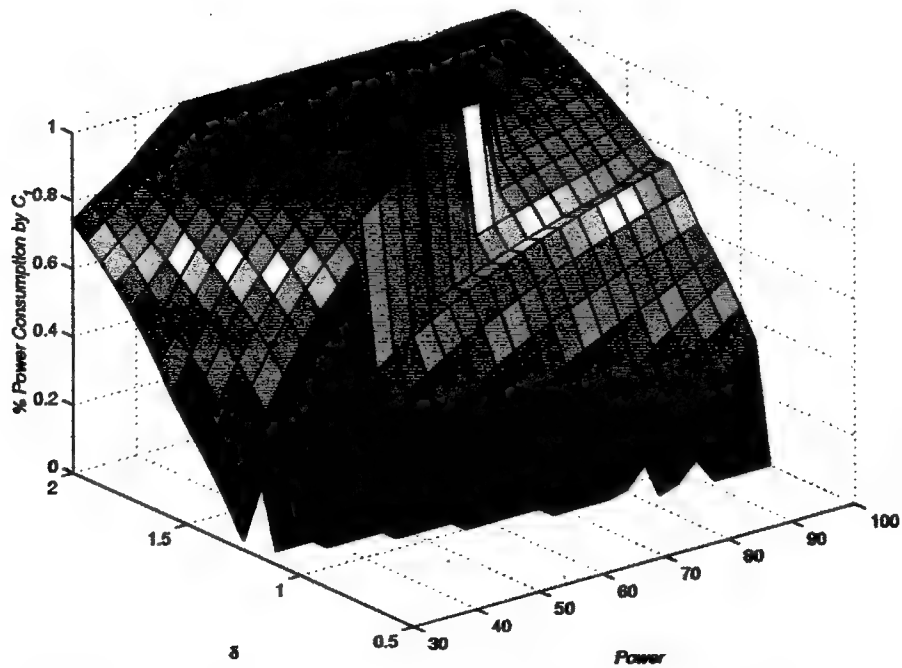


Fig. 5.29: Percentage power consumption by S2T16B in optimal mixed configuration.

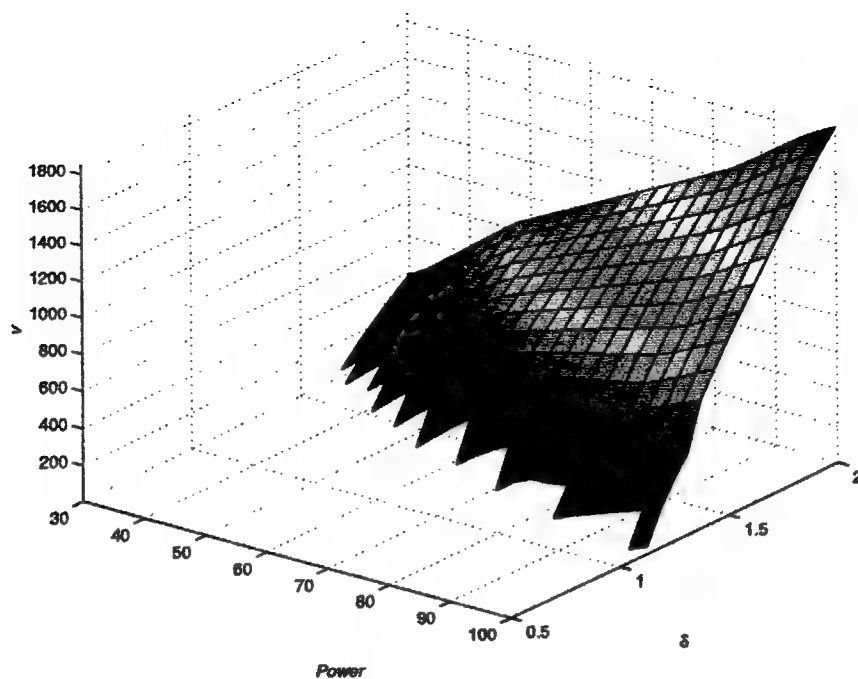


Fig. 5.30: Maximum velocity with S2T16B-only configuration.

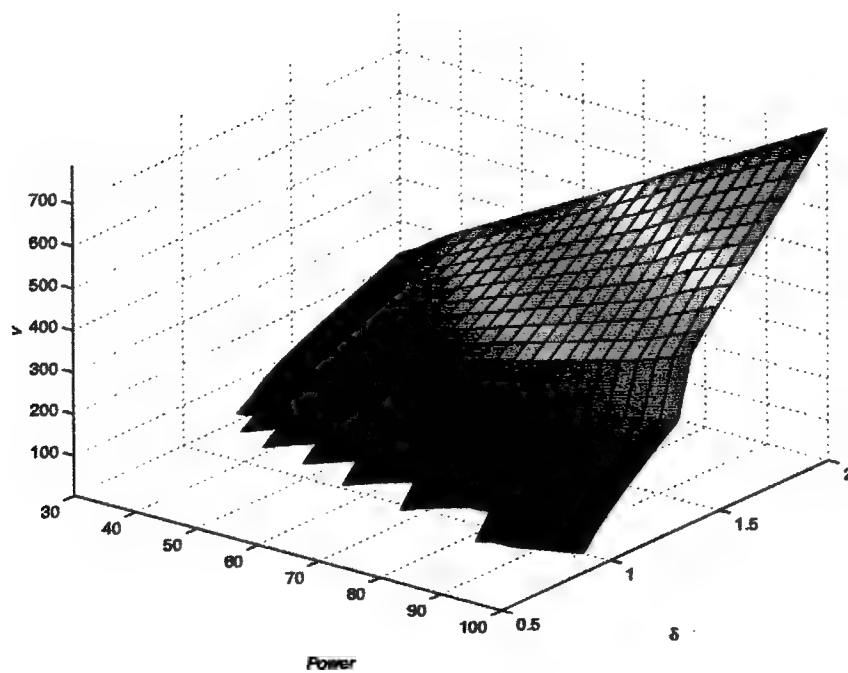


Fig. 5.31: Maximum velocity with S1D64B-only configuration.

the S1D64B. However, the above statistic only considers the average across the feasible solutions for that card type. The S2T16B provides feasible solutions in only 45.9% of the power-resolution pairs, as compared to 68.0% for the S1D64B, which is the same percentage as attained by the optimal mixed configuration. This outcome results from the exclusive employment of the S1D64B in the fine resolution region by the mixed configuration. Although the S1D64B is not ideal in the majority of cases tested, it can always provide a feasible solution whenever the S2T16B can.

The FFT size employed in both single card type configurations follows the pattern expected by the respective memory-processor ratios of the daughter-cards. Of the feasible solutions, the FFT size ranged from 512 to 2048 for the S2T16B and from 1024 to 4096 for the S1D64B. The graphs of the section size for both daughtercards are shown in Figs. 5.32 and 5.33. Note that the graph for the S2T16B closely resembles a shifted and scaled version of the graph for the S1D64B. The shift would be in the resolution dimension by about 0.5 m and the scaling in the  $S_a$  dimension by one half. This phenomenon results from an active memory constraint in the optimization problem up to the point of feasibility for the S2T16B and an active processor constraint thereafter.

### 5.2.1.3 Nominal Mixed Card Type Configuration

The nominal section size with optimal card configuration problem evokes some interesting variable relationships. Fig. 5.34 graphs the maximum velocity attainable under this configuration. The points of discontinuity in the graph correspond to jumps in the FFT size, as illustrated in Fig. 5.35. The point of interest in these two graphs is that as the velocity increases,  $F_a$  decreases (note that Fig. 5.35 is reversed in regards to Fig. 5.34). It could be expected that maximizing velocity, being processor intensive, would call for large FFT sizes for

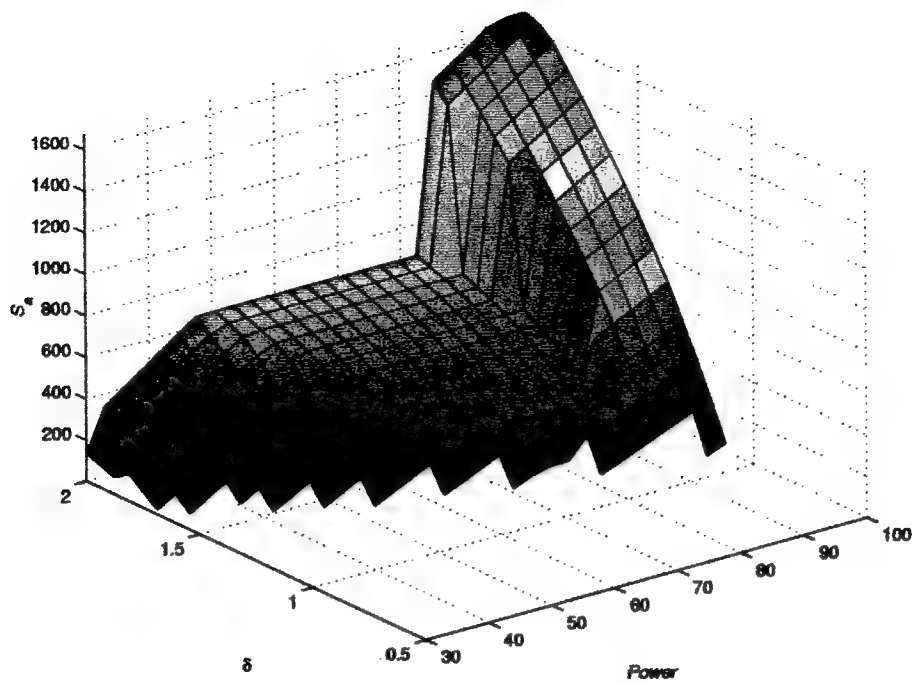


Fig. 5.32: Section size of S2T16B configuration in maximum velocity problem.

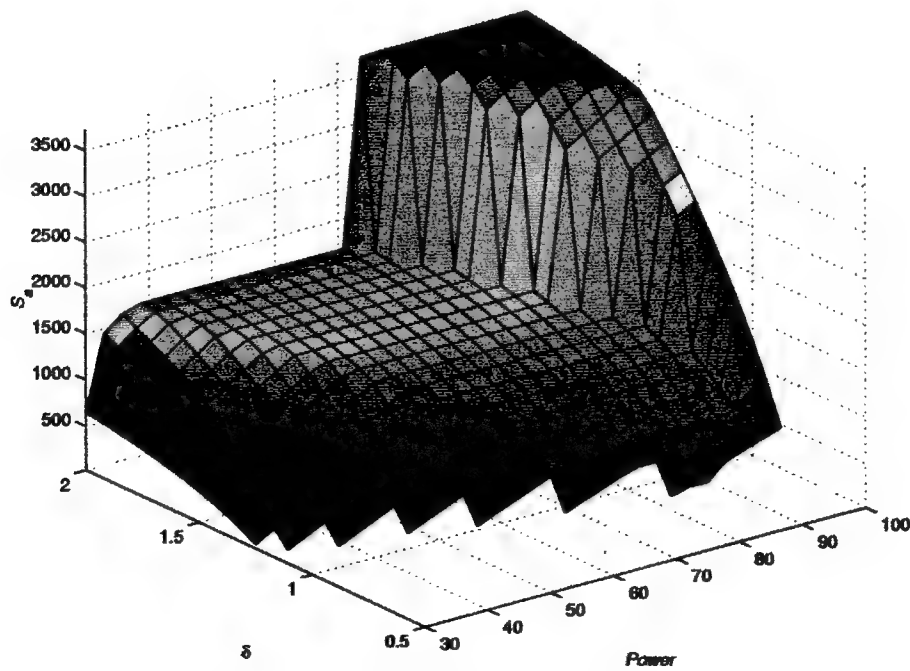


Fig. 5.33: Section size of S1D64B configuration in maximum velocity problem.



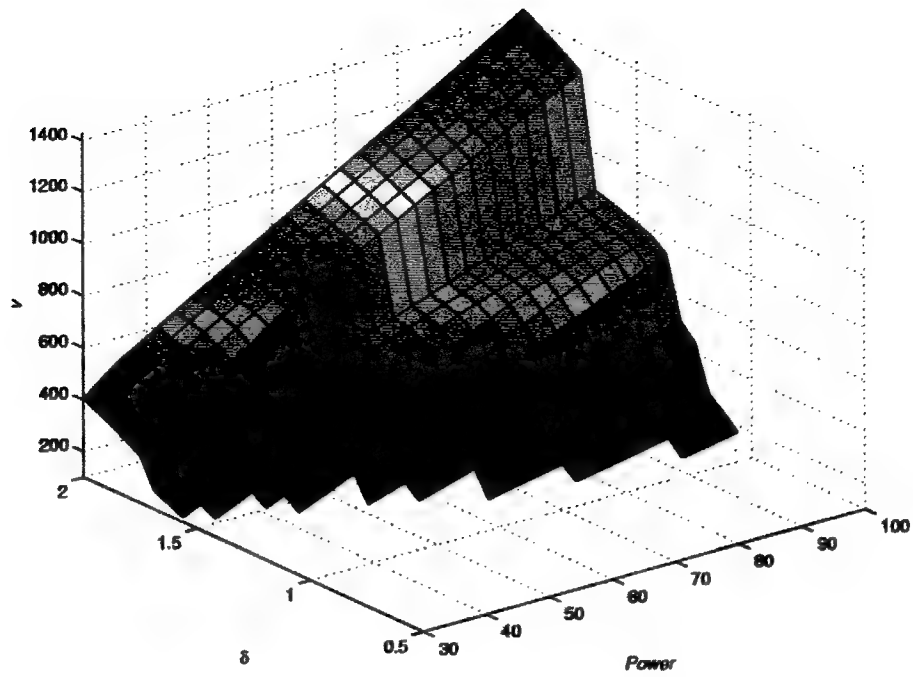


Fig. 5.34: Maximum velocity attainable in nominal mixed configuration.

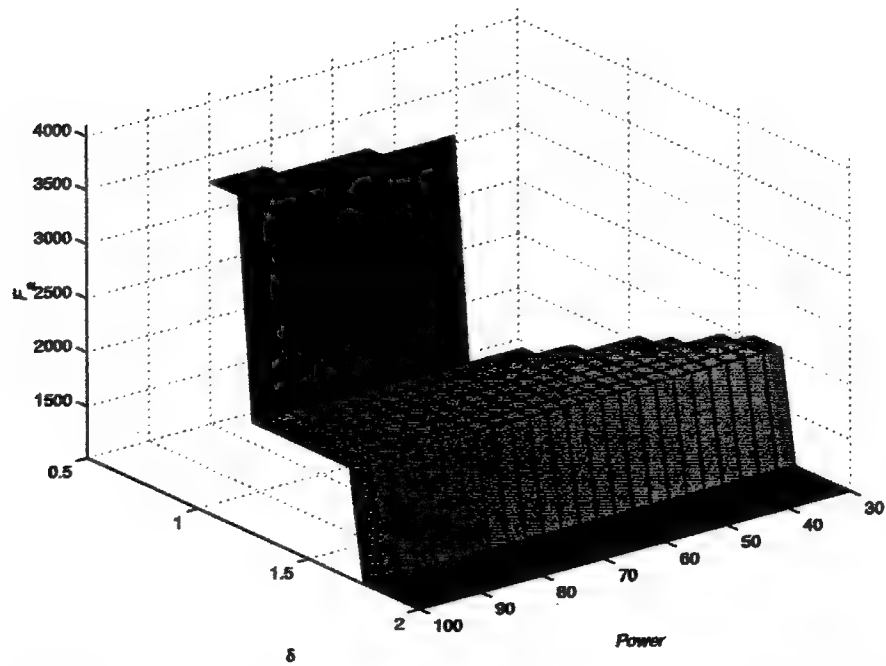


Fig. 5.35: FFT size for maximum velocity attainable in nominal mixed configuration.

efficient processing as in the optimal mixed configuration (Fig. 5.26). However, just the opposite is true in this case. The nominal section size forces the FFT size to be much smaller than is optimal because the section size is equivalent to the kernel size (Fig. 5.10), and the kernel size decreases as resolution becomes coarser. Compensation for this counterproductive section size trend is made by employing a larger percentage of the S2T16B card (Fig. 5.36). The processor rich and memory poor S2T16B card can afford to do rather inefficient processing with the small FFT size yet still provide higher velocities than could the S1D64B.

#### 5.2.1.4 Nominal Single Card Type Configuration

Finally, the nominal single card type configurations are investigated. Figs. 5.37 and 5.38 depict the maximum velocities and feasibility regions for the nominal section size configurations of the two card types. The relationship between these two configurations is very similar to that of the optimal single card type configurations but with decreased velocities and regions of feasibility.

#### 5.2.1.5 Comparison of Maximum Velocity Configurations

Table 5.2 compares the different configurations for the maximum velocity problem. Note that the minimum velocity statistic is not meaningful because each configuration theoretically at some point provides a maximum velocity of  $0+\epsilon$ , where  $\epsilon$  is a very small number. However, from graph to graph the minimum velocity varies because the discrete sampling points disallow the occurrence of the real minimum velocity in each case. Although the percentage of area with feasible solutions statistic approaches 100% as the resolution and power approach infinity, the statistic is meaningful for the sampling space because these values are deemed as representative of values of a real system. Average velocity statistics are given both over the total area and over the feasible area only. The average

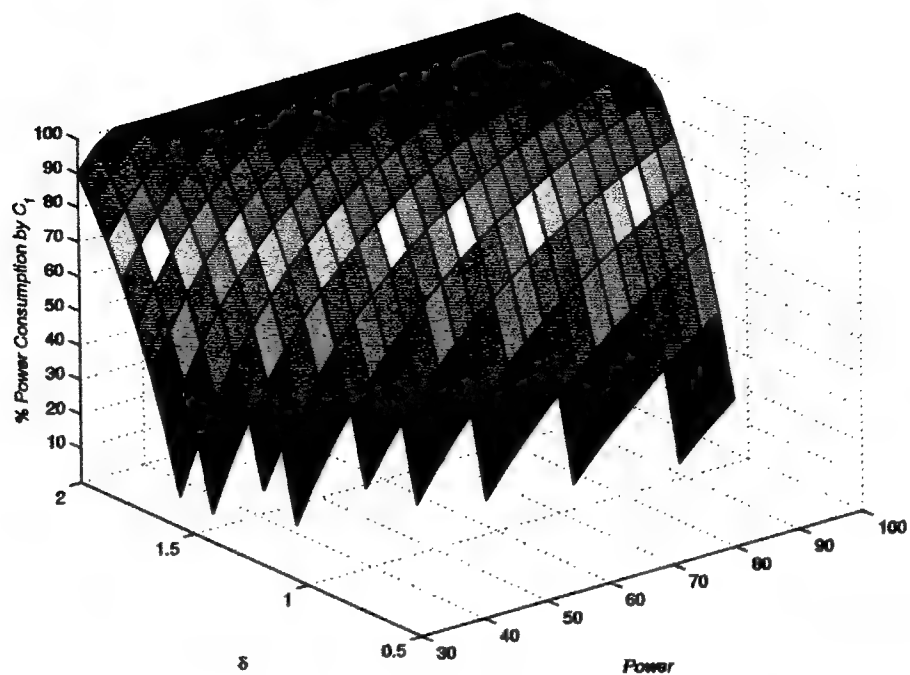


Fig. 5.36: Percent power consumption by S2T16B in nominal configuration for maximum velocity.

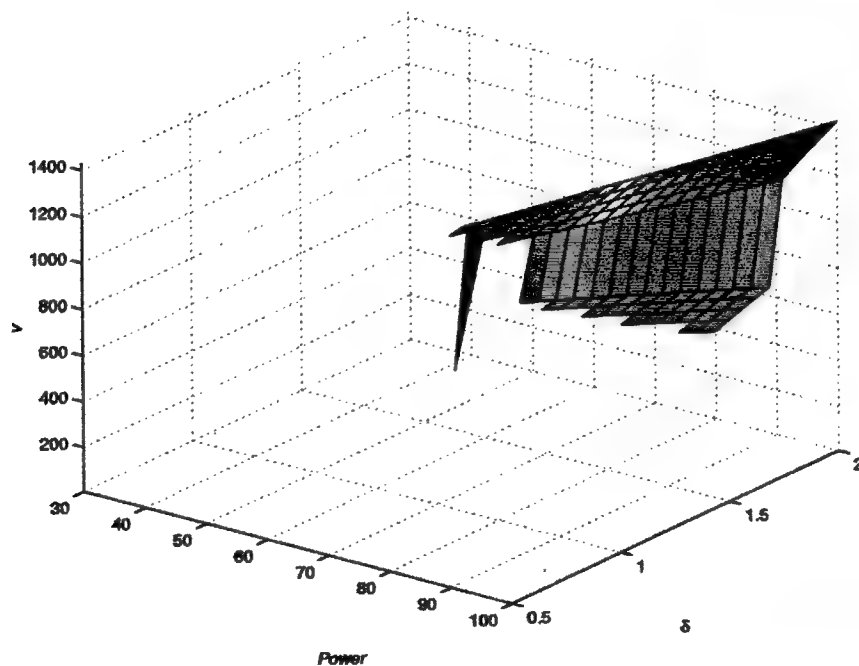


Fig. 5.37: Maximum velocity attainable in nominal S2T16B configuration. The lowest value represents a physically impractical velocity of 7.6 m/s.

velocity over the feasible area is not a valuable statistic alone in the design of a system, although it does provide insight into the performance of a configuration once the feasible solution boundary is crossed.

### 5.2.2 Configuration with Set Number of Cards

Constraining the problem further, the number of each card type is also fixed. Although this model is much simpler to optimize because there are two fewer optimization variables ( $C_1$  and  $C_2$ ), this model may represent a frequently occurring situation for a system engineer: The hardware is already decided, whether because it was the only option in purchasing or because it is being reused from a previous purpose, and now the software must be configured to make the system work at optimal performance. The power then is set ( $\Pi = 12.2C_1 + 9.6C_2$ ) and the only variables left to optimize are  $v$  and  $S_a$ . The objective function and constraints remain the same as in the set power problem except for the omission of the power constraint (Eqn. 5.10).

Fig. 5.39 compares the optimal and nominal configurations of two different systems. The first system has five each of the two daughtercard types. The second type has seven of the S2T16B and two of the S1D64B. Note that the power consumption of both systems is slightly different: The 5:5 system requires 109.0 w and the 7:2 requires 104.6 w. The results were similar to those above of the fixed power but variable card-configuration model. As expected, the configuration with the greater proportion of S2T16B cards performed better at coarse resolution and provided fewer feasible solutions at fine resolutions.

A revealing point in the plot is where resolution is approximately 1.35 where there is a sharp point of discontinuity. Unlike the power minimization problem, the discontinuities do not result from jumps in the FFT size. Inspection of Figs. 5.40 and 5.41 shows no corresponding FFT size movement at  $\delta = 1.35$  m. In

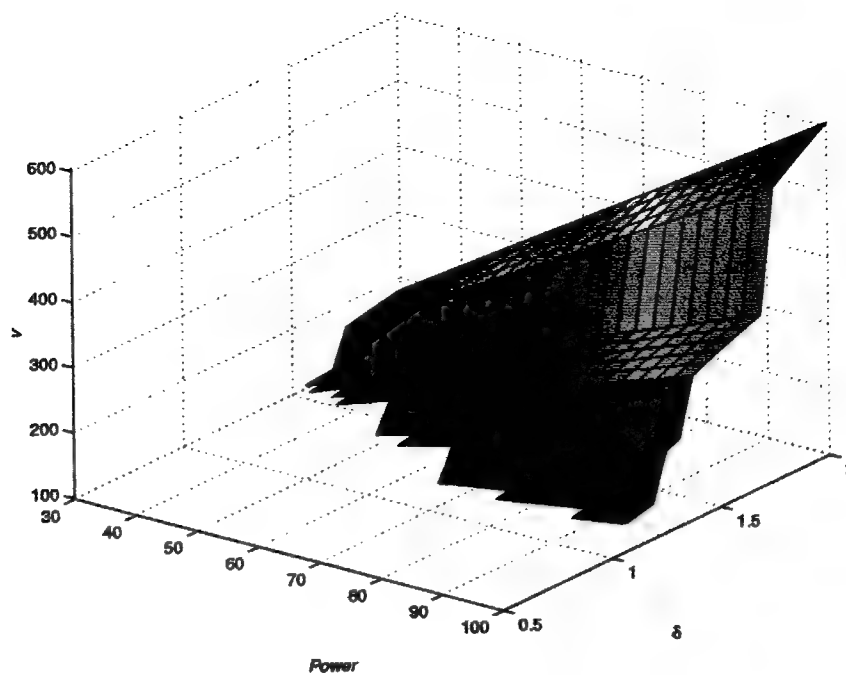


Fig. 5.38: Maximum velocity attainable in nominal S1D64B configuration.

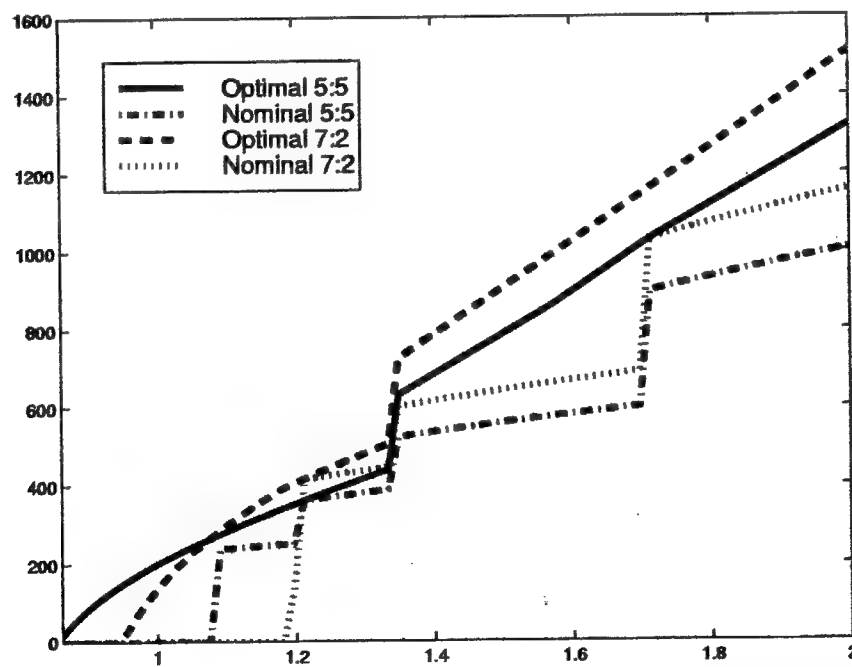


Fig. 5.39: Maximum velocity by nominal and optimal configurations in two systems: one having five of both type daughtercards and the other having seven S2T16Bs and two S1D64Bs.

the 7:2 configuration,  $F_a$  even remains at a constant 2048. Instead, the reason for the discontinuities in the maximum velocities for the optimal configurations is due to a jump in the range FFT size. The range FFT size has played an insignificant role in the optimization problem up to this point in the investigation. With the number of cards and resolution set, the fall of  $F_r$  from 32768 to 16384, caused by the increase in resolution coarseness, spurred a sharp increase in maximum velocity because an additional seven processors became available for azimuth processing. Recall that  $F_r$  is computed as the next power of two greater than the sum of  $S_r$  and  $K_r$ , both of which are functions of resolution and radar parameters, and are therefore not optimized in the maximum velocity problem. As a result, the effect of  $S_r$  is much more poignant in the present problem than in other problems.

Also note that a major disadvantage with the nominal section size heuristic in the maximum velocity problem is that as the section size optimally needs to be increasing as resolution becomes coarser, the decreasing  $K_a$  forces  $S_a$  to also decrease. As a result, the disparity between the optimal and nominal configurations increases as curves approach the right side of the plot where resolution becomes coarser.

Table 5.2.2 summarizes the two set hardware configurations discussed above. Dissimilar to the power minimization problem, where the optimal section size was usually much smaller than the kernel size, the optimal section size in the present case averages two to three times the nominal section size.

### 5.3 Minimization of Resolution

The minimization of resolution (i.e., making resolution finer) is the most computationally intensive of the optimization problems. Resolution must be known before any of the following expressions can be calculated:  $K_r, S_r, P_r, M_r, K_a, P_a$ ,

Table 5.2: Comparison of configurations showing the average velocity over the total sampling area ( $\bar{v}_t$ ), maximum velocity, the average velocity over only the feasible solutions ( $\bar{v}_f$ ), and percentage of area with feasible solutions and the percent increase or decrease of each statistic over that of the optimal mixed configuration.

Configuration	$\bar{v}_t$	% +	Max.	% -	$\bar{v}_f$	% +	% Feas.	% -
Optimal Mixed	382	-	1851	-	562	-	68.0	-
Nominal Mixed	309	19.0	1429	22.8	592	5.3	52.3	23.1
Optimal S2T16B	331	13.5	1851	0.0	592	28.2	45.9	32.5
Optimal S1D64B	194	49.2	789	57.5	286	-49.1	68.0	0.0
Nominal S2T16B	211	44.8	1429	22.8	882	56.8	24.0	64.7
Nominal S1D64B	141	63.1	605	67.3	271	-51.9	52.3	23.1

Table 5.3: Comparison of set hardware configurations: (1) five each of both cards and (2) seven S2T16Bs and two S1D64Bs. The table shows the minimum resolution at which a solution was feasible, the maximum velocity, the average velocities over the total range of resolutions ( $\bar{v}_t$ ) and over only feasible resolutions ( $\bar{v}_f$ ), and the average section size  $\bar{S}_a$ .

Configuration	Min $\delta$	Max $v$	$\bar{v}_t$	$\bar{v}_f$	$\bar{S}_a$
Optimal 5:5	0.95	1511	516	679	1932
Nominal 5:5	1.20	1157	379	622	690
Optimal 7:2	0.86	1323	574	821	1215
Nominal 7:2	1.09	1006	415	768	629

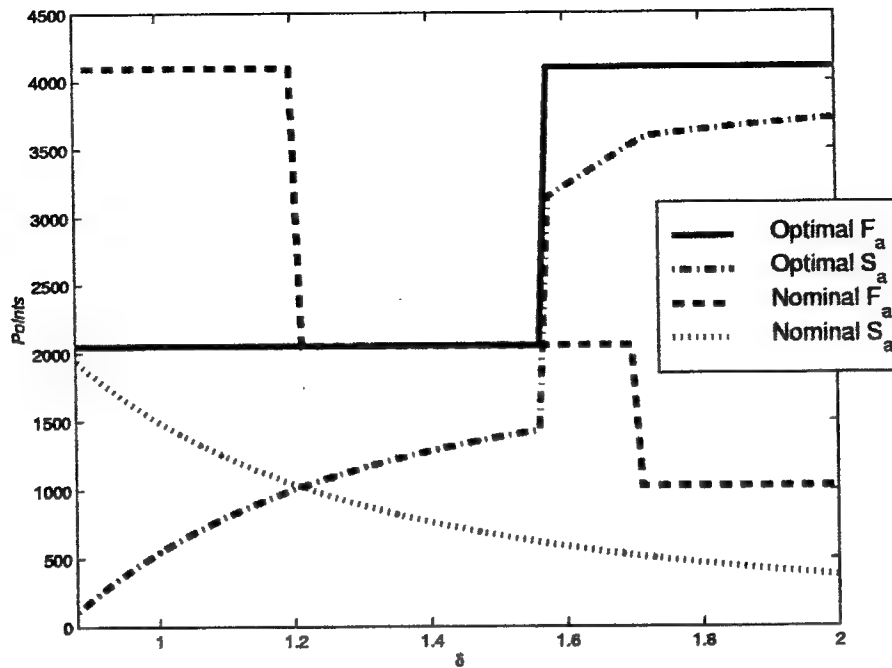


Fig. 5.40: Optimal and nominal FFT and section sizes for the 5:5 system.

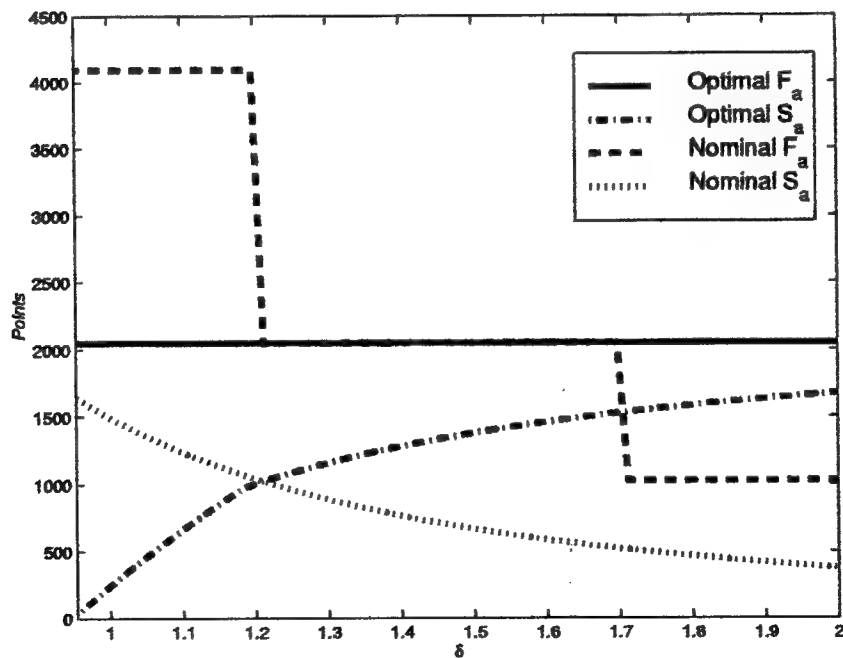


Fig. 5.41: Optimal and nominal FFT and section sizes for the 7:2 system.



and  $M_a$ . The most troublesome of the above variables for formulation is  $K_a$ . Without a value for  $K_a$  when the optimization algorithm is entered, not even a lower bound for  $F_a$  can be determined. Recalling that  $F_a = 2^k$ , where  $k = \lceil \lg(K_a + S_a) \rceil$ , and because  $S_a$  is to be optimized, the first value of  $k$  usually tried is  $k = \lceil \lg(K_a + 1) \rceil$ . Without a value for either  $S_a$  or  $K_a$ , however, the above calculation for  $k$  becomes  $k = \lceil \lg(1 + 1) \rceil = 1$ .

Based on historical data, a slightly larger value for  $k$  can be initially injected into the optimization routine and successively higher values tried thereafter in the same manner as in the other problems. A better initial value could be offered for  $F_a$  if the lowest feasible value for resolution was calculated beforehand for which all the constraints are met, but that in itself is the optimization problem.

As a result of the above difficulty with a lack of an initial  $K_a$ , the first guesses at  $F_a$  tend to be very poor. MATLAB's `constr` function is not always robust enough to handle such poor guesses and in the course of calculating the best resolution for the range of  $F_a$  values tried, `constr` periodically "crashes" on infeasibly low  $F_a$  values, seemingly having entered into an infinite loop. To rectify such a situation, the program must be restarted at the point it failed, incrementing the  $k$  in  $F_a$  by one (only for that power-velocity pair).

Only several of the possible configurations for the resolution minimization problem are investigated here because of the computational intensity and strains on the robustness of the optimization routine for this problem. Furthermore, in some cases solution points are obviously aberrant from their surrounding values. Consequently, the absolute convexity of the solution space for resolution minimization is suspect. In each configuration investigated below, the initial solution surface is shown as for all the power minimization and velocity maximization problems. However, because of the aberrant solution points mentioned, where appropriate the aberrant points in the initial surfaces are smoothed using

a moving average technique. This surface then is also presented.

The objective function for the resolution minimization problem, similar to the velocity maximization problem, is to minimize

$$Z = \delta. \quad (5.14)$$

The constraints are revised to reflect the dependence on  $\delta$ :

$$6C_1 + 2C_2 \geq P(F_a, S_a, \delta) \quad (5.15)$$

$$32C_1 + 64C_2 \geq M(S_a, \delta) \quad (5.16)$$

$$K_a(\delta) + S_a \leq F_a, \quad (5.17)$$

with the standard lower bounds:

$$C_1 \geq 0, C_2 \geq 0, S_a \geq 1, \delta > 0. \quad (5.18)$$

### 5.3.1 Optimal Mixed Card Type Configuration

The initial optimal solution graph for the resolution minimization problem is shown in Fig. 5.42. It would be expected that an optimal surface would be nonincreasing or nondecreasing along each dimension. That is, as velocity increases in one dimension for a set power, resolution should become coarser. Similarly, as power increases for a set velocity, resolution should become finer. Thus it is expected that the optimal solution surface is nondecreasing in the power dimension and nonincreasing in the velocity dimension.

However, it is observed that there are aberrations from this expected characterization in Fig. 5.42. Checking the surface against the characterization described above, a total of twelve deviant points are found, although a cursory visual inspection of the graph reveals four prominent aberrations. For each of

these nonoptimal solutions, it is found that the optimization routine employed a smaller FFT size than in the surrounding points. In some cases, forcing the optimization routine to solve for a higher FFT size results in the optimal solution. In other cases, the optimization routine cannot find the optimal solution without a very precise initial guess and an adjusted step size for the MATLAB function. It is also observed that for some deviant points, the surrounding area, although smooth, does not employ a constant FFT size. Rather, the FFT size oscillates between two values. This phenomenon could suggest a boundary area or even a nonconvex area in the solution space resulting in nonoptimal solutions. Supporting the possibility of nonconvexity, the optimization routine occasionally returns an "infeasible solution" message with some initial guesses.

Due to the time expenditure and unreliability of reoptimizing a particular point in the solution surface, as discussed above, a  $3 \times 3$  neighborhood averaging mask was applied to apparently suboptimal solution points. The resultant surface of this smoothing technique is shown in Fig. 5.43. Note that the smoothing mask is not applied to the entire surface but only to the apparent points of deviation. Such interpolated points should provide a basis from which to calculate an optimal value in the case that the particular power-velocity coordinates are exactly the values that are required on a particular system. Confidence in the overall optimality, previously aberrant points notwithstanding, of the solution graph of Fig. 5.43 is lent both from the characteristics of the surface itself and from informal verification of values by cross checking them against the power minimization and velocity maximization graphs of Figs. 5.4 and 5.26, respectively.

Figs. 5.44, 5.45, and 5.46 illustrate the surfaces formed by the azimuth FFT size, section size, and kernel size, respectively, for optimal resolution. That is, the graphs below are based on interpolated values from the smoothed graph of

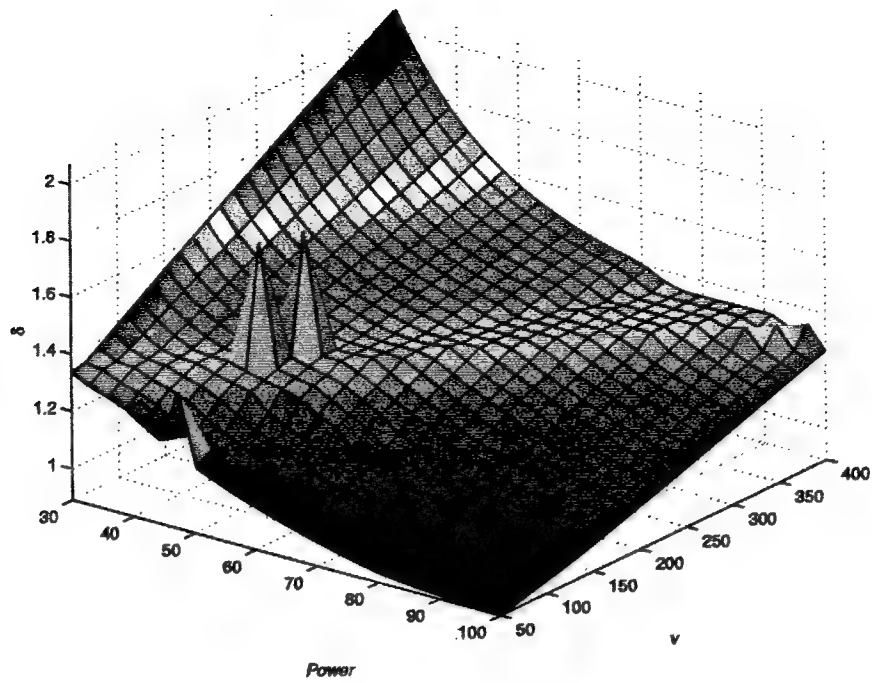


Fig. 5.42: Initial minimum resolution solution in optimal mixed card type configuration.

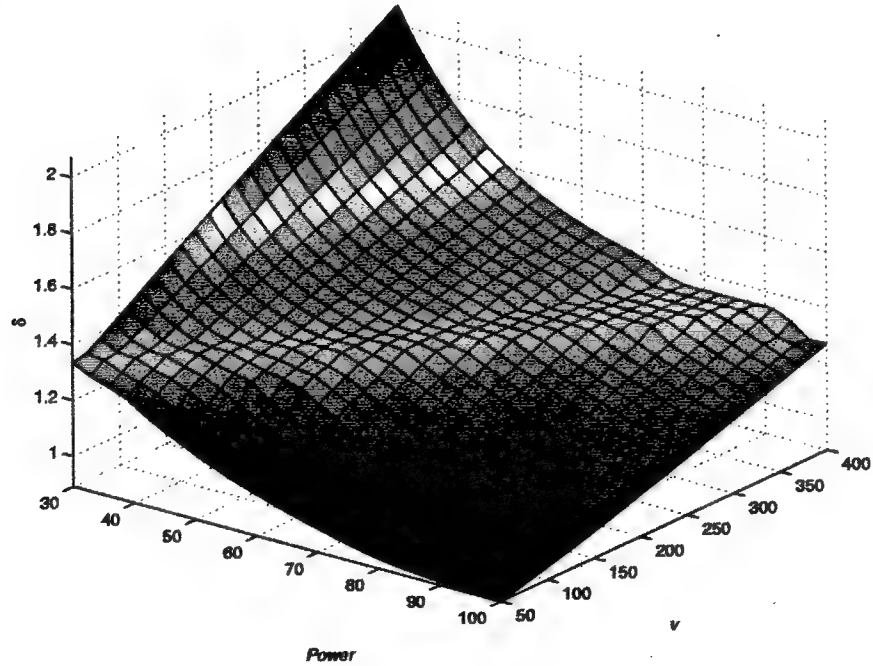


Fig. 5.43: Smoothed minimum resolution solution in optimal mixed card type configuration.

Fig. 5.43. Note that in Fig. 5.44 the surface is consistent with FFT size graphs from previous problems, except for the rift toward the center of the graph. This rift is also reflected in the section size graph of Fig. 5.45, and corresponds to a portion of the level area spanning the graph of minimum resolution in Fig. 5.43. This phenomenon could result from suboptimal solutions in the lower portions of the surface, but it must be kept in mind that there are no infeasible solutions plotted in the graph. Therefore, according to the principle of necessary nonincreasing or nondecreasing functions along each dimension for optimal resolution values, as discussed above, the upper values of the surface can only be in question in that they are too high, not too low. It is assumed at this point that Fig. 5.43 represents a very close approximation to the optimal solution surface. Methods to scrutinize this assumption will be investigated in Chapter VII.

### 5.3.2 Optimal Single Card Type Configuration

Optimization of the single card type configuration for resolution minimization encountered problems. When only the S2T16B was allowed in the configuration, the initial solution surface displays several aberrant points as with the mixed card type graph in Fig. 5.43. See Figs. 5.47 and 5.48 for the initial and smoothed graph for the S2T16B-only configuration.

It would be expected that the single card type configuration using only the S1D64B would display a similar optimization graph with just several anomalies. However, Figs. 5.49 and 5.50, different views of the same graph, do not display isolated points of deviation, but deviant trends. As a result, the smoothing technique is not employed on this graph because not every aberrant point is surrounded by reasonable solution points from which to interpolate a better value. More research into the **constr** function implemented in MATLAB and the solution space of the problem is necessary to surmise why the algorithm

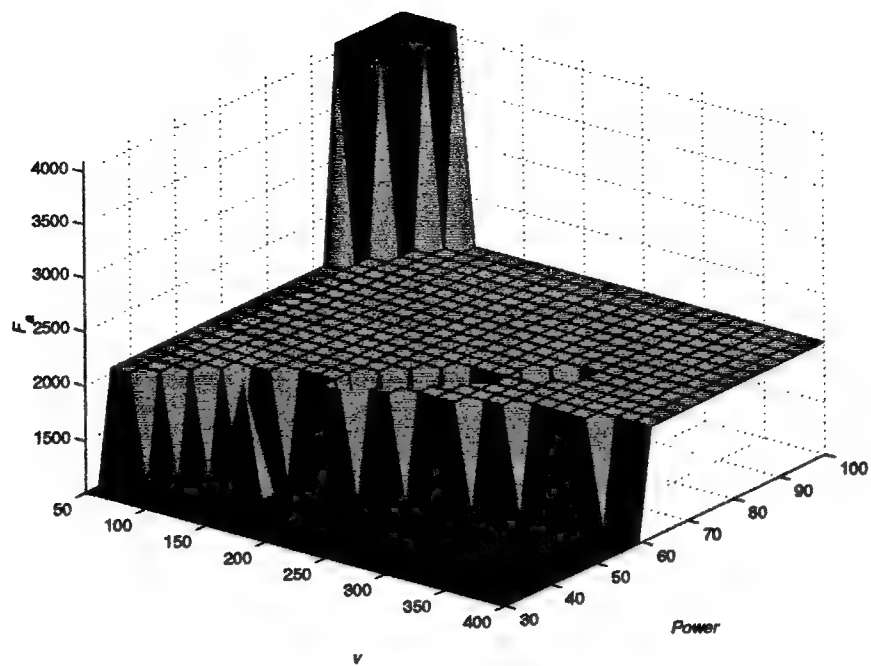


Fig. 5.44: Optimal azimuth FFT size for minimum resolution.

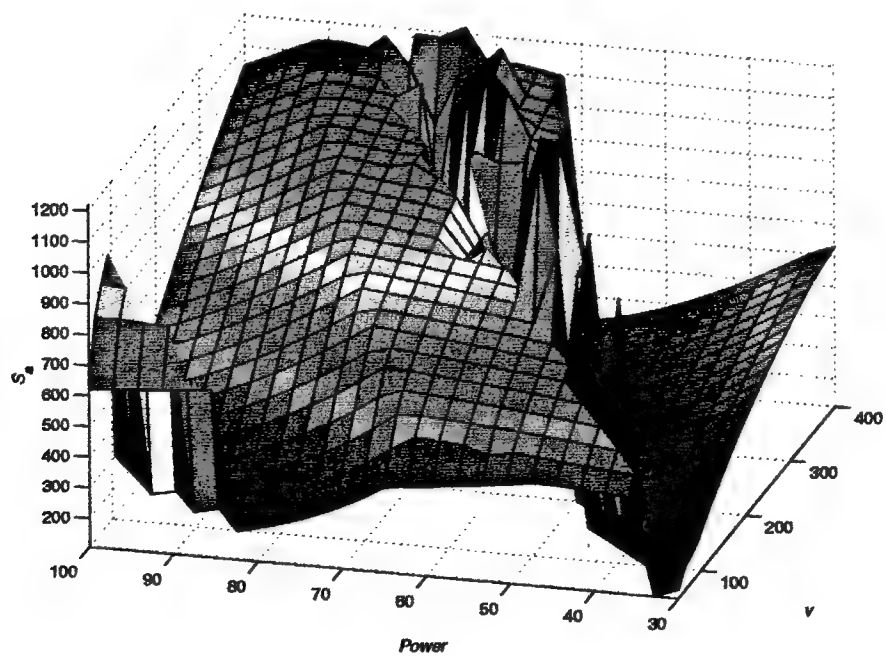


Fig. 5.45: Optimal azimuth section size for minimum resolution.

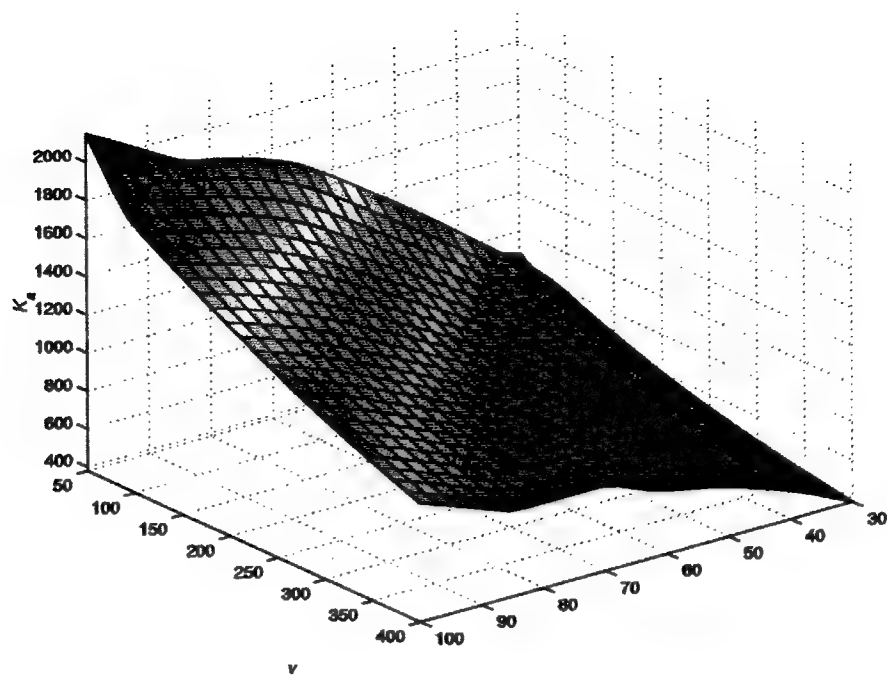


Fig. 5.46: Azimuth kernel size for minimum resolution.

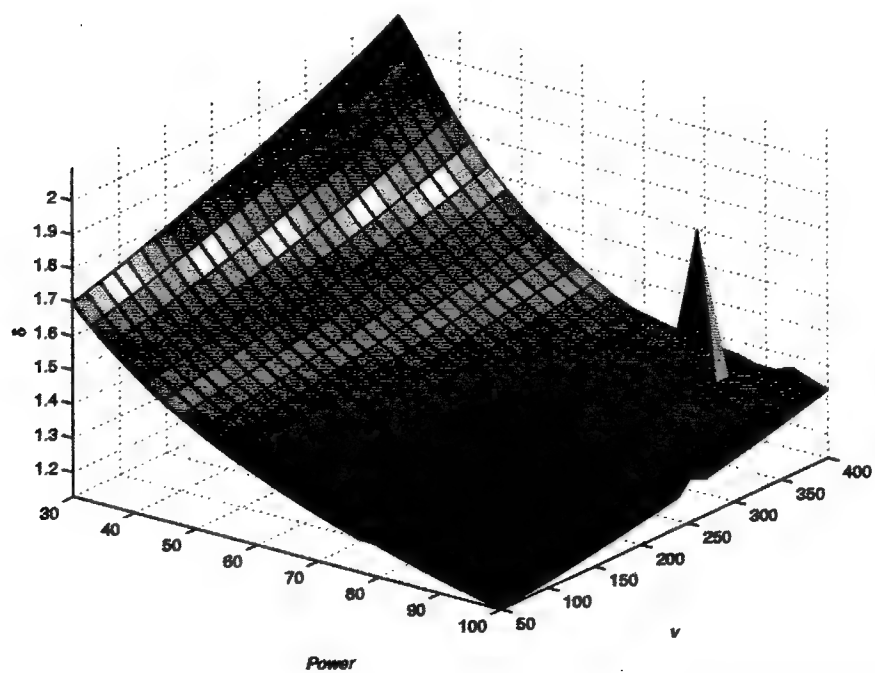


Fig. 5.47: Initial solution graph of the S2T16B-only configuration for resolution minimization.

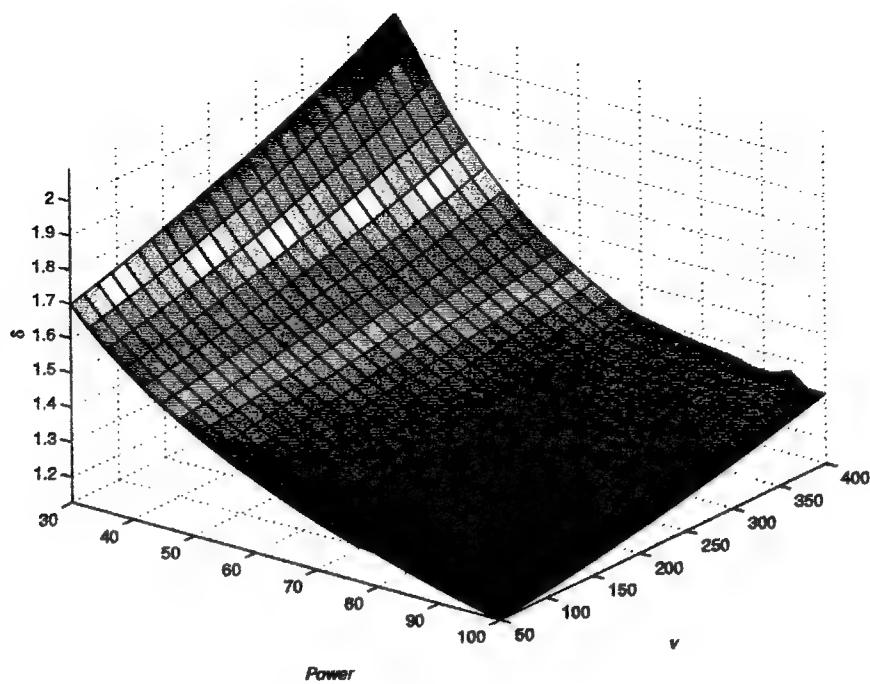


Fig. 5.48: Smoothed solution graph of the S2T16B-only configuration for resolution minimization.

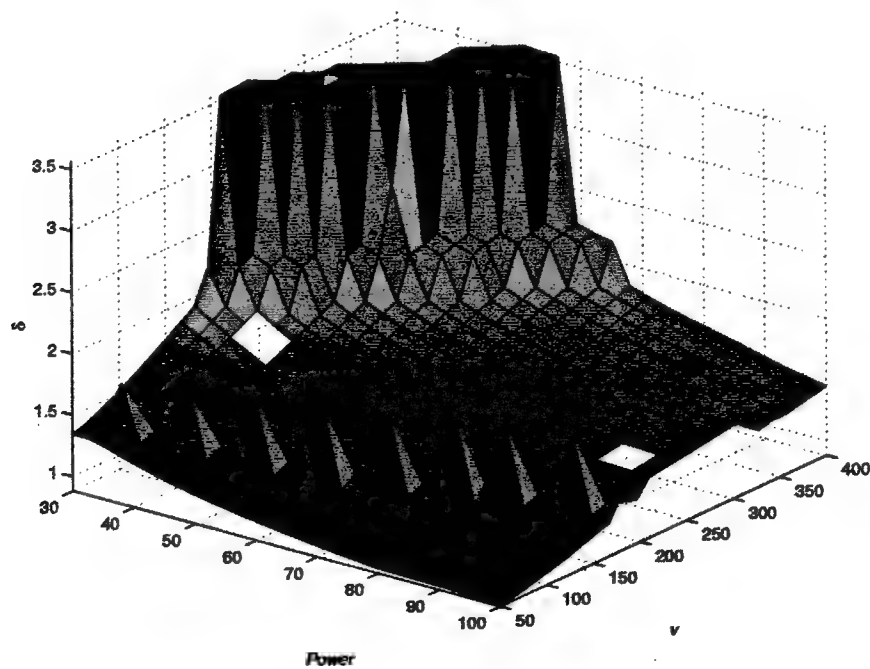


Fig. 5.49: Initial solution graph of the S1D64B-only configuration for resolution minimization.



performed so poorly for this configuration.

#### 5.4 Conclusions

Three distinct optimization objectives have been investigated in this chapter: power minimization, velocity maximization, and resolution minimization. Of the three objectives, most attention has been directed toward power minimization because power is representative of the restrictions concerned in SWAP-constrained systems, as introduced at the beginning of this work. Velocity and resolution optimizations were also investigated, with limited success in the minimization of resolution because of the computational complexity and possible lack of solution space convexity.

For each objective mentioned above, different configurations are explored. Configurations in which the section size is optimized are denoted as optimal, where configurations in which the section size is fixed as the kernel size are denoted as nominal. Both mixed and single card type configurations are investigated. In the mixed configurations, the number of each type of the two available card types are optimized, except for one scenario in the velocity maximization problem where the number of each card type is set.

One of the motivating factors in the outset of this research was to investigate the significance of the arbitrarily-set azimuth section size. It has been shown that proper selection of the section size is crucial to the performance of a system. Without optimization of this parameter, processors or memory can be wasted in a system. The optimal value of the section size is often unintuitively low, conserving memory but causing relatively inefficient use of processors.

The ISMM provides a starting point for system design and performance evaluation. Although some significant assumptions are made in this model to simplify the optimization formulation and concomitant computation, it will be

shown that this simplification provides a reasonable lower-bound for the more involved and accurate model presented in Chapter VI. Furthermore, the simplicity of the ISMM and the associated freedom granted the parameters in each scenario accentuate the interrelationships between the variables, the characteristics of which are otherwise more difficult to discern in the more realistic model. This simplification results in significantly reduced computational intensity and allows for the production of all the data presented in this chapter.

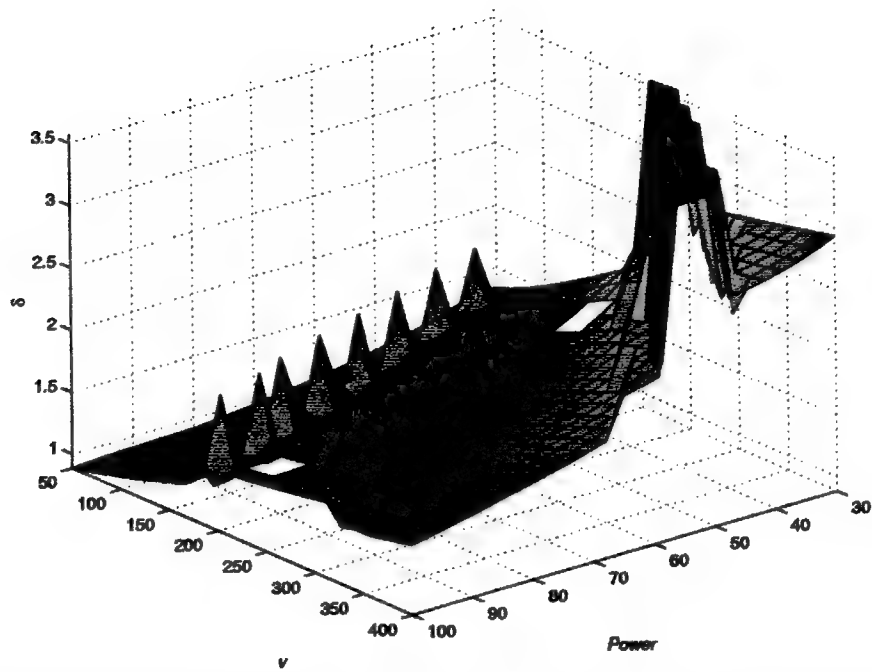


Fig. 5.50: Alternate view of the initial graph of the S1D64B-only configuration.

## CHAPTER VI

### CN-CONSTRAINED MODEL

Increasing the realism of the optimization model, the set of constraints is now revised to ensure that no remote memory accesses occur besides the matrix transposition operation from the range to the azimuth processors. This model is significantly more complex and necessitates the introduction of several new variables. Besides the additional constraints restricting the amount of available memory per processor, the primary difference between this model (henceforth denoted as the CNCM) and the ISMM is in the concept of the fundamental unit of system construction. The fundamental building block shifts from an ambiguously configured daughtercard to a precisely configured CN.

#### 6.1 Formulation

The variables  $C_1$  and  $C_2$ , designating the first and second card types, or number of S2T16Bs and S1D64Bs employed, no longer have meaning in the present model without further refinement. Two new sets of variables, discussed in depth in the next section, are introduced to replace  $C_1$  and  $C_2$ , implementing these refinements. Instead of simple card type variables, the new model requires CN configuration variables. The distinction is made in that the card type is only one parameter in the configuration of a card. In addition, the configuration must specify the number of processors dedicated to range processing and the number of processors dedicated to azimuth processing. Similarly, the configuration description must also delimit the amount of memory dedicated to range and azimuth processors on a given CN. This last detail guarantees the absence of remote memory access during range and azimuth processing. Data must still be transferred after range processing is complete from range to azimuth processors

(the distributed matrix transposition).

With only two processor usages (azimuth and range processing), optimization always will require at most two different card configurations. In this chapter, a card configuration defines the number of processors on each CN type used for range and azimuth processing, and the amount of memory allocated to both types of processing per CN type. Recall that a CN consists of multiple processors sharing a common memory.

Three possible optimization scenarios are possible. The first and most simple scenario occurs when the optimization routine determines that the optimal configuration involves dividing the processors and memory on a card type such that both range and azimuth processing is executed. Furthermore, assuming that whatever division of resources is determined to be optimal, the ratio of range and azimuth processors in the given configuration is equal to the ratio of total range and azimuth processors in the system. In this case,  $N$  such configured CNs are required, providing all the required processors, and thus only one card configuration is demanded. If this mixed CN configuration is optimal (a mixed CN configuration is one in which non-zero fractions of the resources on the CN are allocated for both range and azimuth processing), then no other configuration is necessary. That is, the addition of a second configuration will not improve the performance of the system in any way. (The only time this rule does not hold true is in the optimization of the final CN of a type, which is probably fractional according to the requirements. At this point, however, fractional CNs are permitted in the solution and further discussion of this situation is deferred until later in this chapter.)

To illustrate the first scenario, suppose ten range processors and twenty azimuth processors are optimally required. A possible configuration of the above type might be implemented with S2T16B cards, with one range and two az-

imuth processors assigned per CN. This configuration assumes that the sixteen megabytes of memory on the single CN is sufficient for all three processors. That is, twice the azimuth memory requirement plus the range requirement per processor must be less than or equal to sixteen megabytes. Note that the azimuth and range memory requirements per processor need not be, and most probably will not be, the same.

Consider the next optimization scenario in which the optimization algorithm determines that the best use of CNs is to dedicate all of one type of CN configuration to range processing and another to azimuth processing. In this case, two configurations are necessary for optimality. As an example, one type of CN could be on the S2T16B and all three processors could be dedicated to range processing. Each processor would have for its own exclusive use  $\frac{16}{3} = 5.33$  MB. The azimuth processing could be assigned to CNs of the S1D64B card. Both processors could be utilized, yielding  $\frac{64}{2} = 32$  MB per processor.

The above example coincidentally preserves the convention of the S2T16B as the card type of the first CN configuration and the S1D64B as the card type of the second CN configuration. However, it is important to note that with the new notation, the card types associated with the two CN configurations do not necessarily correspond to the S2T16B and the S1D64B, respectively. The optimization algorithm is given freedom to determine the optimal configuration(s), and the result could be a reversal of the previously designated card types. Although this ambiguity alone could be easily forced into conformity with the earlier definition of *type*, it is important to maintain the ambiguity to allow for the possibility of only one optimal CN configuration, as in the first example, or even to allow for two different CN configurations using the same daughtercard type. The fact that there are two card types and two possible optimal CN configurations is purely coincidental. The latter is due to the presence of two possible programs, or

two types of processing (range and azimuth). Even for any number of available daughtercard types, an optimal configuration still would only require at most two CN configurations.

The third possible optimization scenario resembles the first example in the mixed CN configuration, but with the exclusion of the condition that the ratio of range and azimuth processors on the CN is equivalent to the ratio of the total required range and azimuth processors. Such a situation necessarily occurs when there exists a great disparity in the required number of range and azimuth processors. In such a case, one CN configuration would be a heterogeneous assignment of range and azimuth processors to a single CN, and the second CN configuration would be a homogeneous assignment of whichever processor type was still lacking. For example, if five range processors and twenty azimuth processors were required, the CN configuration of the first example could be employed to incorporate all the range processors and ten of the azimuth processors. The remaining ten azimuth processors would be assigned in a homogeneous CN configuration, either on the same or different type card.

In each case, it is possible that there will be a portion of memory wasted on each CN. In the same way, it is possible that an entire processor is wasted on a CN. If the memory requirements hinder the utilization of all processors, then a processor must be left idle. However, in most cases the optimization algorithm decides against using such a configuration because there is usually a more efficient way of configuring the system, usually by decreasing the section size so that less memory is required and all processors are utilized. In the last example, to accommodate the remaining ten azimuth processors on the same card type, it is probable that only two of the three processors per CN could be utilized because azimuth processors usually require more memory than range processors.

To note the distinction between the ISMM card type variables and the new

CN configuration variables, let  $X$  and  $Y$  abstractly represent the two CN configurations (note that  $X$  and  $Y$  will not be used in the formulation without accompanying subscripts defining specific characteristics of each configuration). Let  $X_T$  and  $Y_T$  represent the daughtercard types of the new configuration variables of the CNCM, where the type can be either the S2T16B or the S1D64B daughtercard. Let  $N_X$  and  $N_Y$  denote the number of each CN required of the corresponding configuration. Note that the combination of the two sets of variables defined above essentially serves the same function as did  $C_1$  and  $C_2$  in the ISMM, with  $C_1$  and  $C_2$  representing the number of cards required themselves and their type implicit in their definition. In contrast, the new variables explicitly define each quantity and quality associated with them.

Two additional subscripts are necessary for the CN configuration variables to complete their description. For notational convenience, let  $I \in \{X, Y\}$ . To denote the number of processors dedicated to range and azimuth processing on a specifically configured CN,  $I_r$  and  $I_a$  are introduced, where the  $r$  and  $a$  refer to *range* and *azimuth*. It might seem necessary also to create a variable to define the amount of memory allocated to each processor of each type, but as shown below, this constraint can be implicitly figured by the ratio of the total amount of memory needed per processor function (i.e., for range or azimuth processing) to the total number of processors (per function) required. Memory thus will be treated as an implicit rather than an explicit optimization variable.

The first two constraints in the formulation ensure that a sufficient number of range and azimuth processors are allocated:

$$P_r \leq N_X X_r + N_Y Y_r$$

$$P_a(S_a) \leq N_X X_a + N_Y Y_a.$$

In contrast to the ISMM, where only one constraint concerned the total number



of processors required, it is necessary to separately calculate and constrain the range and azimuth processor requirements in this model. The above two constraints define the available range or azimuth processors by taking the product of the number of CNs of each type and the number of processors on that CN dedicated to the given type of processing.

The next two constraints in the formulation are the memory counterpart of the first two processor constraints. However, as mentioned earlier, the memory per processor is not an explicit optimization variable as is the processors per CN. Instead, the memory per processor is computed implicitly by the following ratios:

$$M_{CN}(X_T) \geq X_r \frac{M_r}{P_r} + X_a \frac{M_a(S_a)}{P_a(S_a)} \quad (6.1)$$

$$M_{CN}(Y_T) \geq Y_r \frac{M_r}{P_r} + Y_a \frac{M_a(S_a)}{P_a(S_a)}. \quad (6.2)$$

Similar to the formulation in Subsection 5.1.2,  $M_{CN}$  represents the memory available per CN as a function of the configuration type. In the present case, this function is defined as follows:

$$M_{CN}(I_T) = \begin{cases} 16 & \text{if } I_T = \text{S2T16B}, \\ 64 & \text{if } I_T = \text{S1D64B}. \end{cases}$$

An additional basic constraint is necessary to ensure that the number of processors assigned to a CN is physically realizable by that CN. The following constraint ensues:

$$X_r + X_a \leq P_{CN}(X_T)$$

$$Y_r + Y_a \leq P_{CN}(Y_T),$$

where  $P_{\text{CN}}$  designates the number of processors available per CN as a function of the configuration type. Again, as in Subsection 5.1.2, such a function allows the addition of any number of daughtercard types to the hardware choice list without changing the optimization formulation. Only the function definitions would need to be modified to incorporate the addition of daughtercard types. In the present work, the function  $P_{\text{CN}}$  is limited to the following definition:

$$P_{\text{CN}}(I_T) = \begin{cases} 3 & \text{if } I_T = \text{S2T16B}, \\ 2 & \text{if } I_T = \text{S1D64B}. \end{cases}$$

The only additional constraint involves the FFT size and is the same as in the ISMM:

$$F_a = 2^k \geq S_a + K_a, \quad k = 1, 2, \dots$$

The standard lower bounds must also be included:

$$N_I \geq 0, I_r \geq 1, I_a \geq 1, S_a \geq 1.$$

## 6.2 Computational Approach

The CNCM introduces additional variables that must assume only discrete values. Unlike the section size  $S_a$ , which can be computed by merely rounding its optimized value, variables  $I_r$  and  $I_a$ , respectful of  $I_T$ , must be handled in the same manner as the FFT size  $F_a$ . Consequently, many feasible combinations of processor assignments must be tried in order to ensure optimality.

The upper bound on the number of configuration combinations that must be evaluated can be calculated by examining the three optimization scenarios discussed above. In the first scenario, involving only one type of CN heteroge-

neously configured with both azimuth and range processors, all combinations on each daughtercard type in which the sum of the range and azimuth processors is less than or equal to the number of processors available on a given CN must be evaluated. Let it be assumed that the first configuration type variable is optimized for this heterogeneous processor assignment on a single CN configuration (i.e.,  $N_X \neq 0$  and  $N_Y = 0$ , which could be reversed in an actual solution). Let  $\pi_T = P_{CN}(X_T)$ , for  $T \in \{1, 2, \dots, N_d\}$ , where  $N_d$  is the total number of different daughtercard types available, and all daughtercard types are represented by arbitrary consecutive numbers beginning with one. Let  $E_{het}$  denote the set of different combinations that must be evaluated in the single CN heterogeneous scenario. The enumerated triples in the following equation, i.e., daughtercard type (as a number),  $X_r$ , and  $X_a$ , completely specify the set of feasible combinations in the single CN heterogeneous scenario:

$$E_{het} = \bigcup_{T=1}^{N_d} \bigcup_{X_r=1}^{\pi_T-1} \bigcup_{X_a=1}^{\pi_T-X_r} (T, X_r, X_a). \quad (6.3)$$

To sum the total number of feasible combinations that must be tried, Eqn. 6.3 is evaluated as

$$|E_{het}| = \sum_{T=1}^{N_d} \sum_{X_r=1}^{\pi_T-1} \sum_{X_a=1}^{\pi_T-X_r} (1)$$

which also can be expressed by

$$\begin{aligned}
|E_{\text{het}}| &= \sum_{T=1}^{N_d} \sum_{j=1}^{\pi_T-1} \binom{j}{1} \\
&= \sum_{T=1}^{N_d} \sum_{j=1}^{\pi_T-1} j \\
&= \sum_{T=1}^{N_d} \frac{[\pi_T - 1][(\pi_T - 1) + 1]}{2} \\
&= \frac{1}{2} \sum_{T=1}^{N_d} (\pi_T^2 - \pi_T). \tag{6.4}
\end{aligned}$$

To illustrate, suppose that the number of available daughtercard types is  $N_d = 3$  and that the number of processors for each CN associated with each daughtercard is  $\pi_1 = 2$ ,  $\pi_2 = 4$ , and  $\pi_3 = 3$ . Then according to Eqn. 6.4,  $|E_{\text{het}}| = \frac{1}{2}[(2^2 - 2) + (4^2 - 4) + (3^2 - 3)] = 10$ .

In the second scenario, involving homogeneous assignments to CNs of range and azimuth processors, two CN configurations are necessary in which either  $I_r = 0$  in one case and  $I_a = 0$  in the other, or vice-versa. To enumerate the feasible CN configuration combinations, let  $\pi_T$  as used in the heterogeneous scenario above be modified to reflect the letter of the configuration variable in addition to the daughtercard type. That is, let  $\pi_{I_T} = P_{\text{CN}}(I_T)$ , for  $T \in \{1, 2, \dots, N_d\}$ . Furthermore, let  $E_{\text{hom}}$  represent the feasible configuration combinations in the homogeneous case. Assume, without loss of generality, that  $X_a = 0$  and  $Y_r = 0$ , effectively designating configuration set  $X$  as the range CN and configuration set  $Y$  as the azimuth CN. The set of feasible configurations in the homogeneous case is then given by the following expression:

$$E_{\text{hom}} = \bigcup_{X_T=1}^{N_d} \bigcup_{Y_T=1}^{N_d} \bigcup_{X_r=1}^{\pi_{X_T}} \bigcup_{Y_a=1}^{\pi_{Y_T}} \{(X_T, X_r, X_a = 0), (Y_T, Y_r = 0, Y_a)\}, \tag{6.5}$$

where the pair of triples is of the same convention as set in the heterogeneous formulation.

Although Eqn. 6.5 represents a large number relative to Eqn. 6.3, only a small percentage of these combinations must be actually tried for the optimal solution because the configuration of the range CN is independent of the azimuth CN configuration in the homogeneous scenario. That is, the optimal range CN is optimal regardless of the optimal azimuth CN and vice-versa, and thus one CN configuration (range or azimuth) can be optimized without evaluating every combination of the other CN (range or azimuth). Therefore, the quadruple summation can be separated into the range and azimuth CN combinations as follows:

$$\begin{aligned} \text{range CN combinations: } & \sum_{X_T=1}^{N_d} \sum_{X_r=1}^{\pi_{X_T}} (1) \\ \text{azimuth CN combinations: } & \sum_{Y_T=1}^{N_d} \sum_{Y_a=1}^{\pi_{Y_T}} (1). \end{aligned}$$

As a result, Eqn. 6.6 can be reduced to the following:

$$E_{\text{hom}} = \bigcup_{X_T=1}^{N_d} \bigcup_{X_r=1}^{\pi_{X_T}} \{(X_T, X_r, X_a = 0)\} \cup \bigcup_{Y_T=1}^{N_d} \bigcup_{Y_a=1}^{\pi_{Y_T}} \{(Y_T, Y_r = 0, Y_a)\}. \quad (6.6)$$

If  $n$  is the number of combinations associated with Eqn. 6.5, then the number

of evaluations described by Eqn. 6.6 is  $2\sqrt{n}$ . Eqn. 6.6 simplifies to

$$\begin{aligned}
|E_{\text{hom}}| &= \sum_{X_T=1}^{N_d} \sum_{X_r=1}^{\pi_{X_T}} (1) + \sum_{Y_T=1}^{N_d} \sum_{Y_a=1}^{\pi_{Y_T}} (1) \\
&= \sum_{X_T=1}^{N_d} \pi_{X_T} + \sum_{Y_T=1}^{N_d} \pi_{Y_T} \\
&= 2 \sum_{T=1}^{N_d} \pi_T,
\end{aligned}$$

where the last equation employs the notation used in the heterogeneous scenario.

The third scenario, which involves both a homogeneous and heterogeneous CN, is a combination of the first two scenarios. Let this mixed scenario be represented by  $E_{\text{het,hom}}$ . One heterogeneous CN configuration out of all the feasible combinations expressed by  $E_{\text{het}}$  is necessary in this case. Because of the independence of the homogeneous range and azimuth CN configurations, exploited by the reduction of Eqn. 6.5 to Eqn. 6.6, all combinations of  $E_{\text{hom}}$  must also be applied. As a result, the following value for  $E_{\text{het,hom}}$  is derived:

$$|E_{\text{het,hom}}| = |E_{\text{het}}| \cdot |E_{\text{hom}}|.$$

The upper bounds for the total number of processor assignment combinations, respective of daughtercard type, that must be considered in the CNCM optimization is simply the sum of the expressions for the three scenarios already investigated. That is,

$$|E| = |E_{\text{hom}}| + |E_{\text{het}}| + |E_{\text{het,hom}}|,$$

where  $|E|$  represents the total number of evaluations for all scenarios. Note that

the above summation also can be expressed by the following:

$$|E| = |E_{\text{het}}| + |E_{\text{hom}}| + |E_{\text{het}}| \cdot |E_{\text{hom}}|.$$

With the S2T16B and S1D64B daughtercards exclusively as choices,  $E$  can be easily calculated for the model under investigation. Because  $N_d = 2$ , let the daughtercard type be the S2T16B if  $T = 1$  and the S1D64B if  $T = 2$ , preserving the convention of the ISMM. Thus,  $\pi_1 = 3$  and  $\pi_2 = 2$ . With this definition,  $E_{\text{het}}$  can then be evaluated as follows:

$$\begin{aligned} |E_{\text{het}}| &= \frac{1}{2} \sum_{T=1}^{N_d} (\pi_T^2 - \pi_T) \\ &= \frac{1}{2} \sum_{T=1}^2 (\pi_T^2 - \pi_T) \\ &= \frac{1}{2} [(3^2 - 3) + (2^2 - 2)] \\ &= 4. \end{aligned}$$

In the same way,  $|E_{\text{hom}}|$  is evaluated:

$$\begin{aligned} |E_{\text{hom}}| &= 2 \sum_{T=1}^{N_d} \pi_T \\ &= 2 \sum_{T=1}^2 \pi_T \\ &= 2(3 + 2) \\ &= 10. \end{aligned}$$

The total number of necessary evaluations is therefore

$$\begin{aligned}
|E| &= |E_{\text{het}}| + |E_{\text{hom}}| + |E_{\text{het}}| \cdot |E_{\text{hom}}| \\
&= 4 + 10 + (4)(10) \\
&= 54.
\end{aligned}$$

Up to 54 different combinations of processor assignments and card types must be evaluated to ensure optimality. For each combination, the optimization routine must be invoked and the best value, for whatever objective is chosen, of all the combinations and corresponding configuration are declared optimal.

### 6.3 Minimization of Power

Power minimization is the fundamental case of investigation in this work. Because of the increased computational intensity involved with the CNCM, this model is only applied to the power minimization objective. Furthermore, it is deemed sufficient to illustrate the utilization of this new model by applying it only to the two cases of optimal and nominal mixed card type configurations because the mixed configuration is the most general of all the configurations. Analysis of the solutions of both cases will be carried out, followed by investigation of the utilization of the ISMM as a lower-bounds heuristic for the CNCM.

Similar to the convention set in Subsection 5.1.2, power requirements will be represented as functions of the configuration types. Thus the objective function for the power minimization model is as follows:

$$Z = N_X \Pi_{\text{CN}}(X_T) + N_Y \Pi_{\text{CN}}(Y_T).$$

Note that with only the S2T16B and S1D64B available, the power function above is defined as



$$\Pi_{\text{CN}}(I_T) = \begin{cases} 6.1 & \text{if } I_T = \text{S2T16B}, \\ 9.6 & \text{if } I_T = \text{S1D64B}. \end{cases}$$

The above power values reflect the power consumption per CN instead of the power per daughtercard value employed in the ISMM. Although the above function definition of  $\Pi_{\text{CN}}$  consists of only two cases, as with this entire model, the number of different types of daughtercards could increase by adjusting the power function without affecting the optimization formulation.

Some reductions can be made to  $E$  for the power minimization model, exploiting the fixed nature of velocity and resolution. These reductions result from the determination of either infeasible combinations, based on constraints such as memory, or inexpedient combinations, which can be proved to provide sub-optimal solutions. Note that different optimization objectives entail different methods of reducing  $E$  as an upper bound or of reducing the mean  $E$ .

In the case of the homogeneous range CN, only one combination needs to be evaluated. Because the range processor and memory requirements ( $P_r$  and  $M_r$ ) are fixed with resolution and velocity, the memory per range processor is also fixed. Therefore the maximum feasible number of range processors per CN is given by the following expression:

$$\begin{aligned} \max I_r &= \left\lfloor \frac{M_{\text{CN}}(I_T)}{\frac{M_r}{P_r}} \right\rfloor \\ &= \left\lfloor \frac{M_{\text{CN}}(I_T)P_r}{M_r} \right\rfloor. \end{aligned} \tag{6.7}$$

Eqn. 6.7 also expresses the viable minimum number of range processors per CN that should be evaluated because there is no advantage to leaving a processor idle if it can be utilized. Thus Eqn. 6.7 represents the only combination that

needs to be evaluated for the homogeneously configured range CN. As a result,  $E_{\text{hom}}$  is modified accordingly:

$$\begin{aligned} |E_{\text{hom}}| &= \sum_{T=1}^{N_d} \pi_T + \sum_{T=1}^{N_d} (1) \\ &= \sum_{T=1}^{N_d} \pi_T + N_d. \end{aligned}$$

For the heterogeneous case, all combinations that do not meet the following constraint can be removed from consideration before the invocation of the optimization routine:

$$M_{\text{CN}}(I_T) \geq I_r \frac{M_r}{P_r} + I_a \frac{M_a(S_a = 1)}{P_a(S_a = 1)}. \quad (6.8)$$

The above condition is also valid for the homogeneous azimuth CN configuration, where  $I_r = 0$ . No minimum azimuth processor per CN value can be computed as for the range because the expression is only a lower bound for the optimization variable  $S_a$ . A low  $S_a$  entails a very high number of processors, but this fact alone does not eliminate any configurations, whereas the memory constraint does do so. However, the upper bound on  $E$  remains unchanged because it is possible that the restriction of Eqn. 6.8 will never be active, although the mean  $E$  should be reduced.

With application of the above restrictions, the lower bound on  $E$  is zero, meaning that the application and radar parameters do not allow for a feasible configuration on the available types of daughtercards. To calculate the new upper bound of  $E$  for the power minimization model, with the two familiar

daughtercards available, the new  $E_{\text{hom}}$  first must be computed:

$$\begin{aligned} |E_{\text{hom}}| &= \sum_{T=1}^{N_d} \pi_T + N_d \\ &= (3 + 2) + 2 \\ &= 7. \end{aligned}$$

Note that with a larger selection of daughtercard types, the effect of this reduction would be greater. Considering this change in  $E_{\text{hom}}$ , the new value of  $E$  is found to be

$$\begin{aligned} |E| &= |E_{\text{het}}| + |E_{\text{hom}}| + |E_{\text{het}}| \cdot |E_{\text{hom}}| \\ &= 4 + 7 + (4)(7) \\ &= 39. \end{aligned}$$

This new value for  $E$  results in a 28% reduction in the upper bound of the number of evaluations necessary in the optimization routine.

### 6.3.1 Optimal Mixed Configuration

Results of the optimization for the optimal mixed card type configuration are given in Fig. 6.1. The smooth surface, nonincreasing in the resolution dimension and nondecreasing in the velocity dimension, is characteristic of an optimal power surface. Although comparison of the CNCM power graph to that of the ISMM will be presented later in this chapter, it is noted that the graph is almost identical in shape to that of the ISMM (see Fig. 5.4). However, important differences resulting from the additional constraints and variables introduced in the present model necessitate new approaches to analysis of the data.

The most important new information available from the new model is the

configuration variables  $I_T$ ,  $I_r$  and  $I_a$ . It is feasible to present all the information from all three of these variables concisely on one graph because only a subset of the total possible permutations is found to include optimal solutions in the range of values investigated. For labeling purposes, let the following notation be used: a configuration is represented by one or two three-digit numbers. Each number is constructed digit by digit with the CN type first, then the number of range processors per CN of that type, and then the number of azimuth processors per CN of that type. Both of the two possible CN configurations are encoded in this way as part of a total system configuration. That is, a system configuration is denoted by the following:  $X_T X_r X_a \quad Y_T Y_r Y_a$ , where  $I_T$  may be represented by a '1' or a '2' in the present case, denoting the S2T16B or the S1D64B, respectively.

For example, if the optimization routine determined that the optimal system configuration consisted of assigning three range processors and no azimuth processors to the S2T16B, and two azimuth processors and no range processors to the S1D64B, the resultant representation is: 130 202. In the purely heterogeneous scenario, an optimal system configuration might assign one processor each on the S1D64B to range and azimuth processing. The representation for this case would be simply: 211.

The corresponding optimized system configurations for the power surface of Fig. 6.1 is shown in Fig. 6.2. The configuration notation explained above is employed in the legend of the graph.

The majority of system configurations are purely homogeneous CN configurations in which each CN is dedicated to either range or azimuth processing. As would be expected, for this configuration the S2T16B is used for range processing and the S1D64B for azimuth processing. As resolution becomes coarser and velocity decreases, other optimal configurations appear. Primarily motivated by decreasing memory demands resulting from coarser resolution, the heterogeneous

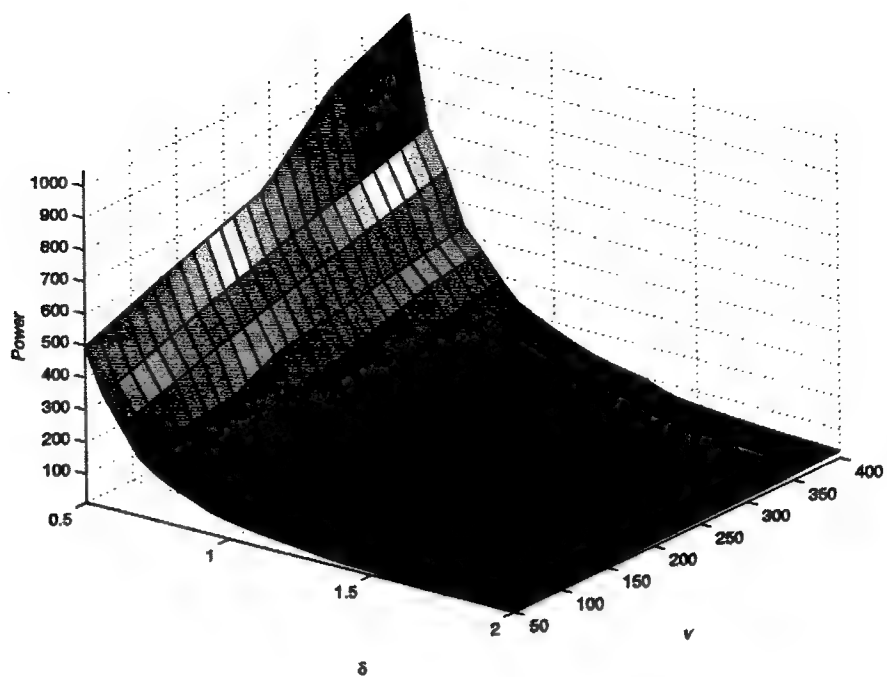


Fig. 6.1: Minimal power for optimal mixed configuration.

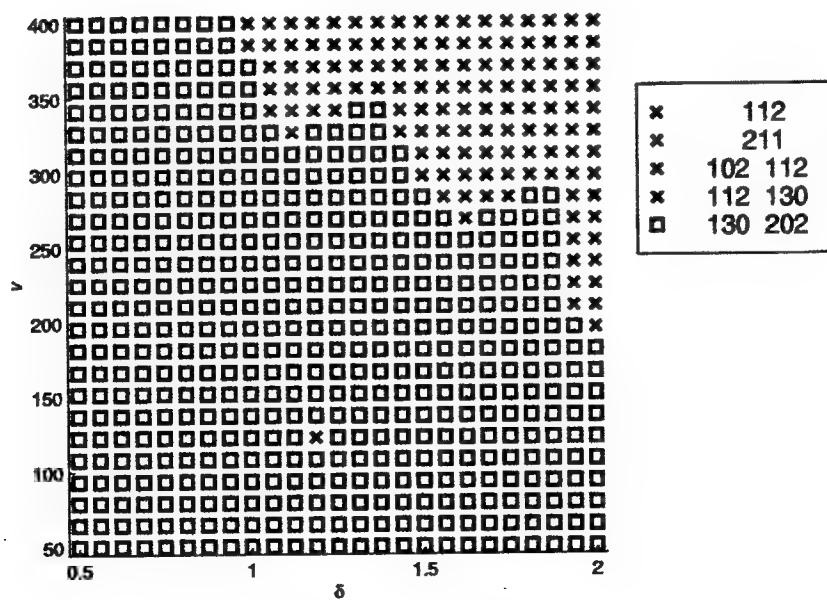


Fig. 6.2: System configurations for minimal power in optimal mixed configuration.

configuration is optimal in which only the S2T16B is utilized with one range and two azimuth processors. It is expected that if power were optimized over even lower resolutions than 0.5 m, the heterogeneous configuration employing only the S1D64B would become more common.

The optimal azimuth FFT size  $F_a$  in the present model is very similar to that in the ISMM. Fig. 6.3 illustrates this resemblance. However, high velocity in conjunction with fine resolution entails a decreased azimuth memory to processor ratio relative to lower velocities at the same resolution. With no advantage in conserving memory that no other processors can use, the two processors on the S1D64B (note that the corresponding configuration at these resolution-velocity pairs is: 130 202) optimize memory usage by increasing the FFT size.

The azimuth section size  $S_a$  closely resembles that of the ISMM just as does  $F_a$ . Fig. 6.4 shows the surface of  $S_a$ . Note the corresponding additional peak in  $S_a$  as in  $F_a$  at the high performance velocity-resolution pairs. The graph of the azimuth kernel size, independent of optimization variables, is the same as in the ISMM (see Fig. 5.10).

The ratio of the azimuth kernel size to the section size ( $K_a/S_a$ ) differs slightly from that of the ISMM because of the motivation to not waste memory available on a CN. Fig. 5.20 shows that this ratio tends to be lower in the coarse resolution area because  $S_a$  is increased to utilize all available memory.

Azimuth memory and processor requirements are slightly different from their counterparts in the ISMM. Fig. 6.6 shows an increase in memory over the ISMM at the high performance corner of the surface, corresponding to the increase in FFT size at the same location observed in Fig. 6.3.

Azimuth processor requirements (Fig. 6.7) exhibit a premature rise and abrupt fall in the area of high velocity and coarse resolution. This seemingly anomalous feature is explained by the configuration graph (Fig. 6.2). Note that this raised

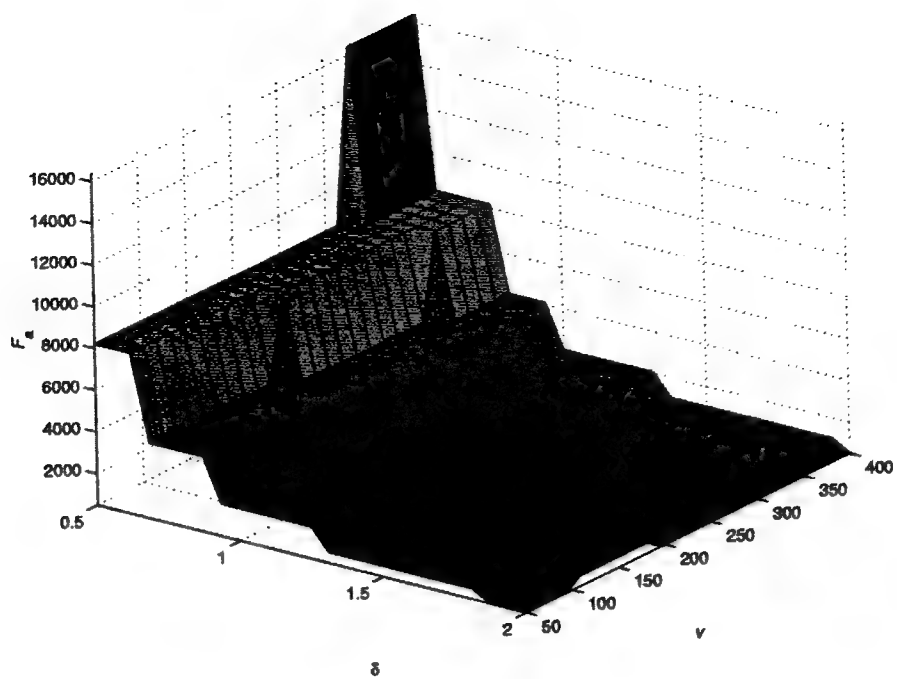


Fig. 6.3: FFT size in optimal mixed configuration.

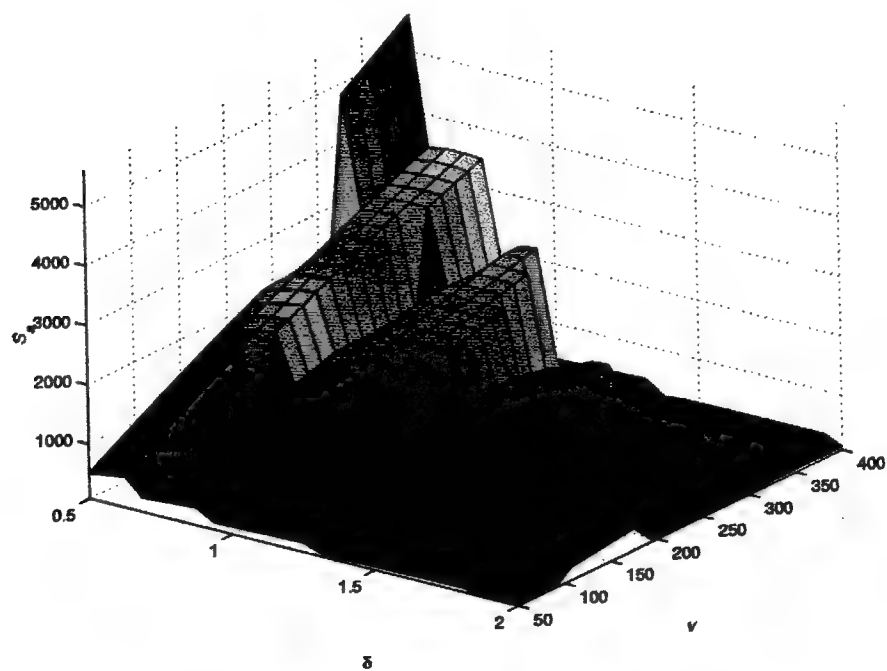


Fig. 6.4: Section size in optimal mixed configuration.

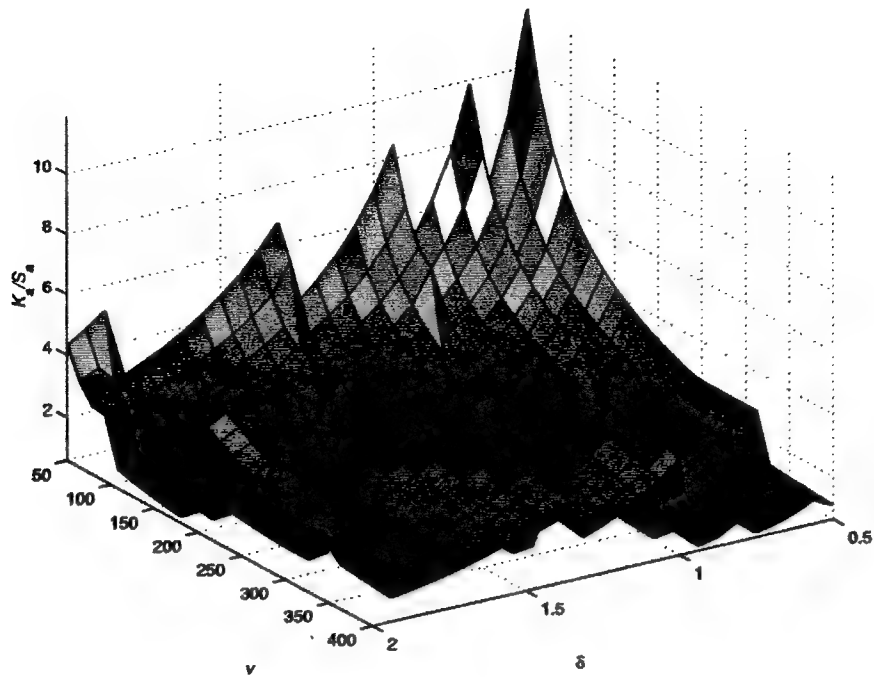


Fig. 6.5: Ratio of kernel size to section size in optimal mixed configuration.

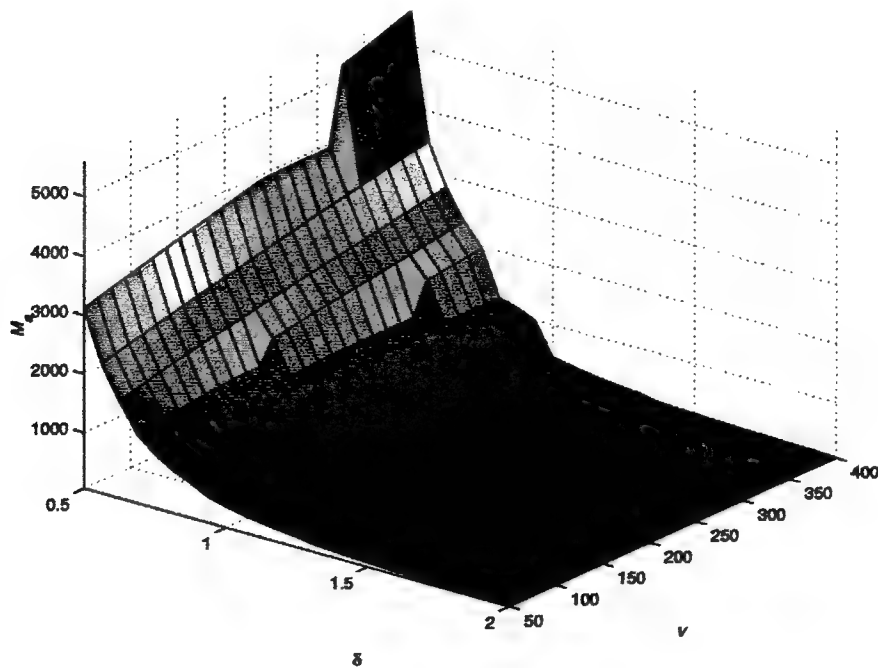


Fig. 6.6: Azimuth memory requirements in optimal mixed configuration.



area on the processor graph corresponds to the area on the configuration graph in which the memory-poor S2T16B is employed for both range and azimuth processing.

The ratios of azimuth memory and processors to range memory and processors, respectively, are shown in Figs. 6.8 and 6.9. Note that the discontinuities in the memory ratio graph is more accentuated in the CNCM than in the ISMM, while the undulations in the processor ratio graph are smoother (compare to Figs. 5.8 and 5.9). One explanation for this observation involves the optimization motivation to utilize all the available memory on a CN, combined with the restriction of using only the memory on a single CN. With the range processing and memory requirements the same for both models, only azimuth processing can be modified to take advantage of extra memory. As a result, the azimuth memory per processor tends to be more uniform. Fig. 6.10 illustrates this tendency, with a lone spike corresponding to the configuration in which the S1D64B was employed with one range and one azimuth processor. This uniformity in azimuth memory per processor entails greater discontinuities in the memory requirements, dependent on the FFT size and section size. The number of azimuth processors also becomes slightly more uniform because optimization of available memory on a card prevents extremely inefficient processing, as might be optimal in the ISMM.

Azimuth memory per processor is one of the most noticeable differences between the CNCM and the ISMM. In the ISMM, memory per processor was not a concern because memory is pooled. For the sake of comparison, the azimuth memory per processor for the ISMM is plotted here in Fig. 6.11. The familiar rolling effect is evident here, associated with the different FFT sizes. The surface shape of Fig. 6.11 is almost identical to that of the ratio of the section size to the FFT size, with the reversal of the velocity axis (see Fig. 5.12).

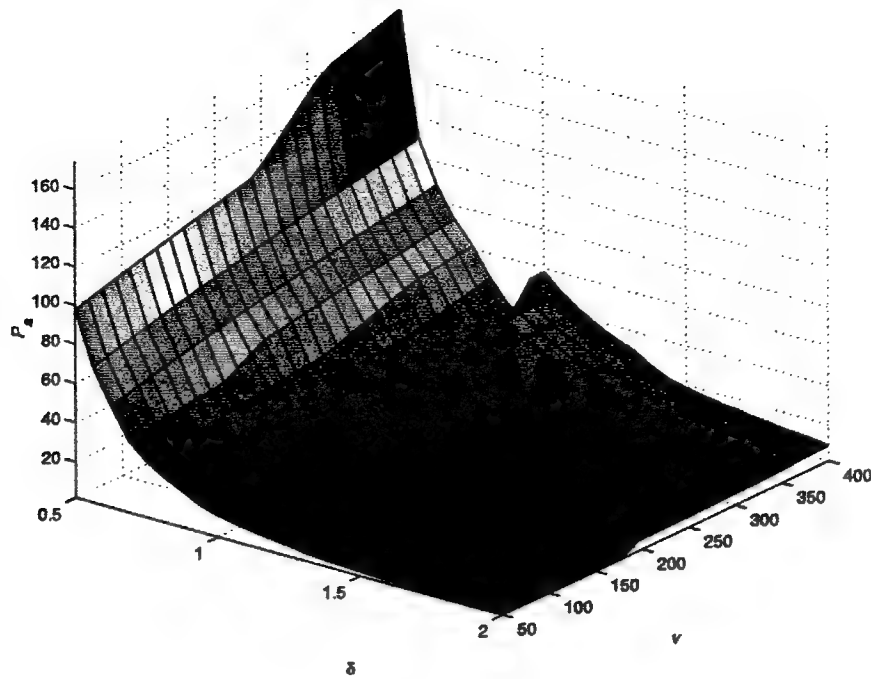


Fig. 6.7: Azimuth processor requirements in optimal mixed configuration.

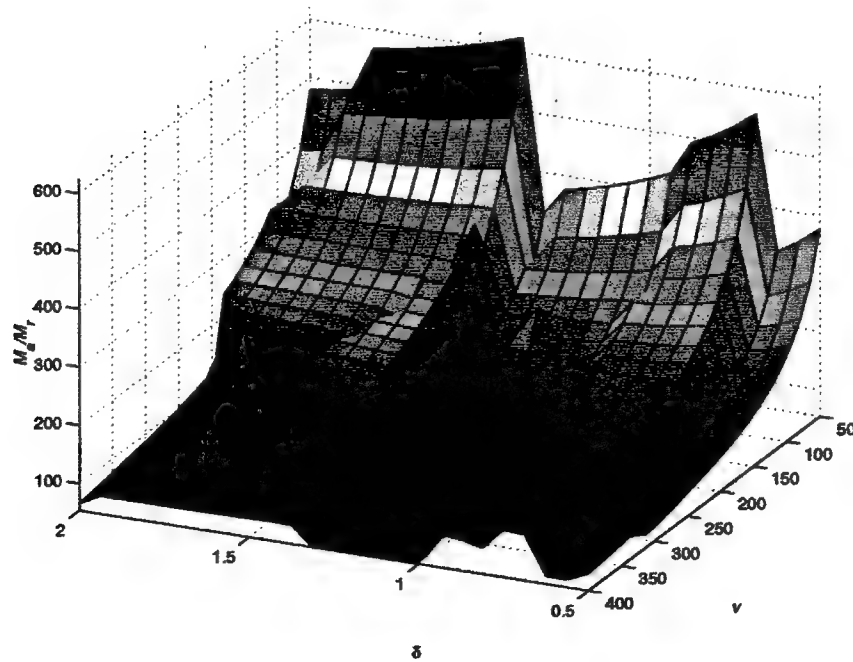


Fig. 6.8: Ratio of azimuth to range memory in optimal mixed configuration.

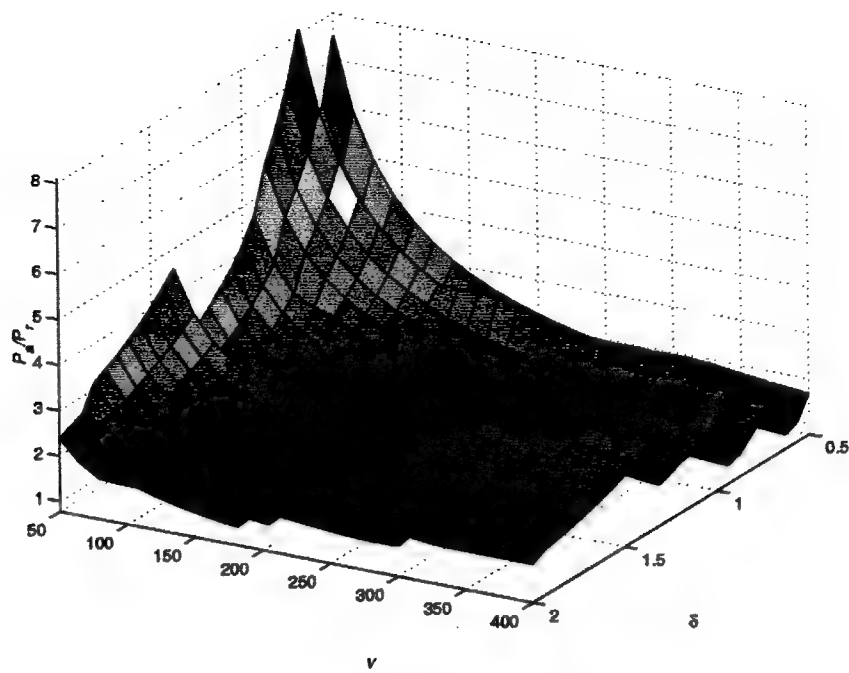


Fig. 6.9: Ratio of azimuth to range processors for optimal mixed configuration.

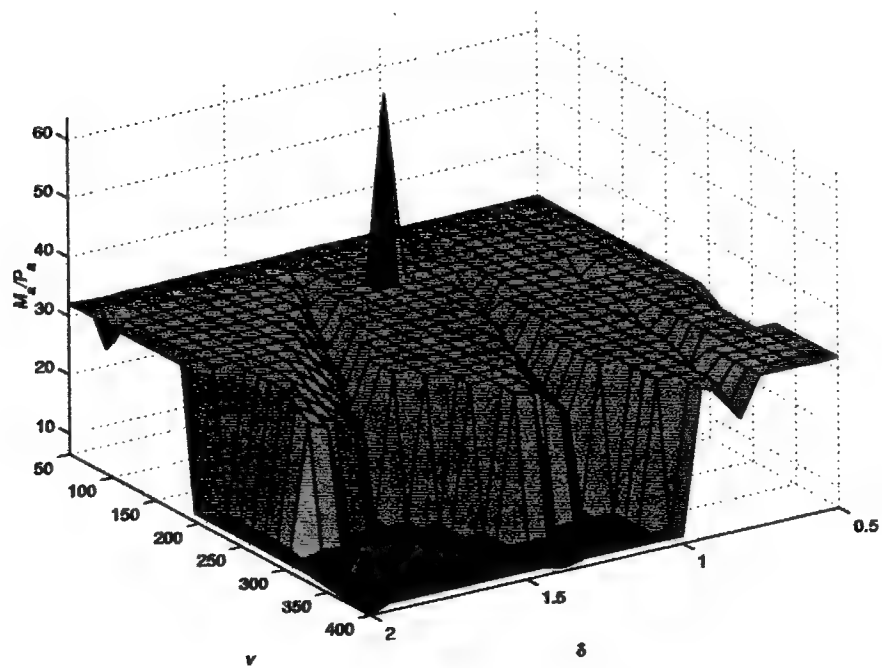


Fig. 6.10: Azimuth memory per processor for optimal mixed configuration in CNCM.

The other dramatic difference in corresponding optimization variables between the two models is the usage of the two types of daughtercards. As opposed to the undulating characteristic apparent in the ISMM (see Figs. 5.13 and 5.14), the CNCM favors the S1D64B, except in a small area of coarse resolution and high velocity, where the S2T16B is exclusively employed. Figs. 6.12 and 6.13 illustrate this trend. Because azimuth requirements dominate the system, homogeneous use of the S1D64B for azimuth processing results in this favoritism, although the S2T16B is homogeneously employed for range processing. In the corner of the graph where velocity is high and resolution coarse, reduced azimuth memory requirements allow implementation of azimuth processing as well on the S2T16B.

### 6.3.2 Nominal Mixed Configuration

As for the ISMM, comparison of the optimal configuration to the nominal configuration is used to measure the utility of optimization. However, unlike in the ISMM, in which the nominal configuration was merely a simplified and specific case of the optimal configuration, with the section size removed as an optimization variable and set equal to the kernel size, the nominal configuration in the CNCM calls for special attention and a slightly more complicated formulation for the nominal configuration to compete with the optimal. Both the simplistic approach, in which no special attention is given, and the more complicated approach will be examined. Because of this difference in formulation, the two approaches to the nominal configuration presented here will be denoted as the *naive* and *sophisticated* approaches. The general formulation of the nominal problem is the same as for the optimal configuration but with the aforementioned removal of  $S_a$  as an optimization variable.

The added complexity in the nominal configuration formulation results from

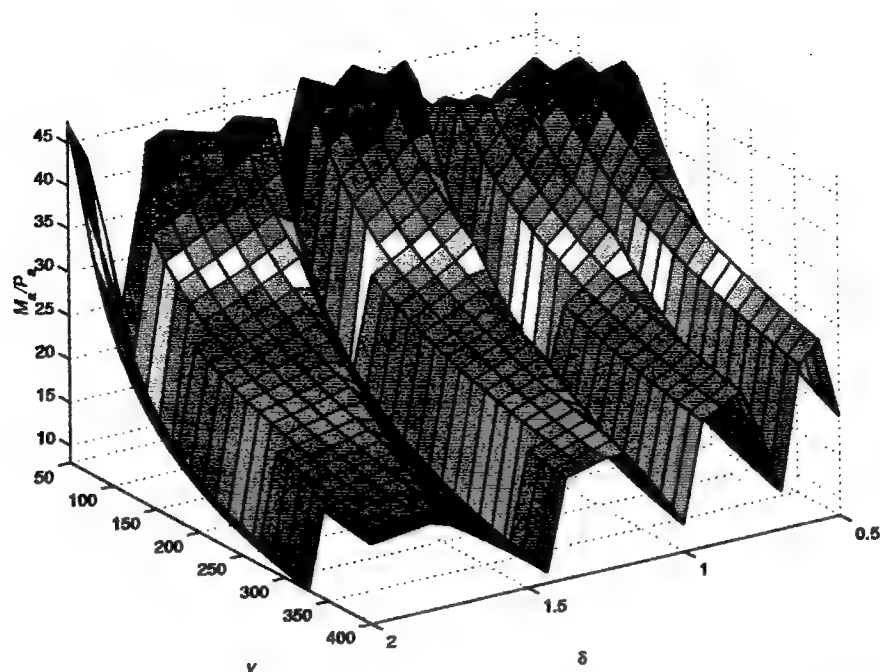


Fig. 6.11: Azimuth memory per processor for optimal mixed configuration in CNCM.

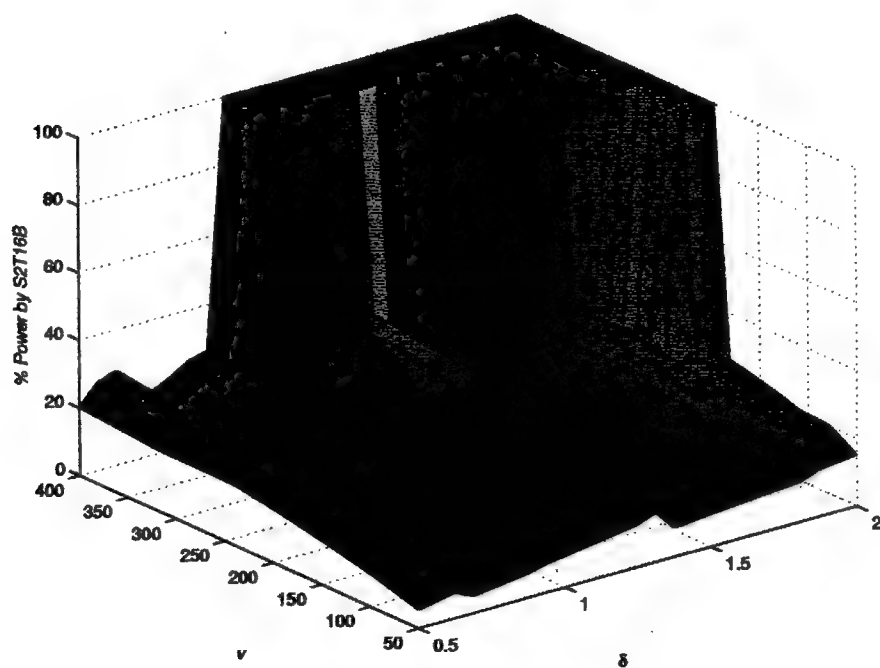


Fig. 6.12: Percentage power consumption by S2T16B for optimal mixed configuration.

the lack of optimizing the section size so as to correlate the employed processors with the available memory. Reliance by the present formulation on the memory per processor values to enforce the local memory access constraint (Eqns. 6.1 and 6.2) entails a serious problem for the nominal configuration. Because both azimuth processor and memory requirements are fixed, the ratio of memory to processors is also fixed. Although this fact alone does not prohibit optimization in most cases, there is a set of resolution and velocity pairs that do not permit a feasible solution regardless of how much power is allocated because the memory per azimuth processor exceeds 64 MB, the upper limit on memory per processor for the two daughtercards under consideration. As it might be surmised, in many cases even the solutions that are feasible are rather poor because there is such an inefficient use of memory. Fig. 6.14 displays the excessive power requirements of the nominal configuration, with approximately 15% of the area investigated infeasible.

This problem at first seems to be a reasonable and even expected penalty for not optimizing the section size, which has been seen to be so critical in the ISMM. However, inspection of Fig. 6.14 reveals a disturbing observation: the area of infeasibility does not occur at the highest performance corner of the graph but in the area at which resolution is fine and velocity is low. At increased velocities, solutions become feasible. This unintuitive result calls for a new formulation. Such a formulation still may not find feasible solutions in every case, but it should not produce feasible solutions by increasing the requirements.

Fig. 6.15 depicts the optimal card and processor assignments with the nominal section size. It is noted that only one azimuth processor is allocated on the S1D64B in the feasible solution area immediately outside the infeasible solution area. This assignment infers a shortage of memory, as is expected by the nom-

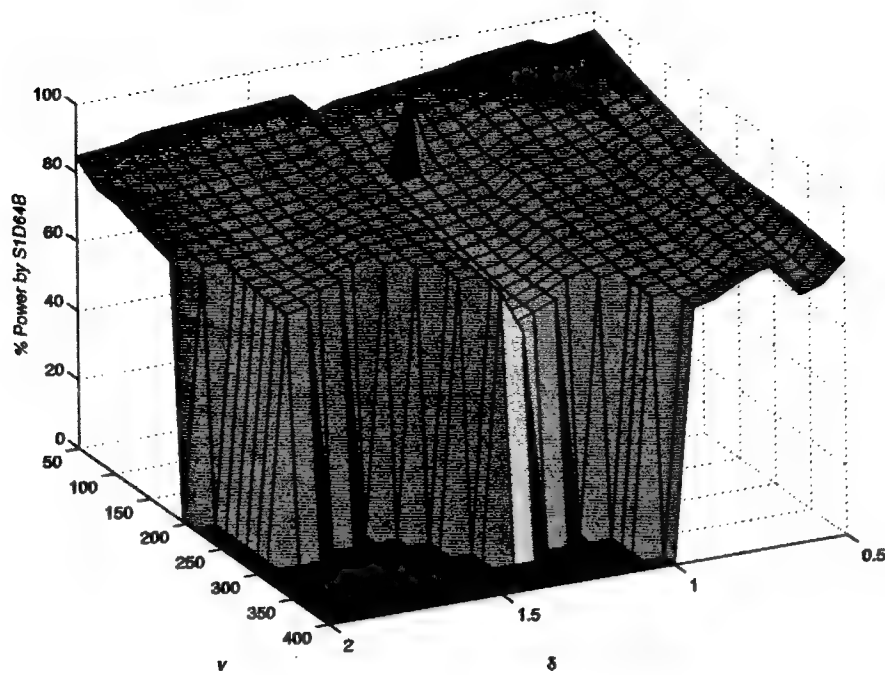


Fig. 6.13: Percentage power consumption by S1D64B for optimal mixed configuration.

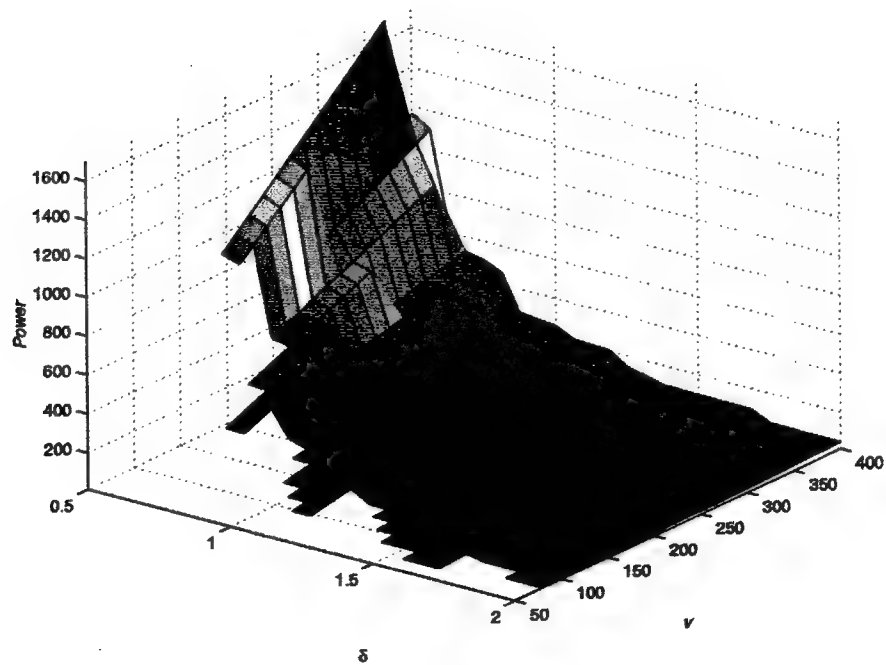


Fig. 6.14: Power consumption in naive approach to nominal mixed configuration.

inal configuration in which the kernel size is too large to act as a section size. Fig. 6.16 confirms this suspicion that the memory per processor is greater than 64 MB at the points at which no solutions were found.

The extreme memory per processor values in Fig. 6.16 result from a high memory requirement due to the fine resolution but yet a low processor requirement due to the low velocity. In the research presented thus far, it has been accurate to assume that 100% processor utilization occurs. In the ISMM, this point is not relevant because a high memory to processor ratio never explicitly causes infeasibility. In the optimal configuration of the CNCM, the section size is always optimized such that resources are not idle. (In tests conducted after the original data was collected for the previous subsection on the optimal configuration, 100% processor utilization was shown to be optimal for every resolution and velocity value investigated. However, it is plausible that for more extreme application parameter values, allowing less than 100% processor utilization might be necessary even with an optimized section size.)

To rectify an excessive memory to processor ratio, either the required memory must be decreased or the number of processors increased. Because the memory required is dependent on section size, which in the nominal case is fixed, memory requirements cannot be modified. However, the number of processors can be increased, thus reducing the individual processor utilization percentage.

One method to achieve feasible solutions in the nominal case is to calculate the actual number of processors  $P_{act}$  (as opposed to the required number of processors  $P_a$ , assuming 100% utilization) by the following equation:

$$P_{act} = \frac{P_a}{U_p}, \quad (6.9)$$

where  $U_p$  denotes the processor utilization as a ratio, and has a range (0,1]. However, the necessary  $U_p$  somehow still must be computed.



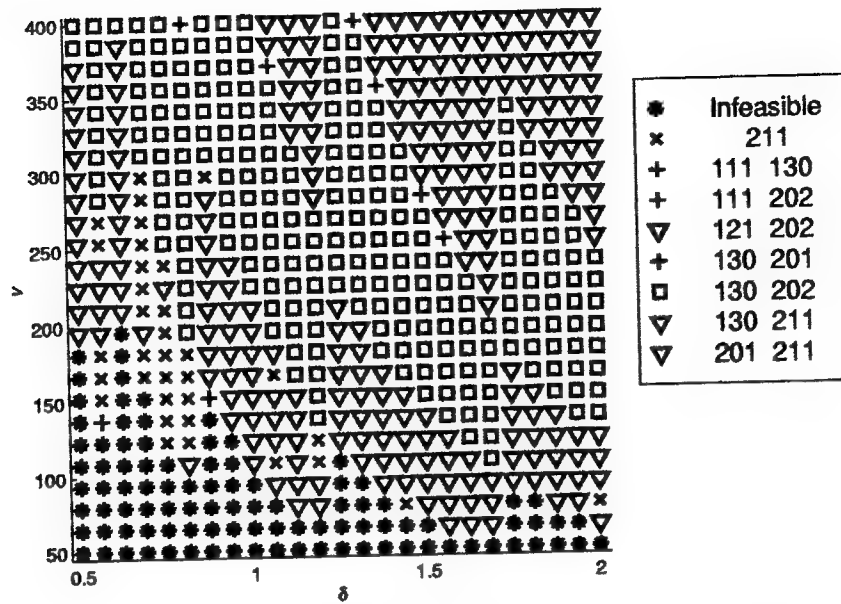


Fig. 6.15: Configurations in naive approach to nominal mixed configuration.

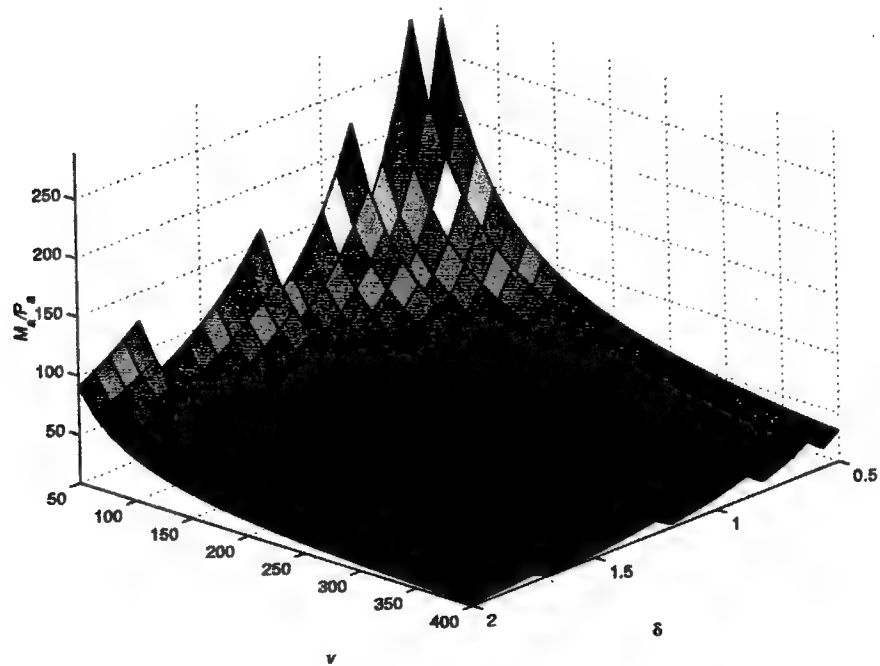


Fig. 6.16: Azimuth memory per processor in naive approach to nominal mixed configuration.

The most basic approach to calculating  $P_{act}$  is to consider the ratio of the required memory and the maximum available memory per CN. Let  $M_{max}$  represent this maximum available memory. If  $T$  is the target card type, then  $M_{max} = M_{CN}(T)$ . If the target card type is not previously determined, and the most versatile (in terms of feasibility) card type is desired, then the card type with the greatest amount of memory per CN should be employed. That is,  $M_{max} = \max\{M_{CN}(T) : T = 1, 2, \dots, N_d\}$ , where  $T$  represents the daughtercard types denoted by consecutive integers beginning with '1'. Thus, one solution to the actual number of processors is

$$P_{act} = \max \left\{ P_a, \frac{M_a}{M_{max}} \right\}. \quad (6.10)$$

With the two daughtercards under consideration,  $M_{max}$  is 64.

Eqn. 6.10 implies the following expression for  $U_p$ :

$$U_p = \min \left\{ 1.0, \frac{P_a M_{max}}{M_a} \right\}. \quad (6.11)$$

With this formulation, it is assured that if a feasible solution exists with the nominal section size, the lack of a large number of required processors will not prevent finding a solution. The solution found, however, may be poor since  $U_p$  is set to its maximum possible value. In cases where all the memory is dedicated to one processor on a card, regardless of how many processors are located on the card and how little memory range processors may require, lack of consideration of less than maximum values of  $U_p$  will often entail additional wasted resources on other cards.

To compare the nominal section size with the optimal section size without this added disadvantage of the nominal formulation, either  $U_p$  or  $P_{act}$  must be optimized. Instead of only two optimization variables in the nominal configu-

ration, a third variable is introduced into the formulation. Let  $P_{act}$  be the new optimization variable so that the formulation may be modified by replacing every occurrence of  $P_a$  in the constraints with  $P_{act}$  and setting  $P_a$  as a lower bound for  $P_{act}$ .

The power requirement results of such a formulation are shown in Fig. 6.17. Notice that they more closely emulate the relationship between the optimal and nominal configurations in the ISMM, with the nominal requiring approximately 30% more power. Furthermore, solutions exist for the entire range of resolution and velocity values.

The configuration graph for the sophisticated nominal model is presented in Fig. 6.18. Comparing this graph to that of the naive approach, it is observed that where previously there were infeasible solutions and solutions that dedicated the entire S1D64B to one azimuth processor, now the S1D64B is used to accommodate one azimuth processor and one range processor. This heterogeneous use of the CN greatly reduces the overall power consumption because a range processor can be assigned to a CN on the S1D64B essentially for "free," because the ratio of azimuth memory to range memory is so high in these areas.

It is obvious that the freedom granted to the formulation to add processors above the strict requirement as computed from the equation for  $P_a$  (see Eqn. 3.3) improves the calculated power consumption of a system. There is a definite penalty paid for this improvement, however, and the penalty is in terms of the extra processors added to reduce the processor utilization and thus the memory per processor. Figs. 6.19 and 6.20 show the number of added processors (i.e.,  $P_{act} - P_a$ ) and  $U_p$ , respectively. Note that as the performance requirements of the system decrease, the utilization also increases because the memory-per-processor constraint becomes inactive.

Comparing the two approaches, the ratio of the power requirements of the

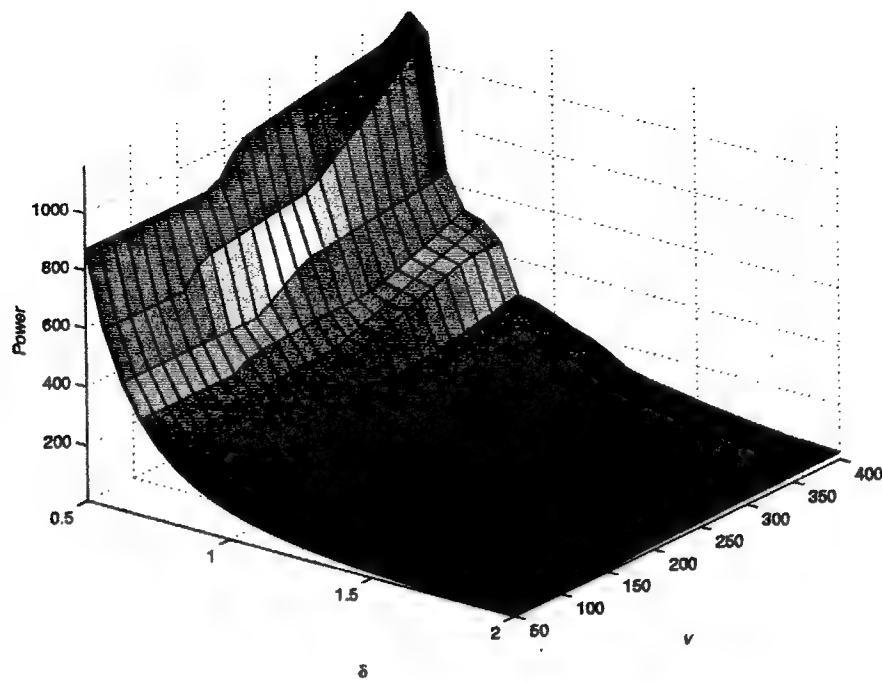


Fig. 6.17: Power requirements for the sophisticated approach to the nominal mixed configuration.

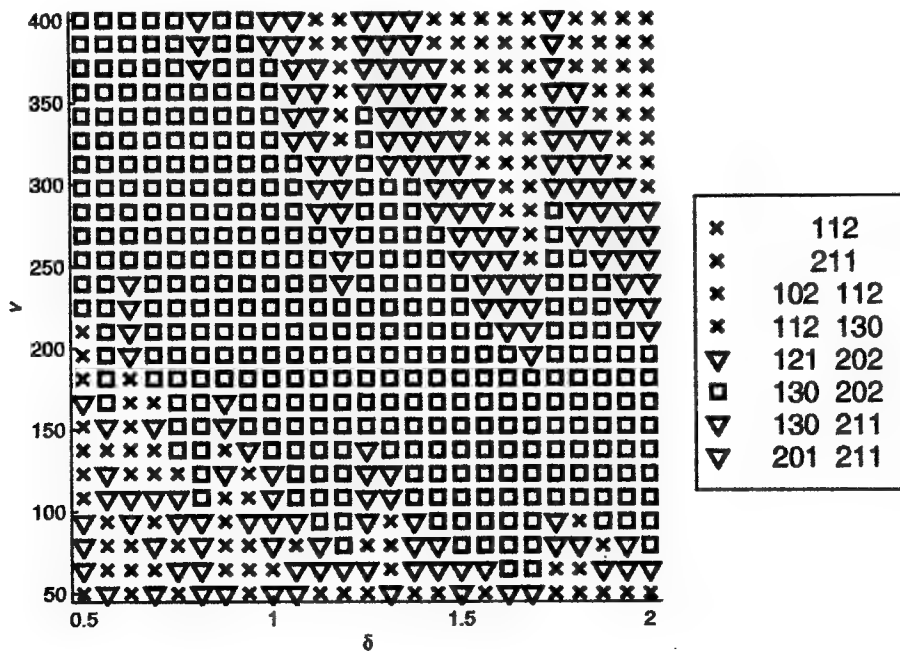


Fig. 6.18: Configurations for the sophisticated approach to the nominal mixed configuration.

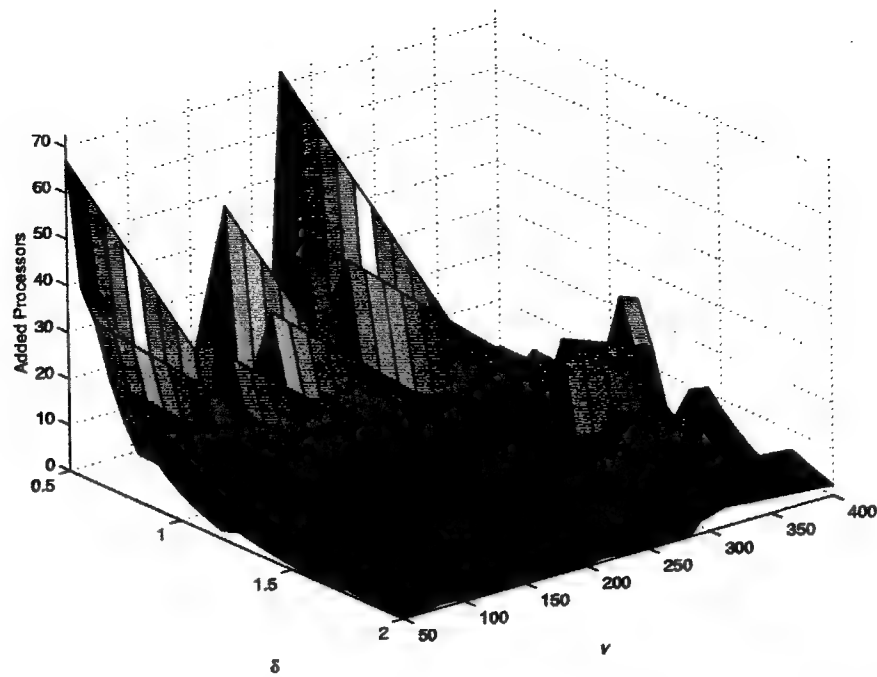


Fig. 6.19: Added processors in the sophisticated approach to the nominal mixed configuration.

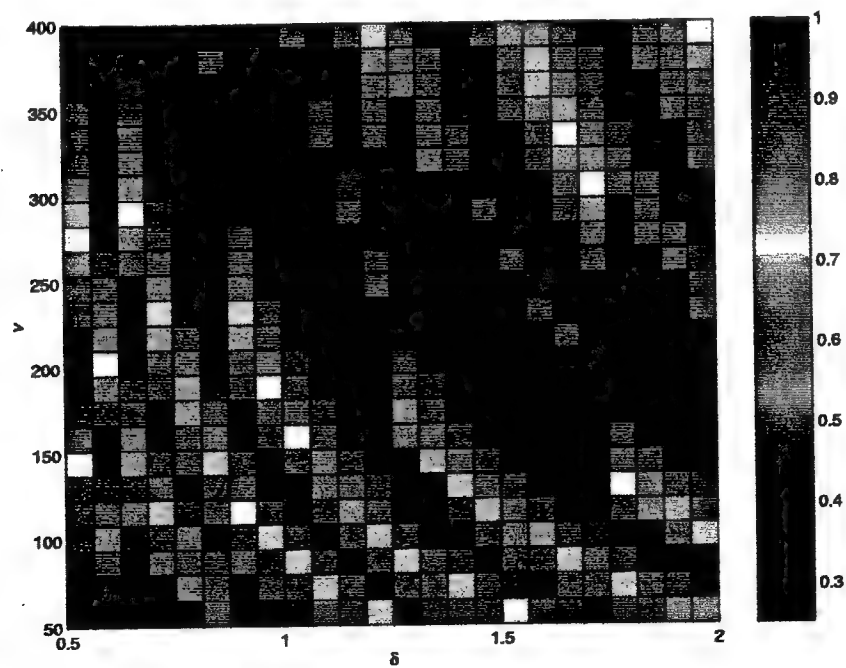


Fig. 6.20: Utilization in the sophisticated approach to the nominal mixed configuration.

naive to the sophisticated approach is plotted in Fig. 6.21. Note that the area which is infeasible in the naive approach cannot be compared. The maximum ratio is 1.67 and occurs at the resolution-velocity coordinates  $\{0.75 \text{ m}, 239.6 \text{ m/s}\}$ . The minimum ratio is 1.00, occurring at the coordinates  $\{0.94 \text{ m}, 137.5 \text{ m/s}\}$  and approaching unity in many places. The mean ratio over the graph of mutually feasible areas is 1.30. Note that this figure would be higher if the naive approach found solutions in the infeasible area because its performance in this area would be worst.

### 6.3.3 Comparison of Optimal and Nominal Configurations

Comparisons will be made of the optimal configuration with both the naive and sophisticated nominal configurations. It is expected that the sophisticated nominal configuration more closely resembles the nominal configuration in the ISMM in relation to the optimal configuration. Whether it is reasonable to use a sophisticated optimization approach to the nominal case, possibly violating the term 'nominal', is an issue that will be discussed in the conclusions. In this subsection only the power requirements are briefly considered.

The weakness in the naive approach to the nominal configuration is in its propensity to infeasible or extreme solutions. Fig. 6.22 displays the ratio of the naive nominal power requirement to the optimal power. The plot shows a maximum ratio of 2.15 occurring at the coordinates  $\{0.5 \text{ m}, 50 \text{ m/s}\}$ , and a minimum ratio of 1.08 occurring at  $\{0.63 \text{ m}, 195.8 \text{ m/s}\}$ . The mean ratio over all the area in which the naive approach found a solution is 1.30.

The sophisticated approach to the nominal configuration (see Fig. 6.23) avoids the extreme ratios present in the naive approach by reducing the maximum ratio to 1.76. This figure is still high but closely reflects the values computed in the ISMM. Furthermore, it must be considered that this maximum

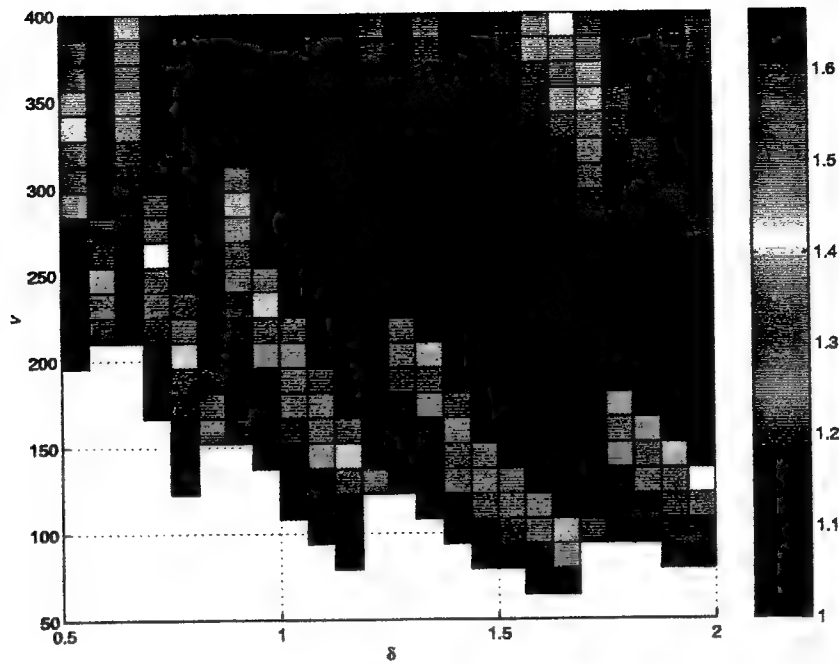


Fig. 6.21: Ratio of the power requirements of the naive to the sophisticated approach to the nominal mixed configuration.

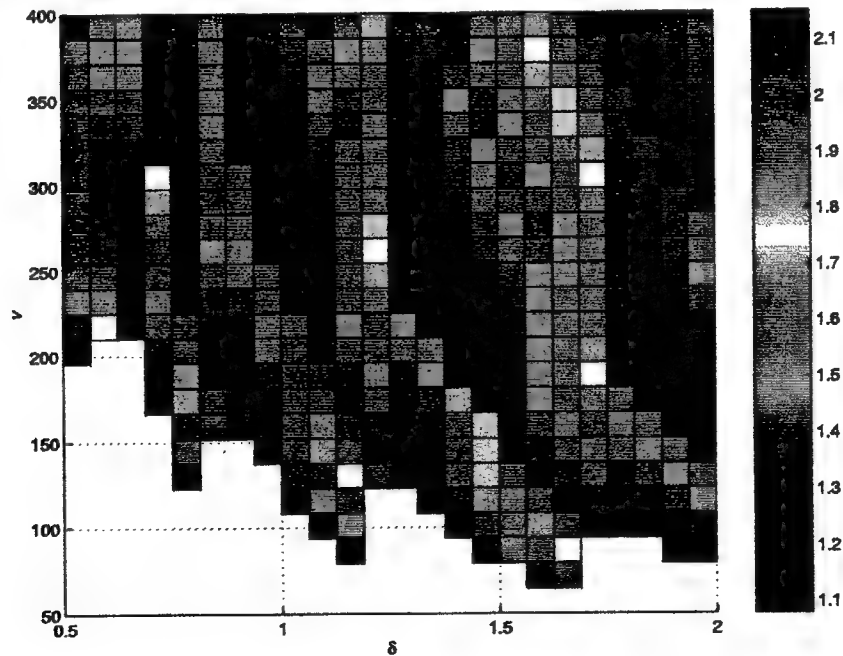


Fig. 6.22: Ratio of the optimal power requirements to those of the naive approach to the nominal mixed configuration.

value occurs at the coordinates {1.69 m, 400 m/s}, which is in the infeasible region of the naive approach. The minimum ratio is approximately the same as in the naive approach, with a value of 1.08 at {0.94 m, 254.2 m/s}. The mean ratio of 1.33, however, is worse than in the naive approach. This aberration is easily explained, though, by taking into consideration the infeasible area of the naive approach that is not calculated in that average, but is calculated in the average of the sophisticated approach. The two approaches cannot meaningfully be compared with these statistics then because of the infeasible region of the naive approach.

#### 6.3.4 Effects of Integer Numbers of Cards

Taking the realism of the formulation a final step farther, the effects of discretizing the number of cards  $N_I$  is explored. Up to this point, real values for the number of daughtercards have been allowed to more accurately view the effects of individual variables and variable relationships. Especially when the total number of cards required is low, forcing the number of cards to be integers greatly alters the overall results. However, to completely specify a system, it must be accepted that only integer cards are installed, even if resources are wasted to some degree.

It is assumed that the optimization routine developed in this chapter is still employed up to the point of determining the best solution out of all the configurations evaluated. Pure integer programming would provide an absolutely optimal solution to the problem, but such an optimization is infeasible in terms of time for most scenarios considered. Such a method could be applied if it is known that the total number of cards required, whatever the exact configuration, is very low. In this case an exhaustive search and optimization of all feasible integer solutions could be applied. This approach is not viable, however, for the



vast number of scenarios that this research purposes to address. Therefore, the following discussion is premised on an available solution involving real values for the number of cards, the raw results of which were presented in the previous two subsections.

One approach to discretization is simply to round the numbers after optimization. Although this approach yields a good average approximation to the actual requirements, rounding off promises neither to be optimal nor even feasible. Rounding off is safe and probably optimal when card number values are near the next integer. In this case, the ceiling of a value is taken. However, in the case that rounding off calls for truncating the decimal portion of a number (i.e., rounding down), there exists the possibility of cutting resources below the required levels. Rounding down is never possible in the case of a single card type configuration or in the case in which both card number values are to be rounded down in a two-card-type (purely homogeneous) configuration.

Rounding up of values is always safe, i.e., permits a feasible value, but does not promise optimality, even in the integer sense. There exists the possibility that in a homogenous configuration the floor can be taken of one of the card number values. If this state is true, a savings of the power consumed by one daughtercard is entailed for the overall power requirement. It is therefore useful to check for this possibility, especially in systems in which the total number of cards required is low.

Disparity in the characteristics of daughtercards, as is true with the two cards under investigation in this research, often causes the probability to be low that one card can be rounded up and the other down. Greater differences in the resources supplied by each card imply lower probabilities that the rounding up of one card can supply what is sacrificed by the rounding down of the other.

To ameliorate this problem, the card number values are fixed at the proposed

rounding values and the problem is then reoptimized. The more appropriate term for this operation is *solving the problem with constraints*. Note that with the card configuration variables set (it is assumed that the processor assignments  $I_r$  and  $I_a$  remain constant) then the only remaining optimization variable is the section size  $S_a$ . The operation does not need to optimize  $S_a$  because it is insignificant whether all or part of the resources on all cards is used. Determining the absolute feasibility of the system with the fixed resources is the only goal sought with this operation.

For each real solution returned by the initial optimization routine, two permutations of rounding the card number values must be evaluated. Assuming a purely homogeneous solution with two card configuration types involved, the permutations that must be considered are the ceiling of one and the floor of the other, or the floor of the first and the ceiling of the second. It is not necessary to evaluate the floor of both card number values because this clearly violates the required resources. Similarly, neither is it necessary to evaluate the ceiling of both card number values because in this case the requirements are clearly met. Nevertheless, the addition of the two permutations that must be evaluated effectively triples the number of total optimizations that must be performed, although these two optimizations converge quickly because there is only one optimization variable.

An additional consideration in the discretization of the number of required daughtercards is the number of CNs per card. Because the optimization routine developed in this chapter returns solutions in terms of CNs and not daughtercards, the number of CNs must be converted to daughtercards, rounded, and then converted back to CNs for the reoptimization (or solving) process. The equivalent technique implemented to collect the results presented consists of rounding the number of CNs to a multiple of the number of CNs per that type

of card. In the case of the S2T16B, values are taken to the nearest multiple of two. In the case of the S1D64B, the floor or ceiling of the number of CNs constitute the same integer value in terms of daughtercards.

With all the considerations mentioned and the discussed technique applied, one card number value allowed taking its floor in approximately 15% of the solutions produced for the optimal mixed configuration in this model. The other 85% of solutions required taking the ceilings of both card number values.

Fig. 6.24 illustrates the effect on power consumption of forcing the number of cards to be integers. As expected, there is no significant effect on the overall graph (compare to Fig. 6.1), especially at the highest performance. Lower performance scenarios in which relatively few cards are present in the system suffer a more dramatic effect. Fig. 6.25 shows the ratio of power consumed by this integer version of the solutions to the real-valued requirements.

A more noticeable effect of the discretization is observed in the graph of the configuration variables (Fig. 6.26). Although the general trends remain unchanged from the original solution (Fig. 6.2), other configurations are found to be optimal where several configurations produce close power requirements in the real-valued solutions. Discretization of the values often changes the initial optimal configuration.

In addition, the intrinsic optimality of some configurations seems to be doubtful. In particular, the configuration '112 201' appears inherently suboptimal. It appears illogical that two azimuth processors can share the 16 MB of memory on a S2T16B, but then only one azimuth processor is assigned to use the 64 MB of memory on the S1D64B. However, it must be noted that only one such card of the '201' type is employed. The optimization routine originally configured the system to employ only the '112' type and had a fractional processor left over on a card upon discretization. The routine therefore determined that one S1D64B

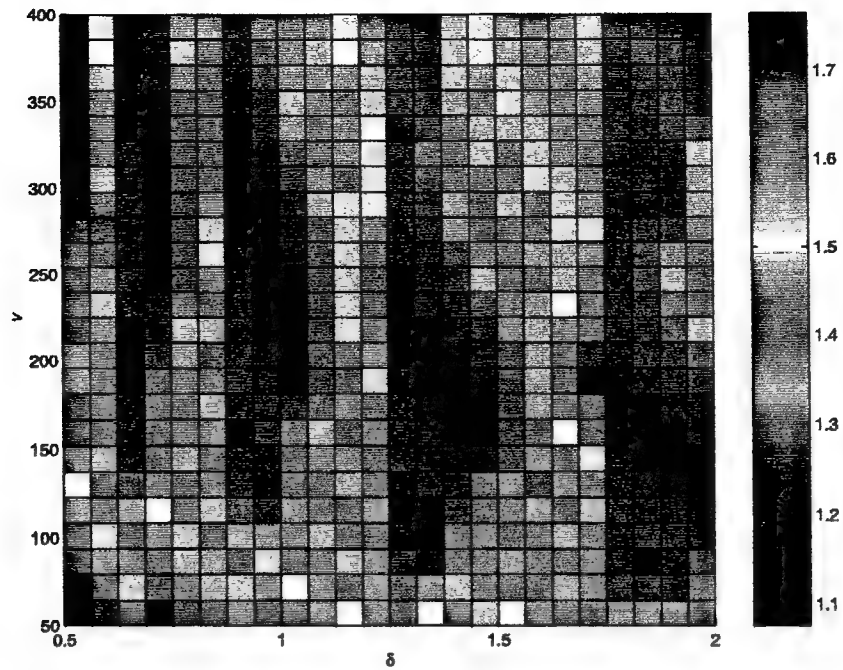


Fig. 6.23: Ratio of the optimal power requirement to those of the sophisticated approach to the nominal mixed configuration.

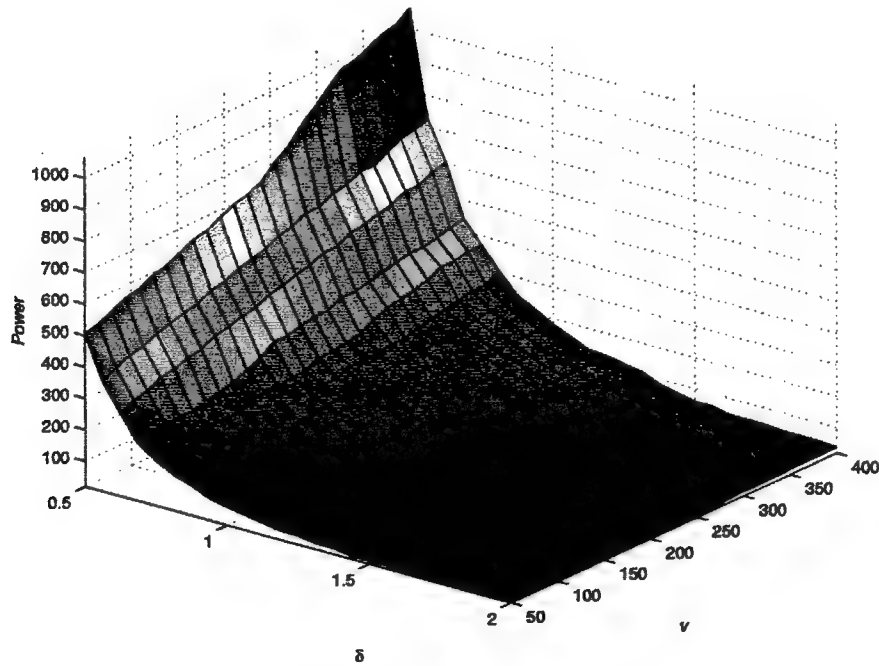


Fig. 6.24: Power requirements of the discrete card number solution to the optimal mixed configuration.

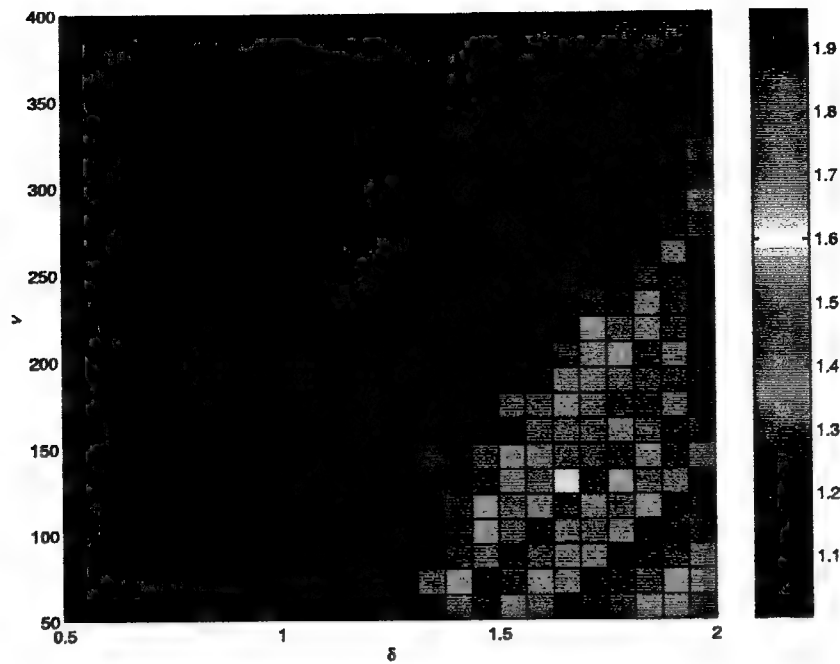


Fig. 6.25: Ratio of power by discrete card number solution to real-valued solution in optimal mixed configuration.

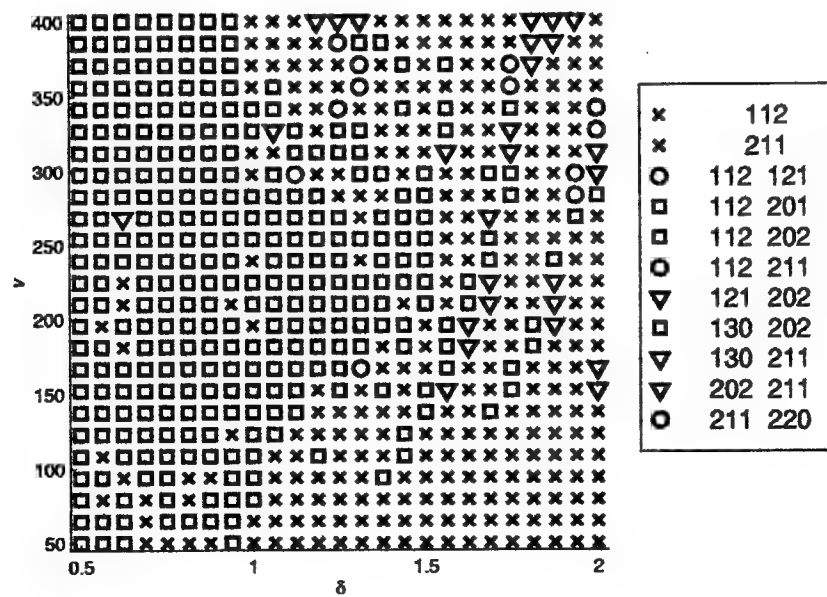


Fig. 6.26: Configurations of discrete card number solutions for the optimal mixed configuration.

consumed less power than one S2T16B, recalling that the discretization is at the daughtercard level and not at the CN level. In terms of CNs, the S1D64B consumes more power per CN than does the S2T16B. The optimization routine also could have determined that this one card should be of the '202' type without affecting any other system values.

### 6.3.5 Comparison of CNCM and ISMM

The approximation of the ISMM to the CNCM is evaluated below. If the ISMM can serve as a method of approximating the more realistic CNCM, a relatively quick approximation can be ascertained for a system before formulating the more rigorous and precise optimization. Focus is simply on the power requirements because other values have little significance in this context outside of the actual system design. That is, there probably will not be a need to accurately approximate, for instance, the section size, if the card configuration data is not yet known.

The graphs of the power requirements for both the ISMM and the CNCM for the optimal mixed configurations have been given in Figs. 5.4 and 6.1, respectively. It is observed that the general shape of the graphs is the same, although the scaling is higher for the CNCM as would be expected. Fig. 6.27 presents the ratio of the CNCM power requirements to that of the ISMM.

Peaks in the ratio graph occur along FFT size discontinuities, as is present in other graphs of the same nature. In the ISMM, azimuth memory per processor is allowed to reach values that are physically unrealizable on the available daughtercards. Hence, the CNCM explicitly limits the memory per processor and suffers from this lack of optimization that is available to the ISMM.

Overall, the approximation is deemed to be good. Statistics of the ratio graph are as follows: a maximum value of 1.31, minimum value of 1.002, mean value of

1.11, and standard deviation of 0.07. It is not feasible to add a constant to the ISMM solution to obtain a better approximation because the minimum value is so close to unity. Because the ISMM solutions by definition never exceed the CNCM solutions, the ISMM can be used as a lower bound approximation to the realistic CNCM. If a lower bound is not necessary, but rather a more accurate mean value, the solution from the ISMM can be multiplied by the mean ratio value 1.11, keeping the standard deviation in mind.

#### 6.4 Conclusions

The CNCM involves a more sophisticated optimization formulation than does the ISMM presented in Chapter V. With this sophistication comes increased computational intensity, one to two orders of magnitude greater than in the ISMM. Benefits of the CNCM include a high degree of fidelity to plausible system realizations and the provision of information lacking in the ISMM to completely specify a system configuration.

Although not investigated in this work, the CNCM can be applied to other optimization objectives such as velocity maximization and resolution minimization for fixed power. Application to other optimization objectives entails obvious modifications to the formulation, as in the ISMM, as well as consideration of special cases that affect the upper bound on or mean number of configuration combinations to be evaluated. This same principle equally applies for other hardware-constrained configurations such as the single card type configuration investigated for the ISMM.

Comparison of the optimal with the nominal configuration in the CNCM presents additional problems. Unlike in the ISMM, the most simplistic approach to the nominal formulation results in an area in which no feasible solutions are possible. This fact alone is not of concern, but when it is taken into consider-

ation that increasing velocity and thus the overall requirements of the system facilitate feasible solutions, the problem calls for more attention. The problem is discovered to result from a high memory requirement and low processor requirement, thus creating an unrealizably high memory per processor value. The problem is rectified by the introduction of additional processors, thus simultaneously reducing processor utilization and memory per processor.

The added complication of the nominal formulation, resulting in three optimization variables just as in the optimal formulation, begs the question whether there is any advantage in employing the nominal section size in the CNCM. The extreme values produced by the nominal configuration probably lead to a negative response to that question. However, the primary reason for including the nominal configuration in this chapter is not to provide an alternative and clearly inferior method of determining a system configuration, but to serve as a point of reference to the advantage of optimizing the section size, which is a fundamental basis of this work.

The utility of the ISMM solution as an approximation to power requirements as determined by the more realistic CNCM has been demonstrated. A mean ratio of 1.11 (CNCM power to ISMM power) and a standard deviation of 0.07 presents the ISMM as a good approximator to the CNCM.

Although not tested in this work, a possible improvement to the ISMM, if used to approximate, is to incorporate into the formulation the same optimization variable that was created for the nominal configuration in this chapter. If the required processors  $P_a$  was used as a lower bound to the number of actual processors  $P_{act}$ , where  $P_{act}$  is an additional optimization variable, the peaks in the ratio graph might be reduced. Although the addition of an optimization variable entails greater computational complexity, overall computation time should not be greatly affected relative to the computation time of the CNCM because



of the lack of integer programming.

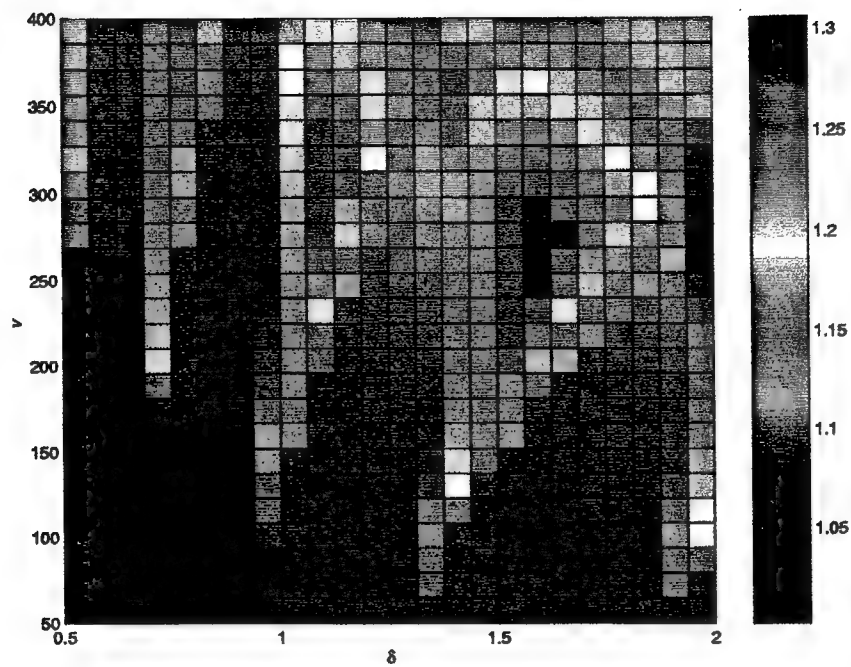


Fig. 6.27: Ratio of power requirements for the CNCM to the ISMM for the optimal mixed configuration.

## CHAPTER VII

### RANDOMLY-GENERATED SOLUTIONS

In this chapter the use of randomly-generated solutions is introduced to verify the results obtained in the previous two chapters. The method is applied only to the power minimization objective of the optimal mixed configuration of the ISMM, this case being the fundamental and most general configuration investigated. After application of this method of verification, the possibility of extending the use of this method as a primary means of optimization is also explored.

#### 7.1 Solution Verification

Although it is impossible to completely verify the optimality of a given solution set without proving the convexity of the solution space, the random solutions provide a means to greatly increase the confidence invested in a solution. Random numbers are generated to form a large number of random solutions, the best of which is compared to the solution provided by the formal optimization formulation developed in Chapter V. If any random solution is superior to one obtained by the optimization formulation, implementation of the formulation cannot claim to produce truly optimal solutions. Consequently, in this case the formulation (inferring also the convexity of the problem) and/or MATLAB routine is suspect.

To produce random solutions, several optimization variable values are randomly generated and values for the remaining variables solved based on the injected random values. Given a resolution, velocity, and proposed optimal power requirement, random values are generated for the section size  $S_a$  and number of S2T16Bs,  $C_1$ .

The range for the section size is set at  $[1, 2^{15} = 32768]$ , encompassing values both much greater than and less than any found to be optimal in results collected. Such a great range increases the probability of poor solutions when a random value falls at one end of the range, but this statement presupposes the optimality of the proposed result. In random testing this assumption cannot be made. The FFT size is then calculated in the standard method based on the value of the generated  $S_a$  and  $K_a$ , which is dependent only on resolution.

$C_1$  is generated with a range that is dependent on the proposed optimal power. Although this dependency initially appears to corrupt the impartiality of the testing, it actually serves to ensure that every solution has at least a theoretical chance of being optimal. Let  $\Pi_{\text{prop}}$  be the proposed optimal power. Then an upper bound is imposed on  $C_1$  equal to the maximum number of S2T16Bs supported by a power value of  $\Pi_{\text{prop}}$ . That is, if  $C_{\text{max}}$  designates this upper bound on  $C_1$ , then

$$\begin{aligned} C_{\text{max}} &= \frac{\Pi_{\text{prop}}}{\Pi_d(\text{S2T16B})} \\ &= \frac{\Pi_{\text{prop}}}{12.2}, \end{aligned}$$

where  $\Pi_d$  is the power requirement per daughtercard expressed as a function of the daughtercard type. Although this restriction does not necessarily improve the mean solution value of the random test, it does insure that at the point of generation of  $C_1$ , the solution is not already inferior to the proposed optimal solution.

With values for  $S_a$  and  $F_a$ , values for  $P_r$ ,  $P_a$ ,  $M_r$  and  $M_a$  can be calculated as usual based on Eqns. 3.2, 3.3, 3.5, and 3.6.  $C_2$  is then calculated by heeding the more demanding of processor and memory requirements in terms of daugh-

tercards. That is,

$$C_2 = \max \left\{ \frac{(P_r + P_a) - \pi_d(\text{S2T16B})C_1}{\pi_d(\text{S1D64B})}, \frac{(M_r + M_a) - M_d(\text{S2T16B})C_1}{M_d(\text{S1D64B})} \right\}$$

$$= \max \left\{ \frac{(P_r + P_a) - 6C_1}{2}, \frac{(M_r + M_a) - 32C_1}{64} \right\},$$

where  $\pi_d$  and  $M_d$  are the number of processors and amount of memory per daughtercard, respectively, as functions of the daughtercard type.

With the randomly generated and calculated values above, the power requirement can be computed, as previously, as a function of  $C_1$  and  $C_2$  and their respective power requirements per daughtercard (Power =  $C_1\Pi_d(\text{S2T16B}) + C_2\Pi_d(\text{S1D64B})$ ). It is clear that any given power calculation based on the random values has a very low probability of being even close to a proposed optimal solution. However, with a very large sampling of random solutions, the probability increases that if there exists a better solution than the proposed optimal power, then random testing will discover it.

In this work, MATLAB's **rand** function is used to generate random numbers. The function produces a vector of uniformly distributed floats in the interval  $[2^{-53}, 1 - 2^{-53}]$ , with a theoretical nonrepeating period of  $2^{1492}$ .

The method described above was applied to the resolution and velocity values of 0.88 m and 297.9 m/s, respectively. The proposed optimal solution was 183.31 w. Table 7.1 displays the results of random testing taken over 1000000 samples. Fig. 7.1 shows the histogram of the tallied results using 100 bins. The spike at the extreme right of the graph represents the sum of the frequencies of values greater than can be displayed on the graph range.

The random tests lend a large degree of confidence to the proposed optimal solution. Considering Table 7.1, it is apparent that the best solution occurs when all parameter values are near the proposed optimal values. It is noted that

Table 7.1: Summary of random solutions.

Case	Power	% Inc.	$F_a$	$K_a$	$S_a$	$C_1$	$C_2$
Optimal	183.31	–	4096	1960	1491	5.43	12.19
Random Best	183.36	0.024	4096	1960	1455	5.63	11.94
Random Worst	94536	51471	2048	1960	1.0	0.477	9847
Random Average	839.3	357.9	–	–	–	–	–

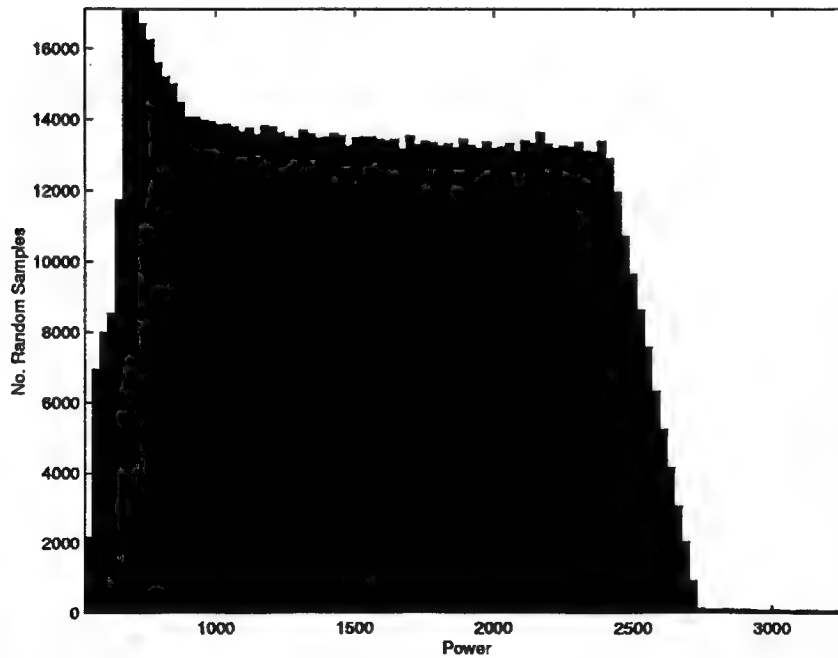


Fig. 7.1: Histogram of results from random testing on minimal power.

random testing did not produce any solution better than the proposed solution, although it approached the proposed solution value and for all practical purposes found a solution of equal quality, differing by only 0.024%. With one million random tests, confidence is lent to the validity of the testing method because a very good solution was indeed found.

Considering the worst case and mean of the random testing, the quality of both the proposed solution and that produced by random testing is placed in a certain perspective. Haphazard assignments of parameter values can be catastrophic.

Although the random testing has been conducted only on one set of resolution and velocity coordinates, the results are representative across the entire range of coordinates. This fact is demonstrated in the next section from a slightly different perspective.

## 7.2 Random Solutions as an Optimization Technique

In the previous section, random solutions are used to sample the solution space to evaluate a proposed optimal solution. This idea leads to the implementation of random sampling as a method of optimization in itself, without the premise of a proposed solution produced by a formal optimization routine.

The only necessary modification to the problem formulation as described in the previous section involves the absence of a proposed solution. Without this value to set the power range for  $C_1$ , another value must be substituted in its place. If historical data is available to provide an upper bound, this data of course could be utilized. However, in the absence of such data, a liberal upper bound for power must be injected into the problem. Even a poor estimate, as long as it is high enough, allows the possibility of finding the optimal solution. The greater the estimate is above the true optimal power, however, the greater

the time the random sampling will take to converge to a solution approaching the optimal.

For each case investigated in this section, the power range for  $C_1$  was set at  $[0, 1200]$  w. It is not completely fair to call 1200 w a "liberal" upper bound since knowledge of the solution surface presented in Fig. 5.4 suggests an upper power value of 868 w. Nevertheless, the mean power consumption of the graph is only 135 w. In practice, optimal values for power are sought for precise resolution and velocity coordinates more often than are power surfaces as has been usually presented in this work. Thus in the highest performance area, an upper bound of 1200 w (and it must be remembered that this upper bound is just for one card with no bounds on the other) is less than 50% higher than the optimal value, which might be considered a conservative and rather accurate 'estimate'. However, for the majority of the resolution and velocity coordinates, 1200 w is a very poor and very liberal power estimate. Furthermore, an auxiliary goal of this exercise is to test all the points of the proposed optimal power surface, and too high a power range would reduce the probability of finding better solutions if they exist.

Random sampling of the solution space was done over all the coordinates evaluated by the optimization methods for power minimization as in the previous two chapters. The range of the random section size was handled slightly different than in the previous section. The range of  $S_a$  was modified as  $[1, F_a - K_a]$ , where  $K_a$  is dependent on the resolution and  $F_a$  is set at  $2^{16} = 65536$ . This modification increases the range of  $S_a$  slightly without increasing the upper bound of  $F_a$ , entailing more 'intelligent'  $S_a$  guesses at high values. Figs. 7.2-7.5 show the results of random sampling for sample sizes per coordinate pair of 100, 1000, 10000, and 100000 respectively. Tables 7.2 and 7.3 display the statistical results.



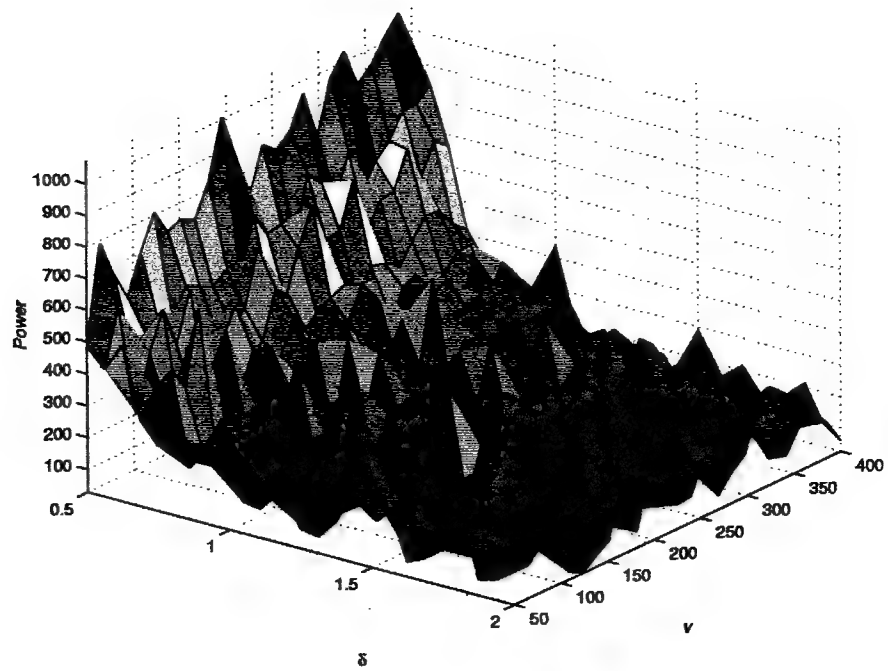


Fig. 7.2: Minimal power requirements by random sampling using sample size of 100.

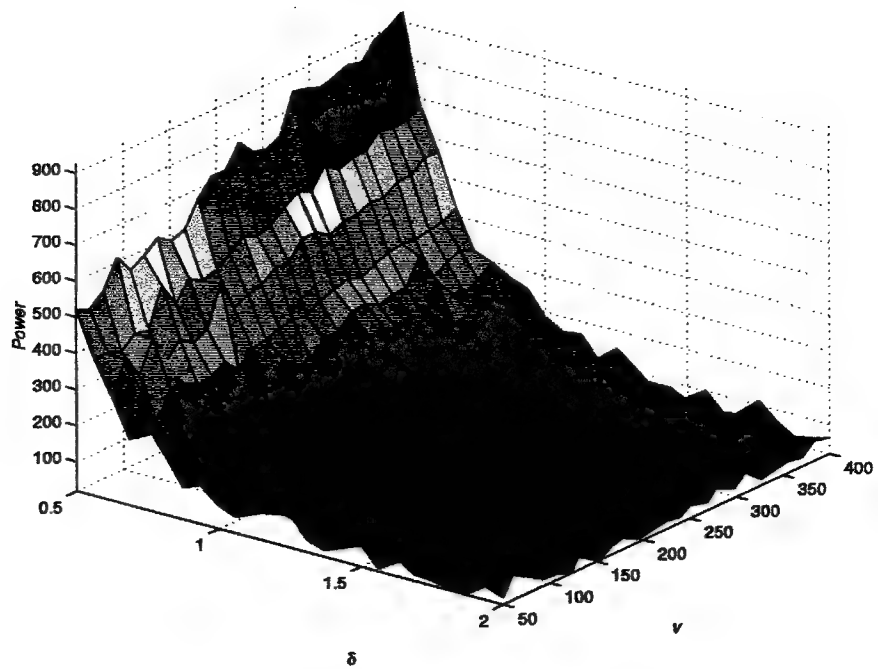


Fig. 7.3: Minimal power requirements by random sampling using sample size of 1000.

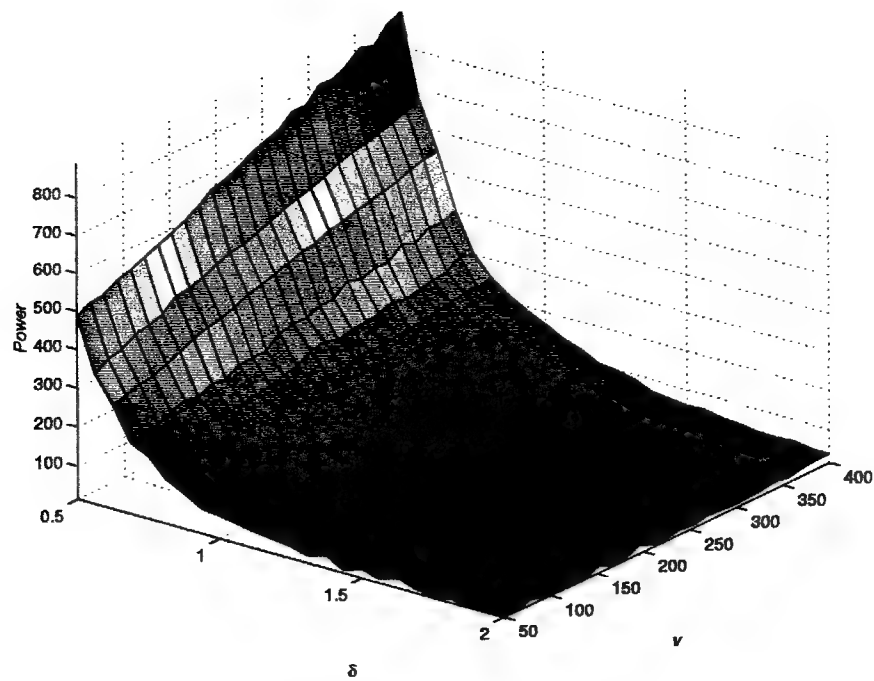


Fig. 7.4: Minimal power requirements by random sampling using sample size of 10000.

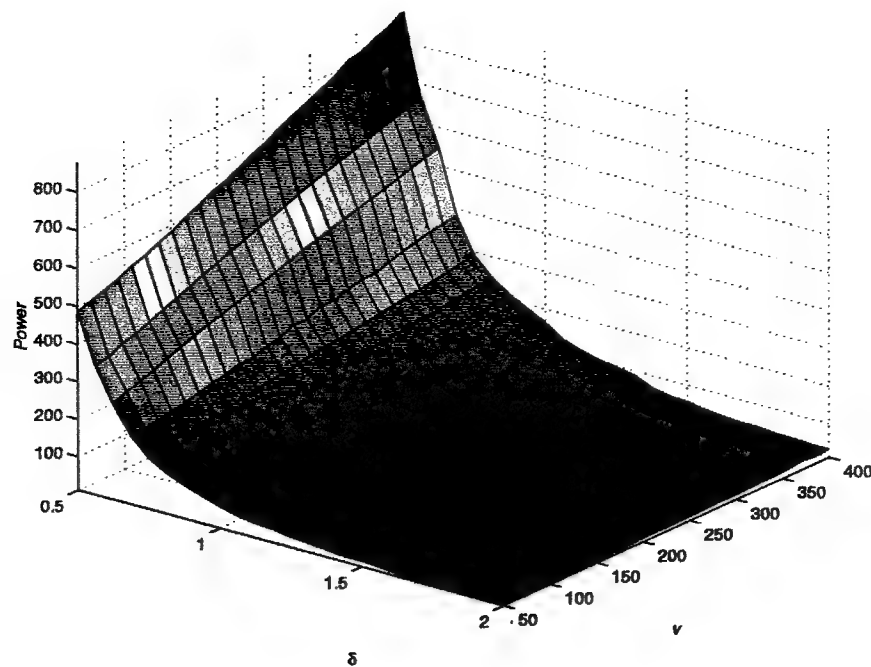


Fig. 7.5: Minimal power requirements by random sampling using sample size of 100000.

Table 7.2: Comparison of optimal and random configuration power consumption.

No. Samples	Minimum	Maximum	Mean
Optimal	9.2	867.6	135.5
100	28.6	1071.7	261.3
1000	18.5	926.1	166.7
10000	15.6	890.9	144.3
100000	10.8	877.4	137.9

Table 7.3: Ratios of random to optimal power.

No. Samples	Minimum	Maximum	Mean	Std. Dev.
100	1.0072	21.365	3.7844	3.1601
1000	1.0061	10.530	1.6493	0.8310
10000	1.0021	2.7504	1.1777	0.2203
100000	1.0010	1.3799	1.0477	0.0580

Table 7.3 shows a steady convergence of the random sampling solutions to the optimal solutions proposed by the formal optimization formulation. However, note that at no point does a random sampling solution discover a better solution than that of the formal optimization. This fact lends confidence to the absolute optimality of the solutions produced by the formulation presented in the ISMM and the associated convexity of the problem (at least local convexity in the range of resolution and velocity values investigated).

Evaluation of the random sampling method involves comparison with the formal optimization solution in terms of the accuracy of the results and calculation time. Using the formal optimization solution as a reference, it is seen that the accuracy of the results (see Table 7.3) depends on the number of samples taken, although even at the 100000-sample level the accuracy is still wanting by approximately 5% and by 18% at the 10000-sample level.

Average calculation times for the random sampling method were 0.011, 0.080, 0.550, and 6.15 s per solution point for the 100-, 1000-, 10000-, and 100000-sample operations, respectively. Relative to the average 1.21 s per solution point for the formal optimization method, the times for the random sampling method are not impressive considering the quality of the solutions. The above times were all collected on the same computer, an Intel Pentium 133 MHz with 80 MB of memory running Microsoft Windows NT 4.0. There was sufficient memory to allow all computations using the 80 MB of main memory only.

It should be noted that the calculation times of the random sampling method are very consistent, since the same operation with a consistent number of iterations is performed for each resolution and velocity coordinate. This consistency is in contrast to the calculation times of the formal optimization method, which varies depending on the accuracy of the initial guess and other factors in the optimization routine such as step size, constraint and objective tolerance, etc.

However, even very poor initial guesses are quickly overcome by the optimization routine and calculation times never vary by more than 300%.

To achieve comparable results with the random sampling method in regards to the formal optimization method, the number of samples would need to be increased beyond the 100000 samples appropriated in this investigation. As observed in the first section of this chapter, the million-sample test provided a solution of essentially equivalent quality as the formal optimization, but this quality was accomplished with the aid of foreknowledge of the proposed optimal solution to set an upper bound on the power consumption by one card. Furthermore, the calculation time on the million-sample test was extremely high (approximately 20 minutes) in comparison to the other solutions, exaggerated by constraints in memory on the local machine performing the computation and resultant reliance on virtual memory.

Thus, in terms of time and accuracy, the random sampling method does not perform well in comparison to the formal optimization method. The random sampling method has the advantage, however, of simplicity in implementation. Even at small samples, the random sampling method produces a basic solution surface clearly similar to that of the formal optimization, so as to be useful for quick approximation.

Random sampling as a method to verify the results of a formal optimization is shown to be useful. Although conclusive verification is impossible, greater confidence in the solution is achieved with larger sample sizes and greater ranges in randomized variables.

## CHAPTER VIII

### CONCLUSIONS FOR PART 1

This work focuses on modeling the processor-memory relationships of an embedded system for synthetic aperture radar (SAR) processing. The Mercury RACE multicomputer, built with commercial off-the-shelf (COTS) components, is the computing platform case study. Within the framework of the models developed in this work, optimization is performed on parameters such as the convolution section size and the choice and number of processor-memory hardware subunits (daughtercards) comprising the system.

Size, weight, and power (SWAP) constraints often motivate the maximization of performance density for a given SAR system, especially in the case of unmanned aerial vehicles (UAVs) or satellites, which often accommodate SAR systems. SAR in itself is an approach to densifying a radar system by substituting a large degree of data postprocessing for radar equipment with prohibitively high size, weight, and power characteristics. Minimization of power is the fundamental objective in this research, although with sufficient parameter guidelines size and weight could also be minimized using the same approach.

The specific mode of SAR investigated in this research is known as *stripmapping*. In stripmapping, successive radar pulses are transmitted and returned in the range dimension, which is orthogonal to the line of flight. Each received series of pulses from an individual transmitted pulse is then convolved with a reference kernel to achieve range compression. The entire range is processed at once in this way.

To create a two-dimensional image, however, processing in the azimuth dimension is also necessary. The azimuth dimension is parallel to the line of flight and is conceptually infinite in length. Thus, processing of the entire azimuth

vector, created from stacked range-processed vectors, is infeasible. To counter this problem, sectioned convolution is employed.

Sectioned convolution extracts a piece (or section) of the azimuth vector, convolves it with a reference kernel as in the range dimension, and then discards a section of the result equal to the length of the reference kernel. Successive processed azimuth sections are then overlapped (with overlaps equal to the discarded section length) to form continuous vectors in the azimuth dimension.

As is intuitive, a large azimuth section length requires more memory than a small section. Correspondingly, a small azimuth section requires more overall processing than does a large section because the percentage of new data processed that is not discarded is low, the size of the reference kernel being fixed.

One major focus of this work is the exploitation of the section size and the concomitant processor-memory tradeoff. Different daughtercards are better suited for different scenarios depending on the memory per processor ratio, the application requirement of which is largely dependent on the chosen section size. The combination of the choices for the section size and number and types of daughtercards employed greatly affects the overall performance and associated power consumption of a system.

Two models are presented in this work that address the problem of determining the optimal values for these variables. Both methods are based on mathematical programming, which provides a method of formulating an optimization problem given an objective and set of constraints. All computation in the work was performed using MATLAB 5.1 and the associated Optimization Toolbox's `constr` function, which implements a Sequential Quadratic Programming algorithm to solve nonlinear constrained minimization problems.

The first model (introduced in [16]) is based on the assumption of an ideal shared memory system. It treats all the memory contributed by individual

daughterboards as a conglomerate block, equally accessible by all processors located on all daughterboards. For the Mercury RACE system, this is an inaccurate oversimplification. However, it is useful to initially investigate the optimization of the SAR system based on such an assumption because it provides clear insight into the interrelationships between variables and the effects of perturbation of other external parameters. In addition, the simplification eases the collection of data because of the relatively low level of computational intensity.

The second model removes the assumption of shared memory and purposes to address system configuration more realistically. With this goal comes an increase in the complexity of the optimization formulation. The constraint set is modified to ensure only local memory access by processors. To accomplish this optimization, a much higher degree of integer programming is required than in the first model, entailing higher computational intensity. The benefits of this second model include solutions that consist of a complete specification of system resources, whereas the first model only specifies which resources are to be employed.

Comparison of the two models shows the first model to be a good approximator to the second model. Furthermore, the first model in its own right is a valid representation of a system in which communication time between daughterboards is only negligibly higher than memory access time by processors to their own memory modules.

The utility of optimization of the section size is demonstrated by comparison of results produced by a heuristic used to determine section size. The heuristic defines the section size to be equal to the kernel size. This section size definition and resultant system configuration is designated as *nominal*. This work finds that the nominal section size, although relatively efficient in processing, is too large for most scenarios because of the excessive memory requirements involved.



Research shows that forcing relatively inefficient processing with an associated reduction in memory requirements is optimal if power is to be minimized. Optimal section sizes thus often are found to be only a fraction of the kernel size, entailing the processing of more old data that is to be discarded than new data.

This work also demonstrates the advantage of employing more than one type of daughtercard in a system. Different daughtercards are characterized by different power requirements, amounts of memory, number of processors, and resultant memory per processor ratios. Optimization exploits these differences and determines the optimal system configurations.

Although minimization of power is the primary objective in this work, other optimization objectives are also considered. Minimization of resolution and maximization of velocity with a fixed power or hardware configuration are also investigated.

Random sampling of the solution space is performed to verify the proposed solutions produced by the optimization formulations. Such testing demonstrated the first model to be trustworthy. Although random testing was not performed on the second model, the same principle may be applied with an associated more complex random testing formulation.

The concept of solution verification with random sampling leads to the proposal of random sampling as a method of optimization. The weakness of this method is in the quality of the solutions and computation time required. Except for very low sample sizes, the required calculation time was equal to or greater than the time required for the formal optimization algorithm. Furthermore, the quality of the solutions was uniformly worse than the formal optimization. The strength of the random sampling method is in its simplicity of formulation. This method can be implemented without any knowledge of mathematical programming or availability to sophisticated mathematical optimization routines.

Generalization of the models developed in this work is straightforward. Especially in the second model, effort is made to avoid use of values specific to the Mercury RACE system. Instead, functions are defined that take as an argument the daughtercard type and return the number of processors, amount of memory, and power. Thus, any system that is constructed with the daughtercard concept, i.e., processor-memory nodes, can be modeled with minor modifications to the formulations presented.

**PART 2:**  
**SIMULATION OF COMMUNICATION TIME**  
**FOR A SPACE-TIME ADAPTIVE PROCESSING ALGORITHM**  
**ON A PARALLEL EMBEDDED SYSTEM [24]**

## CHAPTER IX

### INTRODUCTION TO PART 2

#### 9.1 Background

After taking office, the Clinton Administration launched an extensive investigation researching new methods and procedures for the procurement of federal government goods and services. In an attempt to assist in the reduction of waste and hidden costs, President Clinton, in a 1994 executive order, directed all heads of executive agencies to “increase the use of commercially available items where practicable, place more emphasis on past contractor performance, and promote best value rather than simply low cost in selecting sources of supplies and services” [26].

In addition to re-engineering the policies of governmental acquisition, the Clinton Administration drastically reduced defense expenditures. As a result of the changing, and perhaps advancing, governmental procurement methodology and military cost reductions, the Department of Defense (DoD) is moving towards commercial-off-the-shelf (COTS) products for the design and deployment of military systems. There are a number of embedded military applications such as airborne target recognition systems, undersea sonar platforms, ground processing stations, and command and control systems in which non-commercial resources are being abandoned. In particular, COTS parallel processing systems are replacing custom embedded military sonar and radar systems on ships and airborne aircraft [37].

In contrast to contemporary non-commercial products that involve costly custom engineering, ideally, COTS products offer lower cost hardware, faster development that reduces program lifecycle costs, and higher reliability while adhering to strict size, weight, and power (SWAP) requirements of many military applications. These characteristics of commercial products are achievable simply because of volume production and compatibility with a wide range of applications. Furthermore, the practice of purchasing COTS equipment creates a competitive market that stimulates both technological advancement and decreased costs [37].

As the demand for commercial embedded military parallel processing systems rise, the number of companies producing practical solutions to military-based platforms is also increasing. Mercury Computer Systems, Inc. plays a significant role in providing platforms for DoD computationally-intensive embedded applications. Mercury's primary role for such applications involves supplying very high-performance real-time computing and data I/O capability [29]. Mercury Computer Systems provides state-of-the-art embedded real-time multicomputer systems for typical digital signal processing platforms for intelligence data collection and processing.

Digital signal processing is one of the core technologies central to the operation of military-based radar systems. Digital signal processing is the application of mathematical operations on a digitally represented sequence of samples from an analog signal. Since their emergence in the late 1980s, digital signal processors (DSPs) have experienced tremendous growth rates in areas of signal processing due to reductions in costs, advances in DSP architectures, and improvements in development tools. Simply stated, a DSP is a special purpose microprocessor similar to a traditional microprocessor (e.g., Intel Pentium) that is optimized to perform mathematical operations such as multiplications, additions, and subtractions with greater efficiency. In addition to their increased performance for a class of computations, DSPs are generally silicon conservative and less expensive than general-purpose microprocessors.

Classical signal processing algorithms are characterized by the need for high-performance computing and involve repetitive, numerically-intensive tasks, which are ideally suited to DSP technology. Processing speeds of a single DSP are often insufficient to satisfy the computation demand of military-based signal processing applications. For such real-time signal processing applications, parallel processing is required to meet the necessary performance requirements.

## 9.2 Focus of the Thesis

This thesis involves the investigation of parallelization and performance improvement for a class of radar signal processing techniques known as space-time adaptive processing (STAP). The assumed platform, which consists of multiple DSPs, is

the commercially available Mercury RACE System [29]. The main contribution of the thesis is the design and implementation of a network simulator for the RACE system. This simulator allows for the performance of various parallel STAP algorithm implementations to be predicted for existing or future RACE system configurations.

STAP involves signal processing methods that operate on data collected from a set of spatially distributed sensors over a given time interval. Signal returns are composed of range, pulse, and antenna-element digital samples; consequently, a three-dimensional (3-D) data cube naturally represents STAP data. STAP algorithms can provide improved target detection in the presence of interference through the adaptive nulling of both ground clutter and signal jamming [43]. Typical parallel STAP involves simultaneous processing of the spatial signals received by the distinct elements of an array antenna and the temporal signals received from multiple pulses of a coherent radar waveform.

Typical processing requirements for STAP range from 10-100 giga floating point operations (Gflops), which can only be met by multiprocessor systems composed of numerous interconnected compute elements (CEs) [38]. A CE contains a processor, local memory, and a connection to the network that interconnects the CEs. In most parallel STAP implementations, there are phases of computation in which data must be exchanged among CEs. A major challenge of implementing parallel STAP algorithms on multiprocessor systems is determining the best method for distributing the 3-D data cube across CEs of the multiprocessor system (i.e., the mapping strategy) and the scheduling of communication within each phase of computation. It is important to understand how mapping and scheduling strategies affect overall performance. The network simulator developed in this thesis is used to evaluate the performance of various mapping and scheduling strategies.

The remainder of this thesis is divided eight chapters. Chapter X provides an overview of radar signal processing and a computation complexity analysis of two STAP algorithms, namely fully-adaptive STAP and a partially adaptive heuristic (element-space post-Doppler STAP) used to approximate the optimal solution. Chapter XI briefly introduces the basic components of Mercury's RACE multicomputer including a description of the CEs, the RACEway interconnection network, and network contention

resolution schemes. Chapter XII illustrates the challenges associated with implementing STAP algorithms on a parallel-processing computer. Two basic paradigms for distributing the 3-D STAP data cube among CEs of Mercury's RACE system are described. Chapter XIII presents small-scale examples to illustrate the effects that mapping and scheduling choices can have on network performance. In Chapter XIV, the design of the simulator, using the Unified Model Language (UML), is described and illustrated. Chapter XV presents some numerical studies involving timing information obtained from the simulator. Finally, Chapter XVI concludes the work with a summary of the research and results.

## CHAPTER X

### OVERVIEW OF STAP

Current methods of radar date back to 1924, when the height of the ionosphere was first measured [41]. By 1935, the military started developing radar-based weapon systems, and shortly after, at the outbreak of the war in 1939, military radar stations were in operation. During the war, the military concealed knowledge of radar technology for obvious strategic reasons. Consequently, detailed technical information about radar was not released to the public until after the war. Today, radar technology has become an integral part of real-time signal and image processing for defense and commercial applications. Modern airborne radar systems are required to detect smaller and smaller targets in the presence of clutter and interference. Space-time adaptive processing algorithms have been developed to extract a desired signal from potential target returns comprised of Doppler shifts resulting from radar platform motion, clutter returns, and interference including jamming. The sections below provide a brief overview of radar signal processing and STAP methods. For a thorough theoretical analysis of STAP, the reader is referred to [27, 43].

#### 10.1 Radar Signal Processing

The basic concept of radar is relatively simple, although its practical implementation is not so trivial. In military environments, radar is used to extend the capability of human's senses for observing the environment, especially the sense of vision. The basic purpose of radar is to detect the presence of an object of interest and provide information concerning that object's range, velocity, angular coordinates, size, and other parameters [36]. Radar operates by radiating electromagnetic (EM) energy, oscillating at a predetermined frequency,  $f$ , and duration,  $\tau$ , into free space through an antenna. In general, the radar antenna forms a beam of EM energy that concentrates the EM wave into a given direction [28]. By effectively rotating and pointing the antenna, the transmitted radar signal can be directed to a desired angular coordinate.



An object or target located within the path of the transmitted radar beam will intercept a portion of the EM energy. The intercepted energy will be scattered in various directions from the target depending on the target's physical characteristics. In general, some of the transmitted energy will be reflected back in the direction of the radar. This retro-reflected energy is referred to as backscatter [28]. A portion of the backscattered wave or echo return is received by the radar antenna. The echo returns, which are gathered by a set of sensors, are sampled, and the resulting data is processed to identify targets and perform parameter estimation. The distance to the target is determined by measuring the time taken for the radar signal to travel to the target and back. Furthermore, the angular position of the target may be determined by the arrival direction of the backscattered wave. If relative motion exists between the target and radar, the shift in the carrier frequency of the reflected wave, also known as the Doppler effect, is a measure of the target's relative velocity and may be used to distinguish moving targets from stationary objects [39].

The basic role of the radar antenna is to act as a transducer between the free-space propagation and guided-wave propagation of the EM wave [40]. The specific function of the antenna during transmission is to concentrate the radiated energy into a shape beam directive that illuminates targets in a desired direction. During reception, the antenna collects the energy from the reflected echo returns. Many varieties of radar antennas have been used in radar systems. The type of radar antenna selected for a certain application depends not only on the electrical and mechanical requirements dictated by the radar design specifications but also on its application. In airborne-radar applications, radar antennas must generate beams with shape directive patterns that can be scanned.

The properties offered by antenna arrays are quite appealing to airborne radar systems. Antenna arrays consist of multiple stationary elements, which are fed coherently, and use phase or time-delay control at each element to scan a beam to given angles in space [33]. The primary reason for using radar arrays is to produce a directive beam that can be repositioned electronically. An electronically steerable antenna array, whose beam steering is inertialess, is drastically more cost effective when the mission requires surveying large solid angles while tracking a large number of targets [33]. Additionally, arrays are

sometimes used in place of fixed aperture antennas because the multiplicity of elements allows a more precise control of the radiating pattern.

The purpose of moving-target indication (MTI) radar is to reject signal returns from stationary or unwanted slow-moving targets, such as buildings, hills, tree, sea, rain, and snow, and retain detection information on moving targets such as aircraft and missiles [37]. The term Doppler radar refers to any radar capable of measuring the shift between the transmitted frequency and the frequency of reflections received from possible targets [41]. Relative motion between a signal source and a receiver creates a Doppler shift of the source frequency. When a radar system intercepts a moving object that has a radial velocity component relative to the radar, the reflected signal's frequency is shifted. For example, consider a radar that emits a pulse of EM energy that is intercepted by both a building (fixed target) and an airplane (moving target) approaching the radar. As previously stated, each of the objects will scatter the intercepted radar signal, which will include a portion of backscatter energy. After the reflected radar signal returns to the radar in a certain time period, a second pulse of EM energy is transmitted. The reflection of the second pulse of energy from the building is returned to the antenna in the same time period as the first pulse. However, the reflection of the second pulse from the moving aircraft returns to the antenna in less time than the first pulse because the aircraft is moving towards the radar. This time change between pulses is determined by comparing the phase of the received signal with the phase of the reference oscillator of the radar [37]. If the phase of received consecutive pulses change, the object of interest is in motion.

### 10.2 STAP Algorithms

The objective of many airborne radar systems is to search the given space for potential targets. Future airborne radars will be required to detect increasingly smaller targets in the presence of interference such as clutter, jamming, noise, and platform motion. If the interference is localized in frequency and comes from a limited number of sources, targets can be detected by using adaptive spatial weighting of the data from each element of an antenna array [27]. By applying the computed weights to the data, the effects of interference can be reduced thus increasing the reception of the reflected signal. For an

airborne radar platform that is in motion, the Doppler spread of the clutter returns is significantly wider, and the clutter characteristics are highly variable due to the changing ground terrain. For this reason, the use of an antenna array provides the potential for improved airborne radar performance. Because of the added dimensionality of received data, the weights must now be adapted from the data in both the time and space dimensions. This signal processing method is referred to as STAP, which is an adaptive processing technique that simultaneously combines the signals received from multiple elements of an antenna array (the spatial domain) and from multiple pulses (the temporal domain) of a coherent processing interval (CPI) [43]. The paragraphs to follow provide a general description of the computation complexity involved in implementing two STAP algorithms, namely fully adaptive STAP and element-space post-Doppler. For a detailed theoretical foundation and computational complexity analysis of STAP algorithms, the reader is referred to [27, 43].

Consider an  $N$  element airborne radar array that transmits a coherent burst of  $M$  pulses at a constant pulse repetition frequency (PRF)  $f_r = 1/T_r$ , where  $T_r$  is the pulse repetition interval (PRI). The time interval over which the EM echo returns are collected is referred to as the coherent processing interval (CPI), and the resultant length of one CPI is  $MT_r$ . For each of the  $M$  pulses,  $L$  range samples are collected by each array element. With  $M$  pulses and  $N$  array channels, the return signal for one CPI is composed of  $LMN$  complex signal samples [43]. Because the signal returns are composed of  $L$  range gates,  $M$  pulses, and  $N$  antenna array samples, the data may be visually represented by the three-dimensional data set shown in Fig. 10.1. This  $L \times M \times N$  data set will be referred to as the CPI data cube [43].

Let  $x_{nml}$  represent the  $n^{\text{th}}$  array element and the  $m^{\text{th}}$  pulse at the  $l^{\text{th}}$  range sample time [26]. Next, define  $x_{m,l}$  to represent an  $N \times 1$  column vector, or a spatial snapshot, composed of the complex return signals from each array element for the  $m^{\text{th}}$  pulse and the  $l^{\text{th}}$  range. By combining all of the spatial snapshots at a given range of interest, an  $N \times M$  matrix  $X_l$  can be formed, where  $X_l = [x_{1,l}, x_{2,l}, x_{3,l}, \dots, x_{M,l}]$ . The shaded plane in Fig. 10.1, referred to as a range gate, represents the  $X_l$  spatial snapshot at the  $l^{\text{th}}$  range. To detect the presence

of a target within a range gate, a linear filter or space-time processor combines the data samples from the range gate to produce a scalar output, which is then typically passed through a threshold process for target detection.

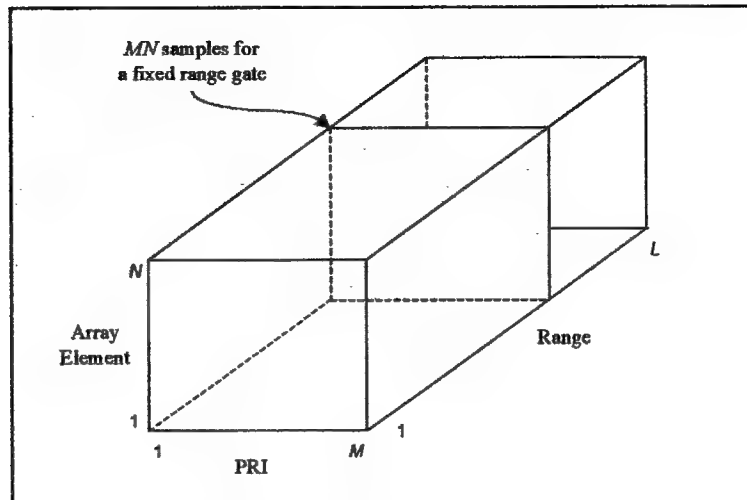


Fig. 10.1 The STAP CPI three-dimensional data cube (derived from [43]).

Three pipelined phases of processing comprise the generic space-time processor (see Fig. 10.2). First, a set of rules called the training strategy is applied to the data to estimate the interference. The objective of training strategy is to provide a good estimate of the interference at a given range gate. Because the interference is unknown, the training data is estimated data-adaptively from the STAP data cube.

The training data computed in phase one is used as input to calculate the adaptive weight vector in phase two. In general, the weight computation phase is the most computation-intensive portion of the space-time processor. Typically, weight computation requires the solution of a linear system of equations [43]. Additionally, each time the training data changes, a new weight vector must be computed. The most common weight computation strategy is called sample matrix inversion (SMI). In an SMI approach, the weight vector is computed from the inverse of the covariance matrix of training data or a QR-Decomposition of the matrix of training data. After calculating a single weight vector, the final phase of weight application commences.

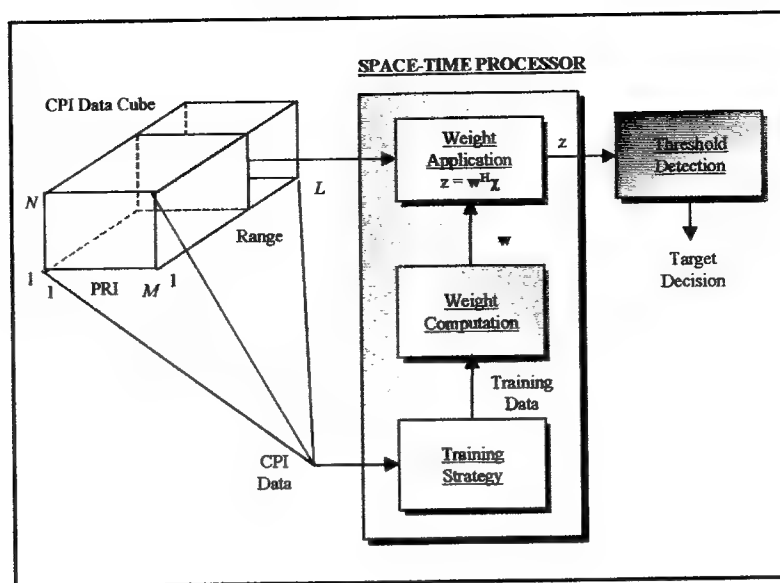


Fig. 10.2 Generic space-time adaptive processor (derived from [43]).

In the final phase, a scalar output is obtained by computing the inner product of the weight vector and range gate of interest. The scalar output is compared to a threshold value to determine if a target is present at a specified angle and Doppler [43]. Because a potential target's angle and velocity are unknown, the space-time processor computes multiple weight vectors to cover all possible target angles, ranges, and velocities at which target detection is to be queried [43].

Fully adaptive STAP refers to a space-time processor that computes and applies a separate adaptive weight to every array element and pulse. The size of the weight vector for fully adaptive STAP is  $MN$ . In order to compute the weight vector, a system of  $MN$  linear equations with dimension  $MN$  must be solved; thus, computing a single weight vector requires a  $O((MN)^3)$  operations [43]. For many conventional radar systems, the product of  $MN$  may vary from several hundred to several thousand with  $M$  and  $N$  both ranging from 10 to several hundred. Furthermore, a weight vector must be calculated for each training set used. The sheer computational complexity necessary to compute the weight vectors for fully adaptive STAP, in real-time, is typically beyond the capabilities of current computing systems (especially in cases where there is limited power and space for the computing

system onboard an aircraft). This fact alone renders fully adaptive STAP impractical and provides adequate motivation for the formulation of alternative heuristic algorithms.

The goal of partially adaptive algorithms is to break the fully adaptive problem into reduced-dimension adaptive problems while maintaining near-optimal results. A partially adaptive processor gathers the large set of input signals from the CPI data cube, transforms them into a reduced number of signals, and solves the reduced-dimension filtering problem with the newly transformed data [43]. Partially adaptive algorithms are classified according to the type of preprocessing performed first. For instance, in element-space pre-Doppler STAP adaptive-processing is followed by Doppler filtering.

In element-space STAP algorithms, every array element is adaptively weighted. The advantage of element-space approaches is that they retain full spatial dimensionality while decreasing the overall problem size by reducing the number of temporal degrees of freedom prior to adaptation [43]. Algorithms belonging to the class of element-space post-Doppler STAP perform filtering on the data along the pulse dimension, referred to as Doppler filtering, for each channel prior to adaptive filtering. After Doppler filtering, an adaptive weight problem is solved for each range and pulse data vector. By using element-space post-Doppler STAP, the computational complexity is reduced to  $M$  separate  $N$ -dimensional adaptive problems. The focus of the proposed research assumes that STAP will be implemented using the element-space post-Doppler partially adaptive algorithm.

## CHAPTER XI

### AN OVERVIEW OF THE PARALLEL SYSTEM

Since the conceptual development and implementation of serial computers in the mid 1940's, their computing speed, complexity, and reliability has steadily and drastically increased to meet the demands of emerging problems. However, the physical constraint imposed by the speed of light limits indefinite improvements in the serial computer domain. Because of the imposed physical constraints, serial computers are unable to meet the throughput requirements necessary to solve certain complex real-time applications such as embedded medical image processing and military signal processing. A natural way to circumvent this problem is to use an ensemble of processors to solve both existing and future problems. The fifth generation of computers is emphasizing scalable parallel processing machines to solve complex large-scale problems. Parallel processing has emerged as a key hardware technology in modern computers, driven mainly by the demand for higher performance, lower costs, and sustained productivity in real-time applications [30].

#### 11.1 Parallel Architectures

In general, parallel architectures may be categorized into two fundamental classes, namely, shared-memory multiprocessors and message-passing multicomputers. Distinguishing the two taxonomies of parallel systems lies in their implementation of memory sharing and interprocessor communication. In a shared-memory multiprocessor architecture, a shared-memory address space is commonly accessible by all processors within the system. Processors communicate with each other by modifying data objects in the shared-memory address space. In a message-passing multicomputer system, each compute node is composed of a processor and its own local memory, unshared with all other compute nodes. Compute nodes are connected with each other via a common data communication fabric or interconnection network. Interprocessor communication is accomplished by passing messages through the interconnection network.

### 11.2 Mercury's RACE Multicomputer

In recent years, Mercury Computer Systems, Inc. has emerged as one of the leaders in the development and manufacturing of commercially available, high-performance, embedded heterogeneous message-passing multicomputer systems. Mercury's RACE multicomputer provides a foundation for parallel systems and offers a set of building blocks that provide upward scalability. A high-level diagram of a typical RACE multicomputer is illustrated in Fig. 11.1. The system's primary components include DSPs and/or reduced-instruction-set-computing (RISC) processors, I/O ports, and a network interface all connected via the RACEway interconnection network.

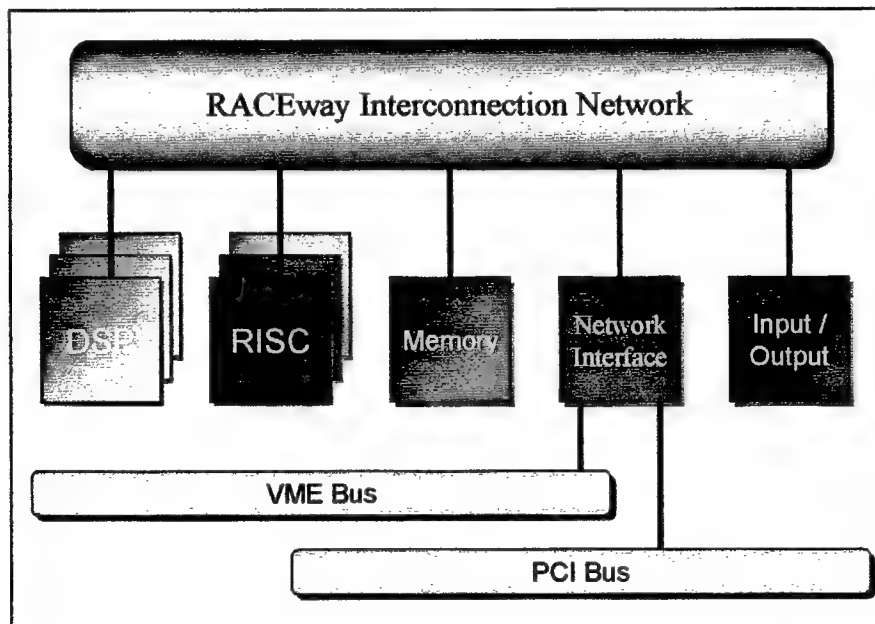


Fig. 11.1 The RACE Multicomputer (derived from [29]).

The RACEway interconnection network is used to provide high-performance communications among the interconnected processors and devices. Each node in the multicomputer interfaces the network through the RACE network chip. The network chip (see Fig. 11.2) is a crossbar with six bidirectional channels consisting of 32 parallel data lines and eight control leads [32]. Each crossbar transfers data synchronously at a clock



rate of 40-megahertz (MHz). Each channel is bidirectional but is only driven in one direction at a time at a rate of 160 megabytes per second (MB/s) [32]. Among the six ports comprising a RACE crossbar, each switch can either interconnect any three port pairs, providing an aggregate bandwidth of 480 MB/s, or can cause data to be broadcast to all or subset of the remaining five ports [29].

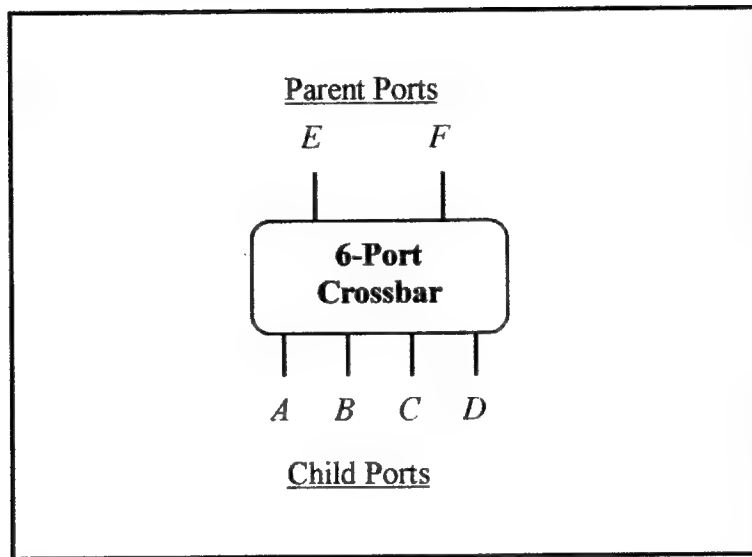


Fig. 11.2 The RACEway six-port network chip (derived from [32]).

The versatility of the RACE network chip allows the RACE multicomputer to be configured into a number of different network topologies. Possible network topologies include two-dimensional (2-D) and three-dimensional (3-D) meshes, 2-D and 3-D rings, grids, and Clos networks; however, the most common configuration is a fat-tree architecture (see Fig 11.3). For a fat-tree configuration, the crossbar switches are connected in a parent-child fashion. Each crossbar has two parent ports, *E* and *F*, and four child ports, *A*, *B*, *C*, and *D* (see Fig 11.2). The crossbars of the RACE multicomputer are connected together to form the branches of the fat-tree. The compute nodes represent the leaves of the tree.

To route a message from one processor to another, the message goes up the tree, selecting one of the two parents as it goes, until it reaches a network chip that is a common ancestor of both the source and destination node [32]. After reaching the common ancestor

network switch, the message travels down the fat tree to the destination compute node.

Fig. 11.4 illustrates a message transfer from two CNs.

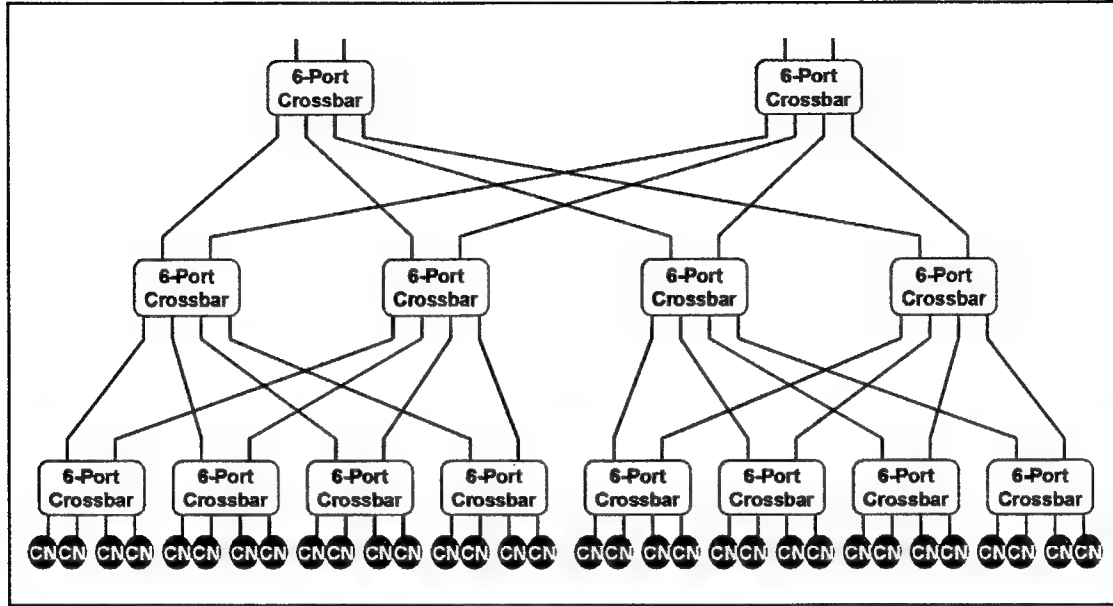


Fig. 11.3 The RACE Multicomputer fat-tree interconnection network.

In conventional tree architectures, there is only one path between any pair of processors. One major problem associated with such conventional networks is that they suffer communication bottlenecks at higher levels in the tree. For example, when several compute nodes in the left subtree communicate with compute nodes in the right subtree, the root node must handle all the messages [31]. This problem can be partially alleviated by increasing the number of effective parallel paths between compute nodes. This type of modified tree architecture is referred to as a fat-tree.

The RACE system is a circuit-switched network. In a circuit-switched network, a compute node establishes a path through the network prior to data transfer. Once the compute node has been granted a path to the destination node, the path is occupied for the duration of the data transmission. Data is transferred from one CN to another across the RACEway interconnect in packets of up to 2048 data bytes in length [45]. Each data transfer initiated by a source CN contains only a single packet consisting of up to 514 32-

bit words. The first two words of a given packet constitute the packet header. The packet header contains the information for routing the packet through the sequence of RACEway crossbars between the source and destination CN, as well as transfer control information such as the packet priority of the transfer [45]. Additionally, the destination memory address of the data transfer is included in the packet header.

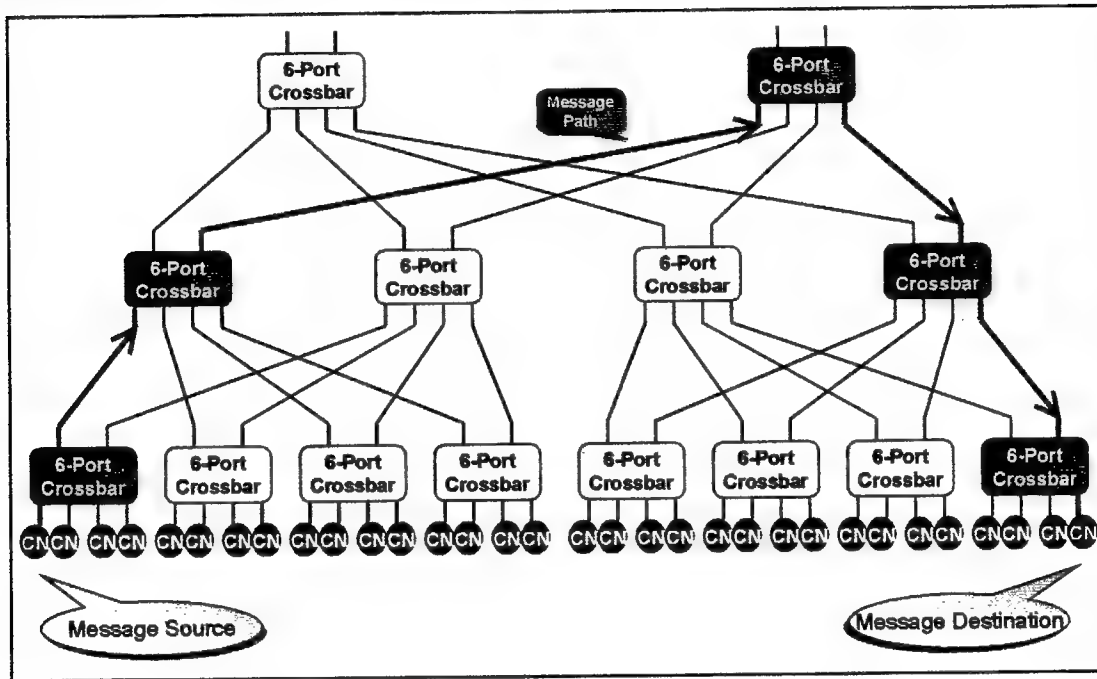


Fig. 11.4 Packet transfer between two CNs.

To send a packet through Mercury's fat-tree network, the first step is to establish a path. To establish a path, a packet header specifying a path is sent through the network along a given channel. A channel's status is categorized as either *free* or *occupied* [32]. The header makes as much progress as possible through the network until blocked. After a packet header has been blocked, it waits until a free channel becomes available. When a free channel matching the path specification (of the packet header) becomes available, the channel is flagged as occupied, and the packet header advances along that path. After establishing a path to the destination node, the packet header sends an acknowledgment to the source along the allocated path. Upon receiving the acknowledgment of a granted network path, the source node sends its packet data down the path in a pipelined fashion

[32]. During the transmission of the last byte of data, each of the occupied channel's status is set to free.

The preceding discussion of path establishment assumed that a clear and contention free path between the source and destination node existed. However, in networks that support a large number of simultaneous packet transfers, a clear path may not exist, thus contention for a crossbar port along a desired path requires arbitration. Clearly, for point-to-point transactions, a given crossbar port can only be part of one transaction per clock cycle [45]. Arbitration between two or more packets is required when the transfer paths pass through common crossbar ports.

When arbitration for a given crossbar port, or sequence of ports, becomes necessary, the arbitration is carried out on the basis of a combination of the user-programmable packet priority and a fixed hardware priority at each crossbar based on the entry and exit ports at the given crossbar [45]. Notably, the hardware priority of a given packet transaction path varies at each crossbar while the user-programmable packet priority is fixed for the duration of the packet's existence. For this work, the user-programmable packet priority is assumed equivalent for all data packets. Consequently, only the hardware priority arbitration rules associated at each crossbar will be used to resolve contention at a given crossbar port between two or more transactions.

In order to implement a fixed hardware priority at a given crossbar, each possible port-port path must be assigned a priority. In general, for an  $N$ -port crossbar there are  $N \times (N - 1)$  unique port-port pairs or possible connecting paths [45]. In this case, the RACE six-port crossbar has 30 (i.e.,  $6 \times 5$ ) possible connection paths. The assignment of the hardware priorities to each of the 30 possible paths through a RACE crossbar is complex and depends not only on the particular path, but also upon both the *status* of the contending transaction and the priority arbitration *mode* of the crossbar.

The hardware priority arbitration process performed at each crossbar to resolve contention for a given port between two or more packets depends on the following three factors. First, the directed path of a given packet through the crossbar; second, the transaction status of the two contending packets; and third, the priority arbitration mode at

the given crossbar [45]. For a more detailed description of the three hardware priority arbitration factors, the reader is referred to [45].

The assignment of hardware priorities to crossbar transaction paths is far from trivial, and the assignment of crossbar path priorities must be such as to guarantee that no *deadly embrace* conditions occur in the system [45]. A *deadly embrace* occurs when two transactions, that proceed in opposite directions along two different paths between the same two crossbars, simultaneously contend with each at both of those common crossbars, with the result that each transaction kills the other [45]. In a fat-tree architecture, the only way to prevent a *deadly embrace* situation is to implement the following rules: First, packets entering port F of any crossbar are given a different priority than those entering port E; and second, for any pair of crossbars that that can be connected via two alternative paths, the path leaving port F of one crossbar must be selected as to enter port F of the other crossbar [45]. For a more detailed analysis of the *deadly embrace* problem and solution, the reader is referred to [45].

Although the crossbars used to implement the fat-tree are physically identical, each crossbar may be configured to perform two different hardware priority arbitration algorithms [45]. The two algorithms are named Top-Level and Standard. The selection of the appropriate algorithm at a given crossbar depends upon the location of the crossbar in the network. For example, crossbars located at the top of the interconnected fat-tree are configured to implement the Top-Level algorithm while the remaining crossbars in the systems are configured to implement the Standard algorithm [45]. The reader is referred to [45] for further detail on both the Top-Level and Standard arbitration algorithms.

The priorities for hardware arbitration of crossbar port contention resolution, as a function of transaction's path through a given crossbar, are different in each transaction status case, as well as for each of the two crossbar arbitration algorithms [45]. The priorities of each of the thirty possible paths are enumerated in Fig. 11.5 and Fig. 11.6 for the case of the Standard and Top-Level algorithm, respectively, in order of priority [45]. As illustrated in the figures, there are a total of 7 different hardware priority levels, with 7 having the highest priority and 1 the lowest [45]. If two contending transactions have different hardware priority levels at a given crossbar, as defined by their respective entry

and exit ports at the crossbar and the transaction status of the exit port, the transaction having the highest hardware priority level will kill the contending lower-priority level transaction [45]. Conversely, if two or more contending transactions have the same priority level, the first one started will hold off any other contending transactions at the same level until the transmission of its data is completed [45]. The objective is to illustrate via Figures 11.5 and 11.6 the complexity of each of the two hardware priority arbitration algorithms. For a complete discussion of each algorithm, the reader is referred to [45].

Hardware Priority	Transaction Status					
	Active		Not Yet Active			
			Port E Involved		Port E Not Involved	
	Entry Port	Exit Port	Entry Port	Exit Port	Entry Port	Exit Port
7	F	A,B,C,D,E	F	A,B,C,D,E	F	A,B,C,D
6	E	F	E	F	A,B,C,D*	A,B,C,D*
5	A,B,C,D	F	A,B,C,D	F	A,B,C,D	F
4	E	A,B,C,D	E	A,B,C,D	-	-
3	*A,B,C,D	*A,B,C,D,E	A,B,C,D*	A,B,C,D*	-	-
2	-	-	A,B,C,D	E	-	-
1	-	-	-	-	-	-

\* - Peer Kill Rules Apply

Fig. 11.5 Standard hardware priority arbitration algorithm [derived from 20].

Hardware Priority	Transaction Status					
	Active		Not Yet Active		Tie	
	Entry Port	Exit Port	Entry Port	Exit Port	Entry Port	Exit Port
7	F	A,B,C,D,E	F	A,B,C,D,E	F	A,B,C,D,E
6	E	A,B,C,D,F	E	A,B,C,D,F	E	A,B,C,D,F
5	A,B,C,D	A,B,C,D,E,F	A,B,C,D	A,B,C,D,E,F	D	A,B,C,E,F
4	-	-	-	-	C	A,B,D,E,F
3	-	-	-	-	B	A,C,D,E,F
2	-	-	-	-	A	B,C,D,E,F
1	-	-	-	-	-	-

\* - Peer Kill Rules Apply

Fig. 11.6 Top-Level hardware priority arbitration algorithm [derived from 20].

As stated earlier in this section, the Mercury interconnection network under consideration is a fat-tree architecture comprised of multiple parallel paths. An interesting feature of the Mercury system is that it provides auto route path selection (i.e., adaptive routing) at the crossbar level, which means the multiple paths in the RACEway network may be automatically and dynamically selected by the RACE network crossbars. For instance, if one path is currently occupied with a data transfer and another path matching the path specification is free, the free path is automatically selected by the crossbar logic [35]. Adaptive routing is used to adaptively route packets that enter on either of the four child ports and exits either of the two parent ports. Auto route path selection frees the programmer from the details of path routing. Additionally, applications that require tremendous interprocessor communication such as distributed matrix transpose and corner turns often benefit from adaptive routing [29].

With the network configured as a fat-tree, the RACEway interconnection fabric provides very good scaling properties. In an  $p$ -processor system, the height of the fat-tree is  $h = \lceil \log_4 p \rceil$ . Thus, the network diameter or maximum number of links traversed is  $D = 2h - 1$ . The bisection bandwidth of a system, which is defined as the minimum number of edges (or channels) that have to be removed along a cut that partitions the network into two equal halves, assuming  $p = 4^i$  processors, is  $B = 160\sqrt{p}$  MB/s [32]. (Each channel in the RACEway system has a bandwidth of 160 MB/s.)

The RACE system may be configured as a heterogeneous multicomputer composed of two or more different types of processors. The potential heterogeneity of the RACE multicomputer includes various possible configurations of i860, PowerPC, and Super Harvard Architecture Computer (SHARC) DSP processors. The SHARC DSP is ideally suited for embedded vector signal processing applications such as Fast Fourier Transforms (FFTs) where physical size and power are at a premium or other similar algorithms that have a high ratio of data-to-computation. Furthermore, the Analog Devices' SHARC processor enables more than twice the physical processor density of reduced instruction set computer (RISC) based CNs. In contrast, the PowerPC and i860, both RICS processors,

are appropriate for executing scalar-type applications, with a low ratio of data-to-computation, generated by arbitrary compiled code.

The CNs in Figs. 11.3 and 11.4 are composed of three basic components: one to three processors (all of the same type), 8 to 64 MBs of dynamic random access memory (DRAM), and a Mercury-designed application specific integrated circuit (ASIC). Each ASIC is composed of address mapping logic, a direct memory access controller (DMA), processor support functions such as timers, and interfacing logic for effective RACEway transfers [29]. The address mapping logic enables local CN access to any DRAM location in any remotely located CN on the network [29]. The DMA engine provides a mechanism for rapid block-transfers between DRAM and other CNs, input/output (I/O) devices, or bridges nodes on the network. There is a unique CN ASIC for each CN processor type.

Because partially adaptive STAP is a signal processing application characterized by a high ratio of data-to-computation, the work to be completed will focus on the use of SHARC CNs. The composition of SHARC CNs includes one to three SHARC processors sharing a common DRAM and ASIC interface (see Fig 11.7). Within a CN, multiple SHARC processors are connected via a common 32-bit bus.

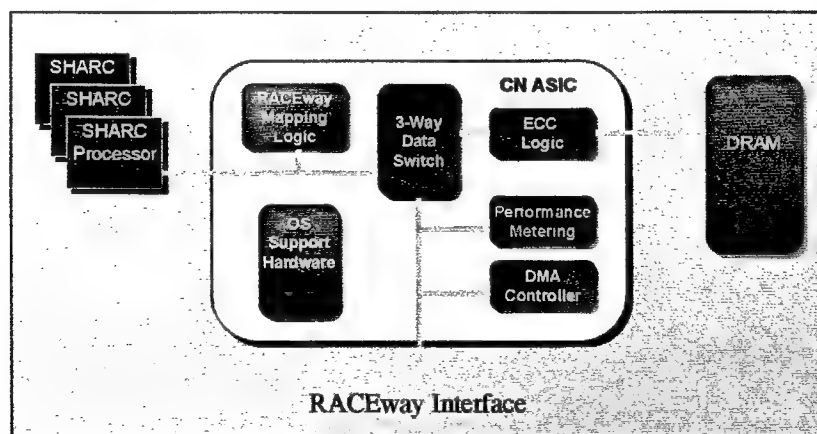


Fig. 11.7 SHARC compute node (derived from [29]).



## CHAPTER XII

### A PARALLELIZATION APPROACH FOR STAP

STAP refers to a class of signal processing methods that operate on a set of radar returns gathered from a set of array channels over a specified time interval. STAP is inherently three-dimensional (i.e., range, pulse, and channel), because the signal returns are composed of range, pulse, and antenna-element digital samples. Thus, a three-dimensional (3-D) data cube naturally represents STAP data. Typical processing requirements for STAP data cubes range from 10-100 Gflops, which can only be met by multicomputer systems composed of numerous interconnected CNs [31]. Imposed real-time deadlines for STAP processing restricts processing to parallel computers.

Developing a solution to any problem on a parallel system is generally not a trivial task. A major challenge of implementing STAP algorithms on multiprocessor systems is determining the best method for distributing the 3-D data set across CEs of a multiprocessor system (i.e., the mapping strategy) and the scheduling of communication within each phase of computation. Generally, STAP comprises three phases of processing, one for each dimension of the data cube. During each phase, the vectors of data along each dimension are distributed among the CNs for processing in parallel. During the processing for each dimension, the entire vector of data along the dimension of interest must reside in local memory at each CN. Additionally, each CN may be responsible for processing one or more vectors of data during each phase.

This re-distribution of data or distributed "corner-turn" requires interprocessor communication. Minimizing the time required for interprocessor communication helps maximize STAP efficiency. To assist in the minimization of interprocessor communication time during the data re-distribution phases, a paradigm for distributing the 3-D STAP data set among CNs of a multicomputer system has been proposed in [38]. Sections 12.1 and 12.2 summarize the work found in [38].

### 12.1 Data Set Partitioning by Planes

At each of the three phases of processing, data access is either vector oriented along a data cube dimension, or a plane-oriented combination of two data cube dimensions. Figure 12.1 illustrates the STAP flow. The three phases of processing include pulse compression, Doppler filtering, and beam weight computation and beam formation. During the first phase, pulse compression, the range dimension is processed. Next, the data cube is corner-turned to process data vectors along the pulse dimension termed Doppler filtering. After a second corner-turn, beam weight computation is performed by implementing a QR decomposition on a data matrix composed of samples from a combination of the range and channel dimensions. Finally, beam formation processing occurs along the contiguous vectors in the channel dimension.

The primary goals of many parallel applications are to reduce latency and minimize interprocessor communication (IPC) while maximizing throughput. It is indeed necessary to accomplish these objectives in STAP environments. To reduce latency, the processing at each stage must be distributed over multiple CNs in a single program multiple data (SPMD) approach. (In a SPMD approach, each CN executes the same program asynchronously.) However, prior to each processing phase, the data set must be partitioned in a fashion that attempts to equally distribute the computational load over the CNs. Furthermore, because each phase processes a different dimension of the data cube, the data cube must be re-distributed in a manner that minimizes IPC.

One approach to data set partitioning is to distribute the data set by data planes (see Fig 12.2). Each data plane is composed of two entire dimensions of the STAP data cube (and one or more elements of the third dimension). For this approach, the number of processors over which the data planes may be distributed is limited to the smallest dimension of the data cube. Shown in Fig. 12.2 is a decomposition of the  $N$  planes, one for each pulse. Data re-partitioning requires IPC between all  $N$  processors, which requires approximately  $N^2$  data transfers.

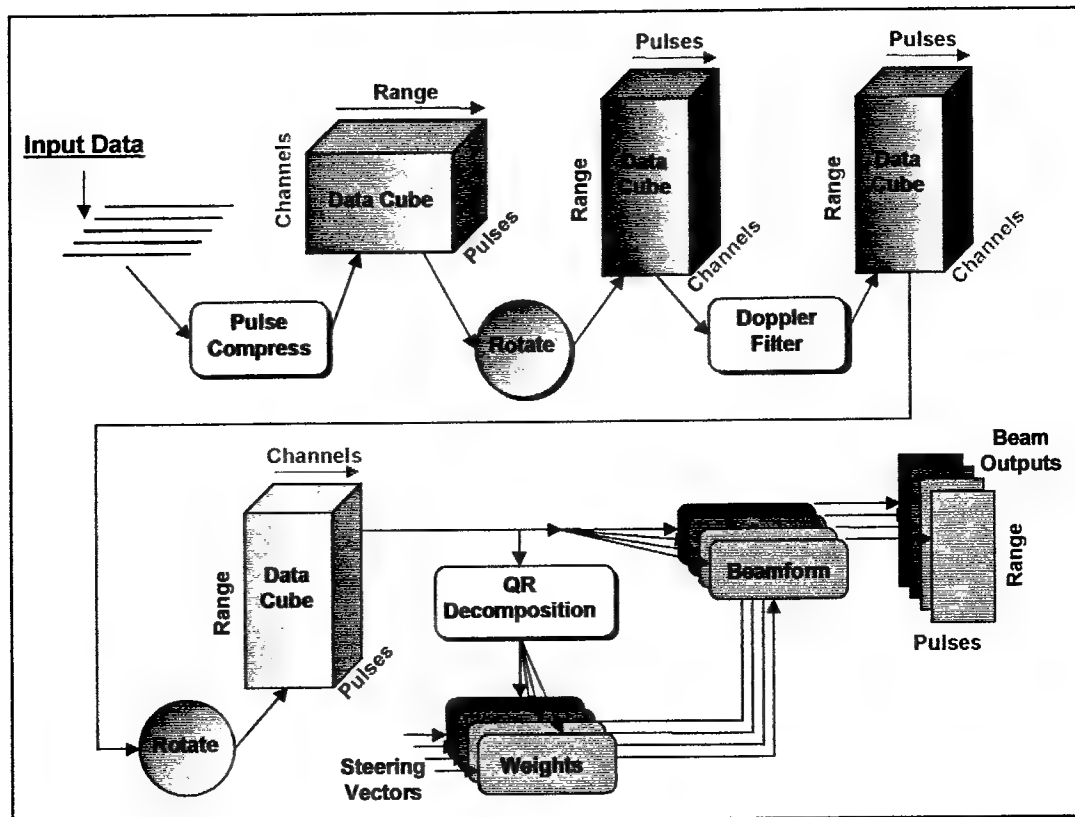


Fig. 12.1 Block diagram illustration of STAP flow (derived from [38]).

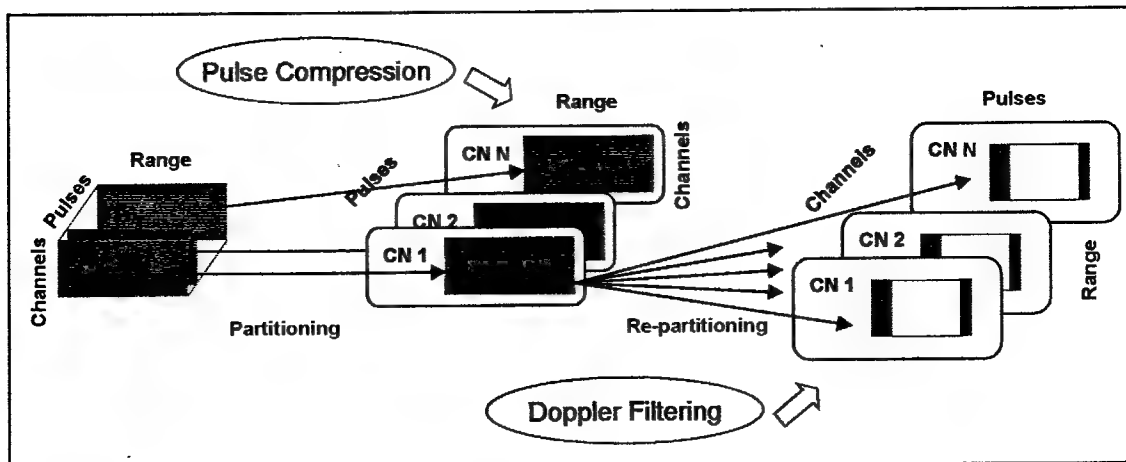


Fig. 12.2 STAP data cube partitioning by data planes (derived from [38]).

## 12.2 Data Set Partitioning by Sub-Cube Bars

A second approach to data set distributing in STAP applications is to partition the data cube into sub-cube bars. Each sub-cube bar is composed of partial data samples from two dimensions while persevering one whole dimension of the data cube as shown in Fig. 12.3. The maximum number of processors over which the data set may be partitioned is equal to the product of the two smallest dimensions of the data cube.

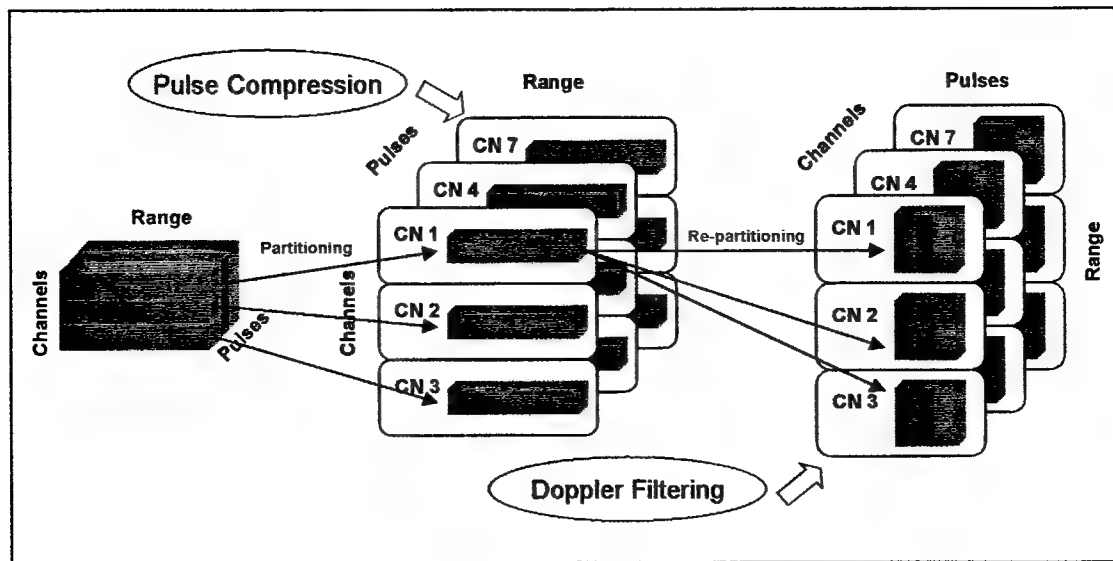


Fig. 12.3 STAP data cube partitioning by sub-cube bars (derived from [38]).

The authors of [38] proposed a five step methodology for effectively distributing and partitioning the STAP data cube across multiple processors. The first step is to partition the data cube over a two-dimensional *process set*. A process set is defined as a logical grouping of processes that can share data and synchronize with each other and other process sets. Data set partitioning is accomplished by dividing the dimensions of the data set by the dimensions of the process set. For example, in phase one of STAP, pulse compression is performed along the range data vectors. By applying a  $3 \times 4$  process set to the STAP data cube, leaving the range dimension intact, the channel and pulse dimensions

become segmented (see Fig 12.4). Implementing this partitioning scheme for the first phase would require twelve processors.

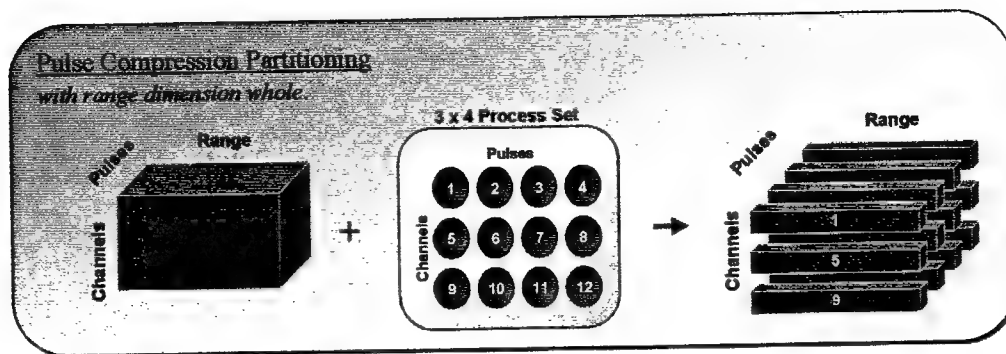


Fig. 12.4 Sub-cube bar partitioning prior to pulse compression (derived from [38]).

Applying the same  $3 \times 4$  process set to the Doppler filtering phase, results in the segmentation of both the channel and range dimensions (see Fig. 12.5). In this phase, the pulse data vectors include every pulse entry for a given array channel and range.

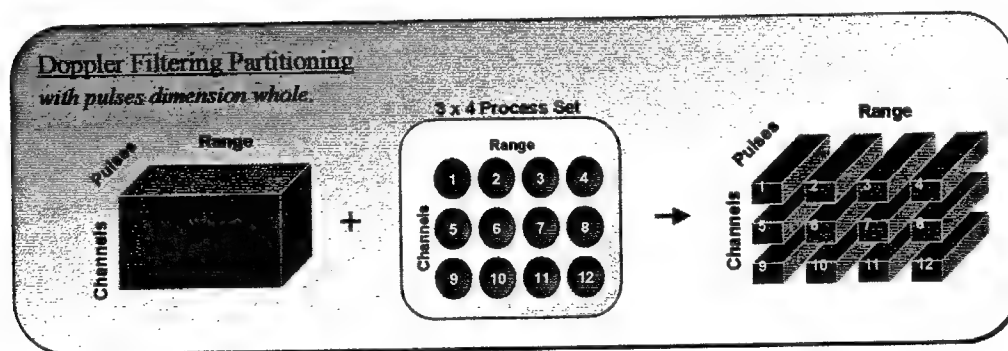


Fig. 12.5 Sub-cube bar partitioning prior to Doppler filtering (derived from [38]).

The second step involves processing in parallel the current whole dimension of the data cube (e.g., pulse compression, Doppler filtering, QR Decomposition). After performing the necessary calculations based on the current whole dimension, the third step entails re-partitioning the data before processing the next dimension. To re-partition the data, the current whole dimension must be exchanged with the next whole dimension (see

Fig. 12.6). As illustrated in Fig. 12.7, the required data exchange occurs only between processors in the same row. For example, process 1 transfers data to process 2, 3, and 4 while process 5 distributes data to process 6, 7, and 8. During this procedure, multiple phases of communication may take place in parallel. Assuming a  $P$ -processor systems, data set re-partitioning would require approximately  $\sqrt{P}$  sets of  $P$  data transfers.

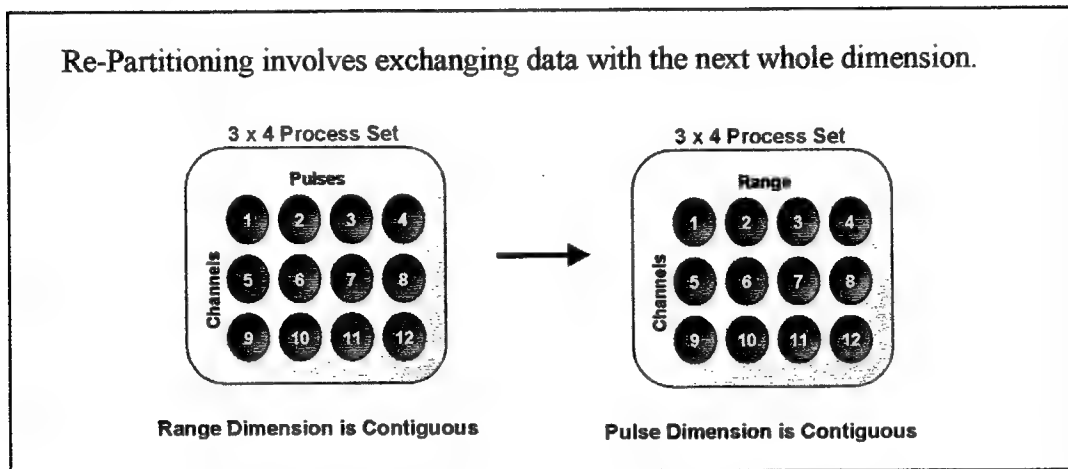


Fig. 12.6 Process set re-partitioning prior to Doppler filtering (derived from [38]).

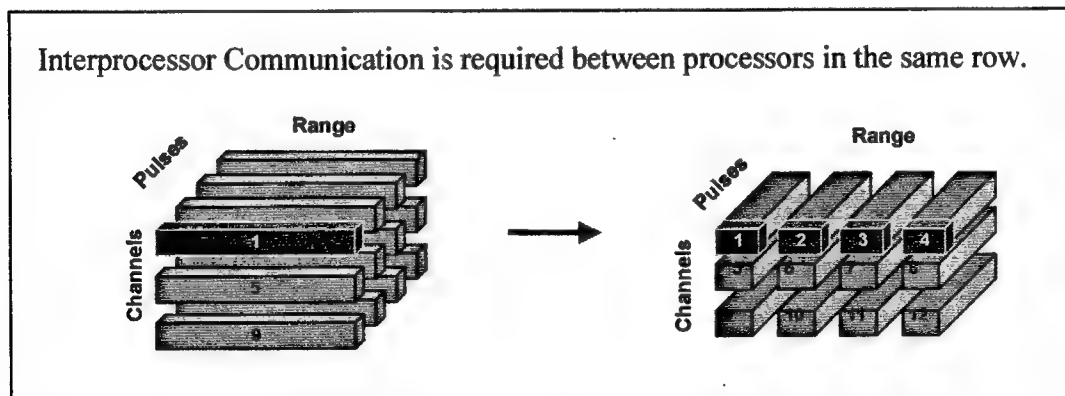


Fig. 12.7 STAP data cube re-partitioning prior to Doppler filtering (derived from [38]).

After re-partitioning the data set, completing step four involves sequentially ordering the data set in memory prior to processing. This local ordering of data, known as data set rotation, does not alter the dimension assigned to the process set nor require any IPC. It

does, however, require a rotation local to each processor. The final step is to repeat each of the first four steps on each dimension of the STAP data cube. Fig 12.8 illustrates STAP processing using sub-cube bar partitioning.

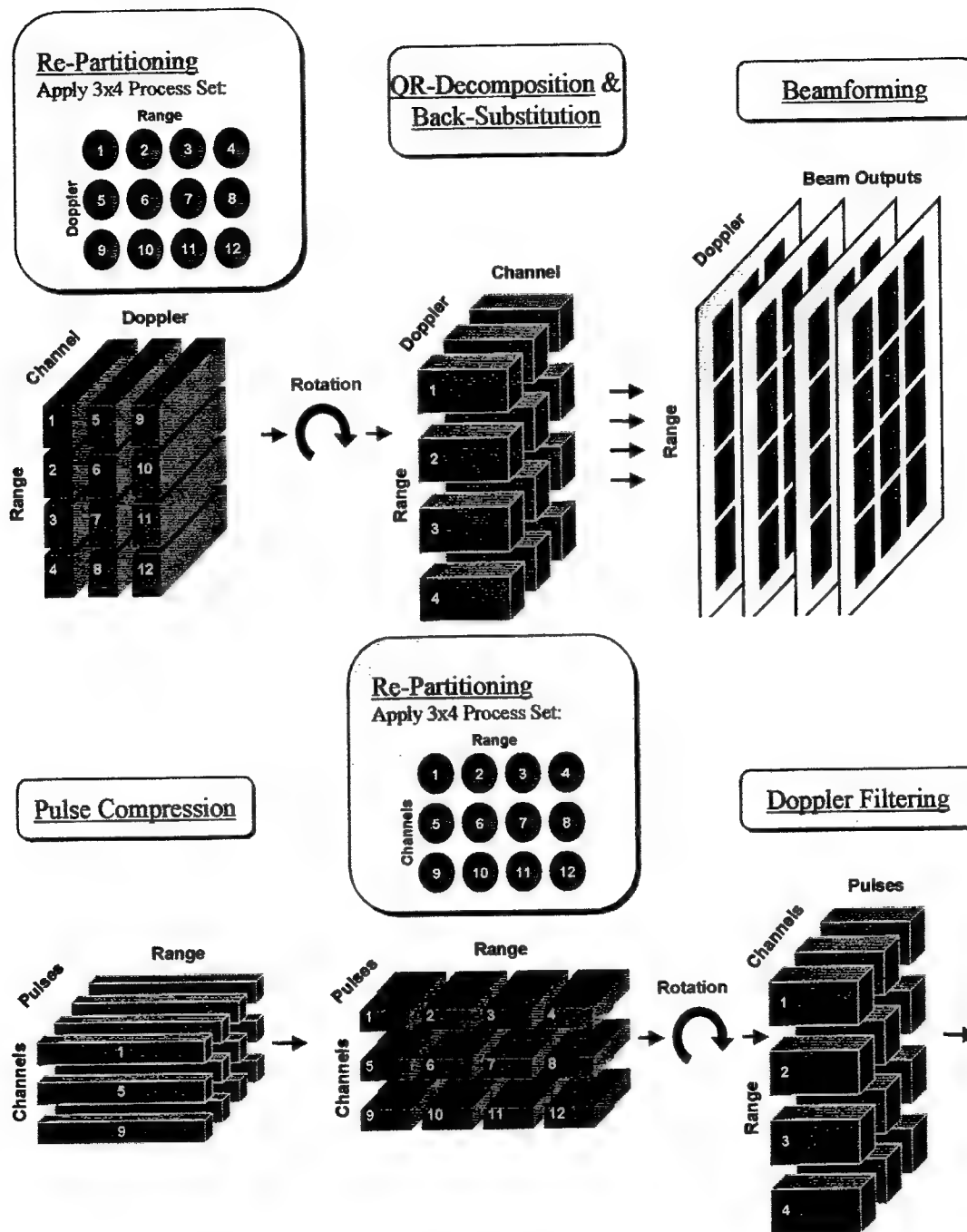


Fig. 12.8 STAP processing using sub-cube bar partitioning (derived from [38]).

### 12.3 Comparison of Data Plane versus Sub-Cube Bar Partitioning

Partitioning the STAP data cube along data planes has one potential advantage over sub-cube bar partitioning. If the initial data plane partitioning is performed along the channel dimension (leaving the range and pulse contiguous), both range processing and Doppler filtering may be performed without a re-partitioning phase taking place between the two steps. By implementing this scheme, only one re-partitioning step needs to take place, which occurs after Doppler filtering. Unfortunately, data plane partitioning has several disadvantages. Partitioning the data set along the smallest dimension or the channel dimension greatly reduces the number of potential processors as compared to sub-cube bar partitioning. Because the number of available processors is smaller, each processor in a data plane partitioning scheme is allocated a larger chunk of the data cube. Besides increasing the local memory requirement at each node, larger data segments demand more processing time thus increasing total completion time. Additionally, data set re-partitioning requires IPC between all processors.

In contrast, sub-cube bar partitioning of the data set provides a method whereby potential gains can be made in scalability and performance while minimizing the IPC time. Also, sub-cube bar partitioning has the potential for finer-grained parallelism because the maximum number of processors over which that data may be divided is the product of the two smallest dimensions of the data cube. Typically, the number of processors allocated to solve a sub-cube bar partitioned data cube is greater than in a data plane partitioning approach. Consequently, each processor performs fewer computations resulting in a shorter completion time. Furthermore, IPC between processing stages is isolated to only clusters of processors and not the entire system. On the other hand, sub-cube bar partition has a few disadvantages. First, this approach requires two separate re-partition and rotation phases. Secondly, scheduling data transfers during the re-partition phase is more complicated because communication is confined to groups of processors. The proposed research is to model, through simulation, the timing effects associated with how data is mapped onto the CNs and how the data transfers are scheduled.



## CHAPTER XIII

### MAPPING DATA AND SCHEDULING COMMUNICATIONS FOR IMPROVED PERFORMANCE

The overall performance of parallel computer systems can be highly dependent upon network contention. In general, the mapping of data and the scheduling of communication impacts network contention of parallel architectures. During phases of data re-distribution on parallel computers, the number of required communications is vastly impacted by the current location and future destination of the data. Determining the optimal schedule of data transfers through interconnection networks is generally an NP-hard problem [30]. Consequently, heuristics are often used to provide sub-optimal solutions. A combination of these two factors, data mapping and communication scheduling, provides the key motivation for the network simulator described in Chapter XIV. The following two sections illustrate the importance of data mapping and the scheduling of communications issues that exist implementing a sub-cube bar partitioning scheme on STAP data cubes.

#### 13.1 Mapping a STAP Data Cube onto the Mercury RACE System

As described in Section 12.2, data set partitioning by sub-cube bars entails partitioning the data cube into bars composed of two partial dimensions and one whole dimension. After partitioning, the sub-cube bars are distributed over a two-dimensional set of processors. Partitioning the STAP data cube across the Mercury System consists of an increased level of complexity because each CN is composed of three SHARC processors (i.e., CEs). (In general, a CN can contain one, two, or three CEs; three CEs are assumed here.) An important issue is how to map the sub-cube bars onto the available CNs on the Mercury System. To illustrate the impact of mapping, consider the examples of sub-cube bar partitioning prior to pulse compression in Fig. 13.1. For this example, assume that the Mercury System is composed of twelve CEs (see Fig. 13.2), and the STAP data cube is divided into twelve sub-cube bars. Additionally, the number on each sub-cube bar

indicates the CE to which the bar is assigned for processing. The left-hand portion of the figure illustrates a mapping scheme where the sub-cube bars are raster-numbered along the pulse dimension. In contrast, the right-hand portion of the figure depicts a mapping scheme whereby the sub-cube bars are raster-numbered along the channel dimension. The coloring code of each bar corresponds to its destination CN (recall that each CN is assumed to consist of 3 CEs). For instance, the three blue sub-cube bars are assigned to the blue CN, while the red CN processes the three red sub-cube bars.

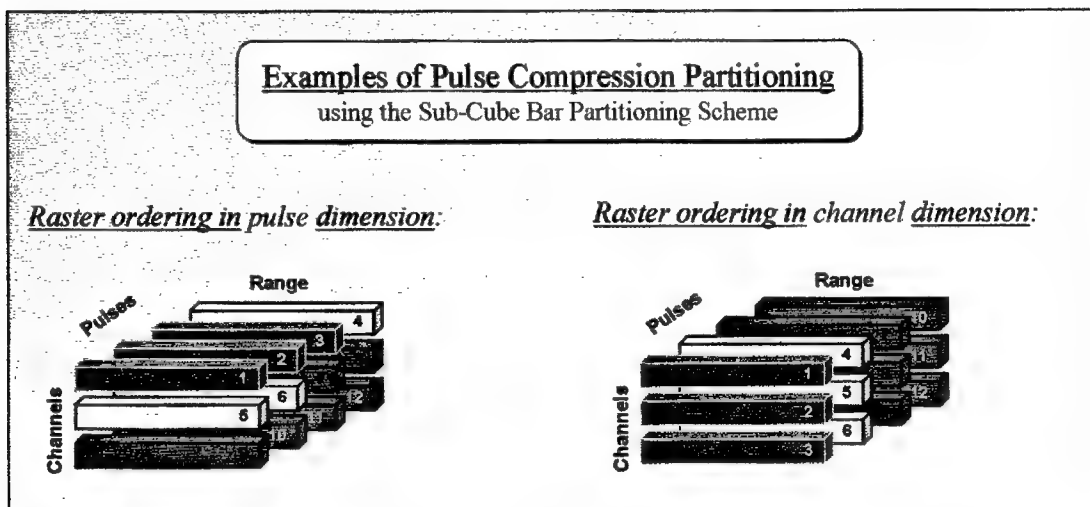


Fig. 13.1 Examples of sub-cube bar mapping schemes prior to Doppler filtering.

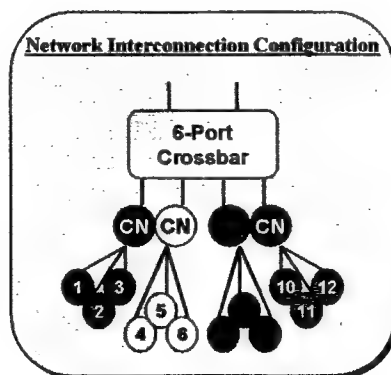


Fig. 13.2 An example configuration of a four CN (twelve CE) Mercury System.

After performing pulse compression on the data samples along the range dimension, the data set requires re-partitioning prior to Doppler filtering. The initial mapping of the data prior to pulse compression affects the number of required communications during the re-partitioning phase. In the case where the data cube is raster-numbered along the pulse dimension, six messages, totaling 20 units in size, must be transferred through the interconnection network (see Fig. 13.3).

Each CN is assumed to be configured as one master CE and two slave CEs. The master CE allocates the entire shared memory pool and distributes memory to the other two CEs. Having a master CE on each CN is advantageous during data re-partitioning phases. When two or more CEs within the same CN have one or more messages to send a common destination CN, the messages may be combined into one message by the master CE's direct memory access (DMA) controller. The newly created message may now be transferred to the destination node by the CN ASIC DMA controller. For increased efficiency, message transfers should be performed by the CN ASIC DMA controller while the CEs are concurrently processing. The reversal of this same process may be applied to message arriving at a CN. Messages arriving at a CN may be composed of one or more messages sent to one or more of its CEs. After receiving the message from the CN ASIC DMA controller, the master CE distributes the message's contents to the appropriate memory location. For this example, the blue CN (or master CE labeled 1) transfers data of size three units to the yellow CN. Furthermore, the yellow CN needs to transfer two messages, one to the blue CN of size three units and one to the green CN of size four units.



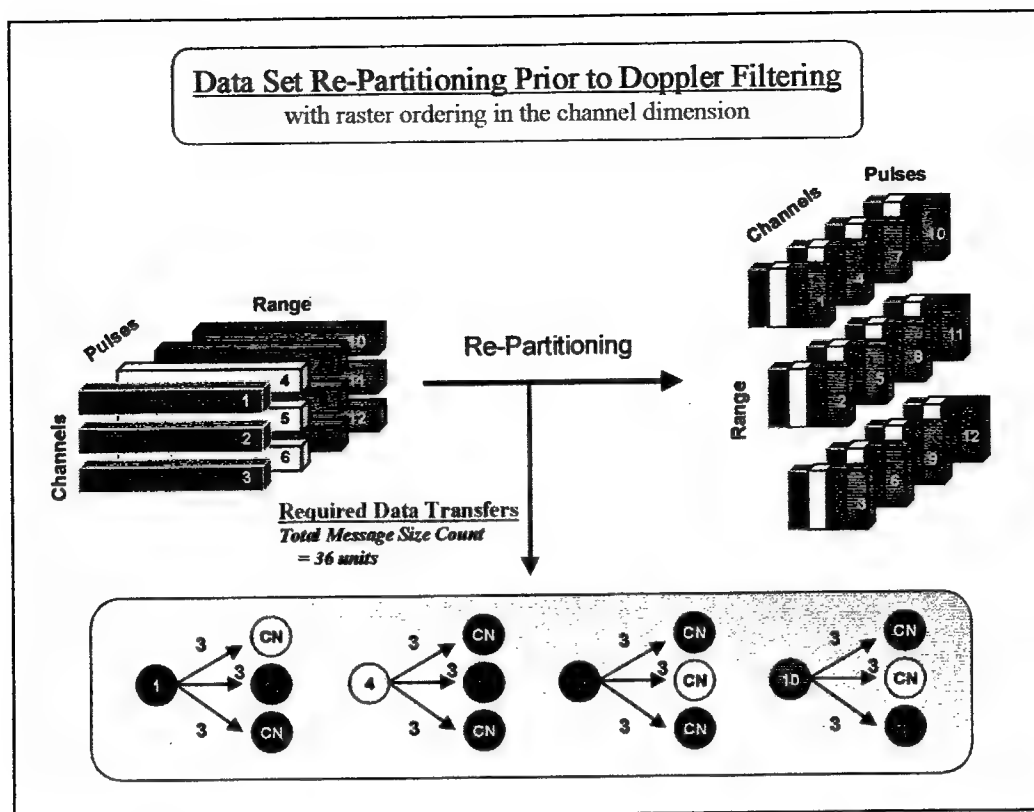


Fig. 13.4 Data set re-partitioning with raster-numbering along the channel dimension.

### 13.2 Scheduling Communications During Re-Partitioning Phases

After processing has been completed on the current whole dimension of the STAP data set, each master CE forms the outgoing messages necessary to replace the current whole dimension with the next processing dimension. Once constructed, the outgoing messages are placed in a queue and await transfer to their corresponding destination CNs. Determining the optimal communication schedule (i.e., ordering) of queued messages in circuit switched networks is a formidable task. Problems of this nature are generally proven to be NP-hard problems.

To illustrate the impact of message scheduling communications during data re-partition phases in partially adaptive STAP algorithms, consider the re-partitioning problem in Fig. 13.3. In this example, the re-partitioning phase involves transferring six messages

through the interconnection network. If the six messages were sequentially communicated (i.e., no parallel communication) through the network, the completion time ( $T_c$ ) would be the sum of the length of each message, which totals 20 network cycles. If two or more of the messages could be sent through the network concurrently, then the value of  $T_c$  would be reduced (i.e., below 20). The purpose here is to illustrate that the order (i.e., the schedule) in which the messages are queued for transmission can impact how much (if any) concurrent communication can occur. Thus, it will be shown that scheduling choices affect the value of  $T_c$ .

An illustration of the required data transfers is depicted in Fig. 13.5. The left-hand portion of the figure shows the current location of the STAP data cube on the given CEs after pulse compression. The coloring scheme of each sub-cube bar indicates the destination CN for the next phase of processing. If part or all of the sub-cube bar is a different color than its current processor color, the data must be transferred to the corresponding colored destination node. The data cube on the right-hand of the figure illustrates the sub-cube bars of the STAP data cube after re-partitioning. Each of the sub-cube bars is composed of sample data of the whole pulse dimension, thus each sub-cube bar is a single color. After re-partitioning the data, Doppler filtering may be applied to each sub-cube bar in parallel.

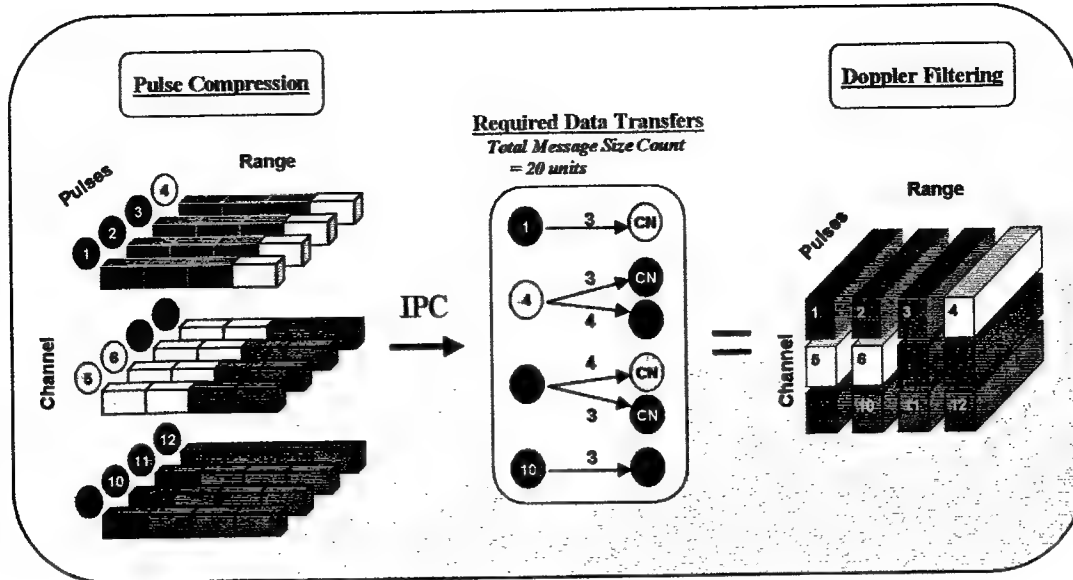


Fig. 13.5 An example of data set re-partitioning prior to Doppler filtering.

Scheduling the communication of each of the six messages through the RACEway network greatly affects the overall performance of the system. Fig. 13.6 shows the six messages, labeled A through F, in the outgoing message first-in-first-out (FIFO) queues of the source CNs. Each message's destination is indicated by its color. For instance, message A, which is colored yellow, is destined for the yellow CN. The destination of the blue message B is the blue CN, and so forth. The number in parentheses by each message label represents the relative size of the message. For example, message A's size is three units. Because of the assumed queues' FIFO implementation, message B must be transmitted before message E on the yellow CN.

The minimum achievable communication time is dependent upon the CN with the largest communication time to send/receive all outgoing and incoming messages. As shown in Fig. 13.6, the minimum possible communication time is the sum of all outgoing and incoming messages on the yellow or green CN, which equals fourteen units. Therefore, any scheduling for the six messages can complete in no less than fourteen

message units. The actual communication time,  $T_c$ , that would result for this example with the given message queue orderings (i.e., scheduling) is 17 network units.

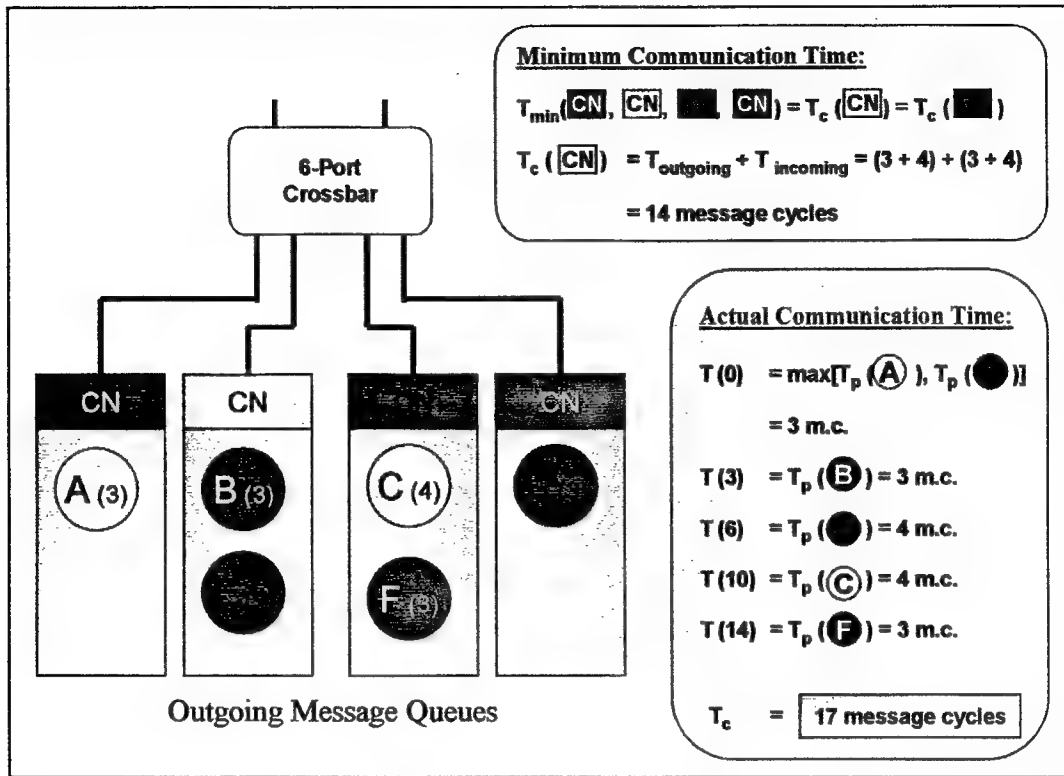


Fig. 13.6 A sub-optimal communication scheduling example.

To understand why the completion time for this example is 17 network cycles, note that at time  $t = 0$ , the first messages in the queue, messages A, B, C, and D, are ready for transmission. In parallel, each CN sends its first message to the six-port crossbar. All four of the incoming messages arrive at the crossbar simultaneous with each message requesting an outgoing channel. The resolution scheme based on port number resolves the network contention conflict. (In this example, each message is assumed to have the same priority.) As stated in Section 3.2, the port number is used as a tie-breaking mechanism for messages with the same priority contending for the same channel (lowest port number goes first). In this example, the lowest port number is associated with the left-most CN, and the highest port number is associated with the right-most CN (i.e., the four “children” ports of the



crossbar are numbered 0 to 3). The crossbar resolves the network contention problem by scanning the child ports from left to right. By using this contention resolution approach, message A is granted access to the link connected to the yellow CN, while message B waits in the queue. Furthermore, message D seizes the link to the green compute node, because message C is unable to establish a link to the yellow CN, which is occupied by message A.

At time  $t = 3$ , messages A and D complete and release the four channels occupied. After the status of the four occupied channels are set to free, queued messages B and C request links through the crossbar to their respective destination nodes. Because messages B and C both require access to the link connecting the yellow CN to the network, only message B, with a lower port number than C, establishes a path through the crossbar at time  $t = 3$ . After message B finishes transfer at time  $t = 6$ , queued messages E and C query the crossbar for a free path to their destination nodes. In this case, messages E and C are contending for the same two channels resulting in a sequential transfer of the two messages. Based on the port numbers, message C follows message E. Furthermore, messages C and F require sequential transfer because they both originate at the same CN. As a result, the remaining three messages are transferred sequential (i.e., no parallel communication) in an  $E \rightarrow C \rightarrow F$  ordering, totaling eleven network cycles. Compared to the minimum possible communication time of fourteen message cycles, the above message scheduling example renders a sub-optimal completion time,  $T_c$ , of 17 message cycles. However, changing the ordering of the messages in the outgoing queues, as described below, will yield an optimal scheduling of the messages.

The message queues in Fig. 13.7 are identical to those in Fig. 13.6 except messages C and F have swapped positions on the green CN. Swapping the ordering of the messages on the green compute node allows for an increase in the number of messages that can be communicated in parallel. To understand how the ordering yields improved performance, note that at time  $t = 0$ , the first messages in the queue, messages A, B, F, and D, are ready for transmission. In parallel, each CN sends its first message to the six-port crossbar. As before, all four of the incoming messages arrive at the crossbar simultaneous with each message requesting an outgoing channel. The crossbar resolves the network contention problem by scanning the port numbers in order, which allows messages A and F to

establish communication links through the network to their respective destination locations. Messages A and F are transferred in parallel and complete in three network cycles. Messages B and C are transferred sequentially and complete in five network cycles. Messages D and E are transferred sequentially and complete in six network cycles.

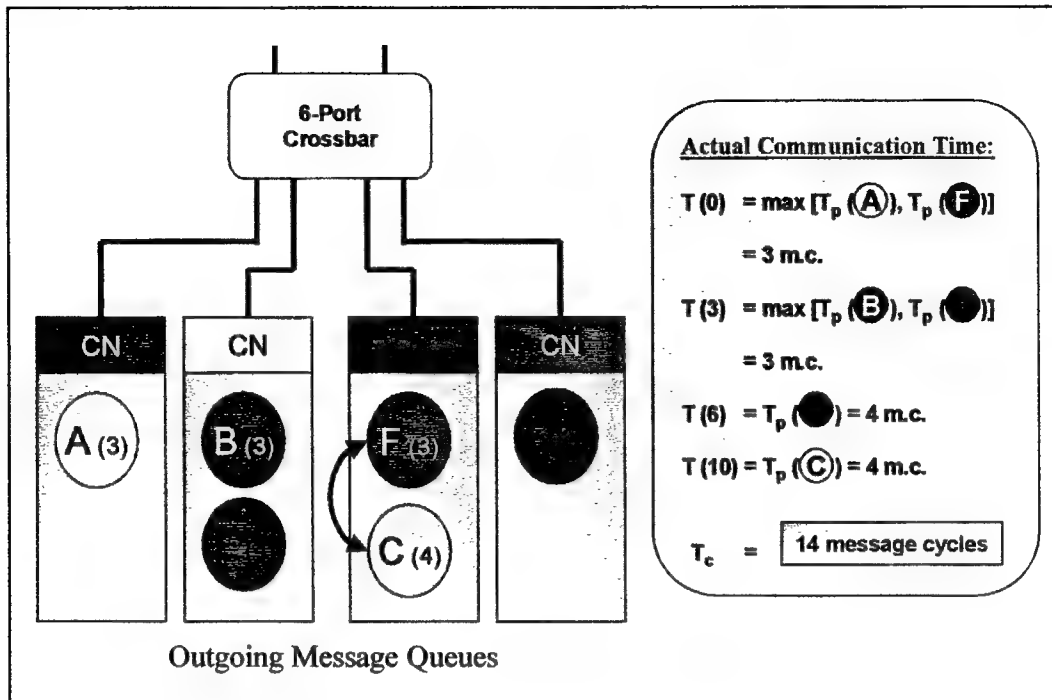


Fig. 13.7 An optimal communication scheduling example.

At time  $t = 3$ , queued messages B, C, and D request communication paths through the six-port crossbar, but only messages B and D are granted network access. Message C remains in the queue because messages B and C are contending for the same link (the link connecting the yellow CN to the network), and message B gains access to the channel because it originates from a lower port number. After messages B and D complete their transmission at time  $t = 6$ , the last two messages, E and C, query the crossbar for path establishment. In this case, messages E and C are competing for the same two channels. Consequently, the remaining two messages are transmitted sequentially, totaling eight network cycles, with E preceding C based on port number priority resolution. For this scheduling example, the actual completion time,  $T_c$ , achieves the optimal completion time of fourteen network cycles.

## CHAPTER XIV

### DESIGN OF THE SIMULATOR

The goal of the simulator's design was to produce an accurate model or representation of the proposed system that could be implemented. As a broader understanding of the underlying details of the Mercury System was developed, the design of the simulator was further refined and modularized. The Unified Modeling Language (UML), a third generation object-oriented modeling language, was utilized to formalize the simulator's requirements in software terms [46]. UML is a language for specifying, visualizing, and constructing the artifacts of complex software systems. It simplifies the complex process of software design and provides a blueprint for construction [46]. The primary goals of UML are as follows: (1) provide an expressive visual modeling language to develop meaningful models; (2) provide specialization mechanisms to extend core concepts; (3) be independent of programming languages and development processes; (4) provide a basis for formal modeling languages; (5) encourage the growth of object-oriented tools; and, (6) integrate the best practices [46].

#### 14.1 UML Class Definitions

The design of the simulator incorporates all the underlying components of the Mercury System. Within the structure of the design, only one definition of a class exists in the model; however, it may appear on several class diagrams. An important aspect of the simulator's object oriented design is modularity. By separating the functional components of the system into classes, the classes and their operations provide inherent modularity as well as information hiding.

The class diagram is one of the core components to a UML model. A class diagram illustrates the important abstractions in the system including relationships. The primary elements included on a class diagram are class icons and relationship icons. Fig. 14.1 shows a suppressed UML class diagram of the network simulator. The rectangular boxes represent the classes, while the lines connecting the classes signify the relationships. A

solid line with a hollow diamond at one end indicates an aggregation relation (i.e., one object is composed of another object). An association (i.e., a dependency) between objects is represented by a solid line. A line with a directed arrow at one end denotes an inheritance relationship (i.e., one object is a specialization or extension of another object). The adornments, such as 1..\*, indicates the number of potential objects participating in the relationship (in this case, the \* means many).

The main class, Network, is composed of a File Output class, a Clock class, a Random Scan class, a Crossbar class, and a Routing Table class. The Network class also *gets data from* the Data Cube class, and the Data Cube class *gets data from* the Process Set class. In these two cases, information relating to the data cube and the process set are passed to instantiated Network objects prior to the start of the simulation. Because both the data cube and process set may change, while the structure of the network remains the same, the two corresponding classes are not contained within the composition of the Network class. This allows a single Network object to operate on different data cubes and process sets without regeneration of the network. A more detailed description of the Network class will be presented in Section 14.2.

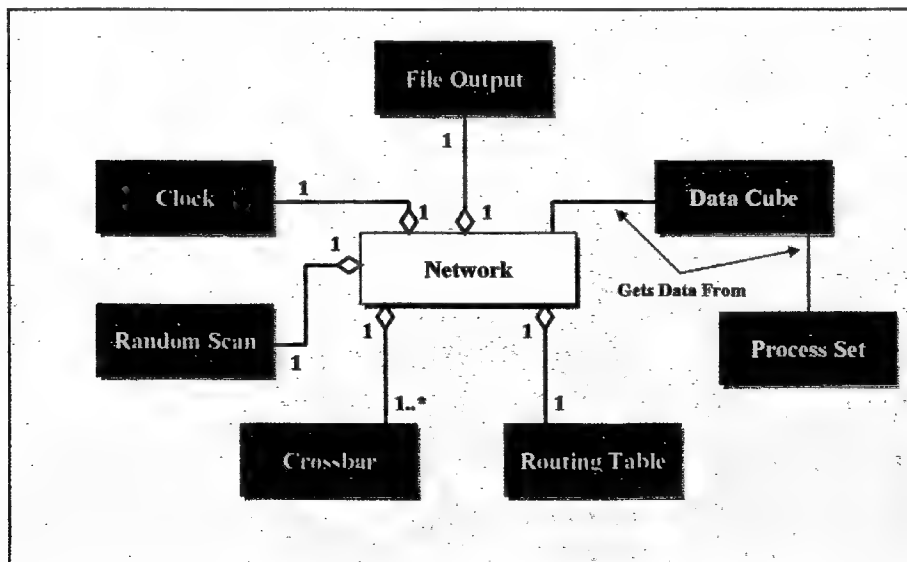


Fig. 14.1 A UML class diagram of the Network class.

A UML class diagram of the Crossbar class is illustrated in Fig. 14.2. The Crossbar class is composed of six Link objects (i.e., two parent links and four child links) and four Compute Node objects. For cases where a Crossbar object is positioned at the lowest level of the fat-tree architecture, the four Compute Node objects are enabled, and the four child Link objects are disabled. Otherwise, the four child Link objects are enabled and the four Compute Node objects are disabled for Crossbar objects not located at the lowest level in the network. Also shown in Fig. 14.2 is a UML diagram of the Compute Node class. Each Compute Node class is composed of two Message Queue objects, one outgoing and one received queue, and two Packet Stack objects, one outgoing and one received stack. A Message Queue object may be composed of zero or more Message objects, and zero or more packets may be included within each Packet Stack object. A more detailed account of each object represented in Fig. 14.2 is discussed in Section 14.2.

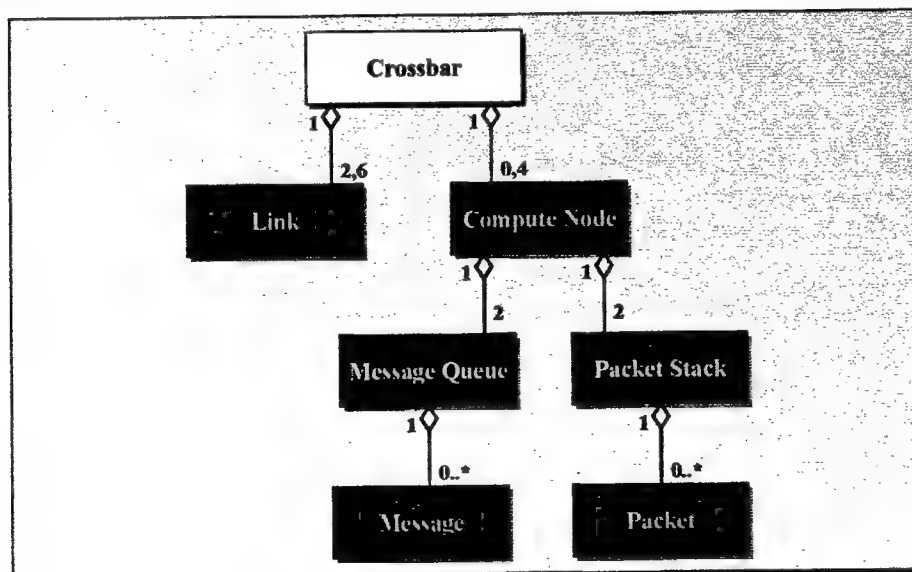


Fig. 14.2 A UML class diagram of the Crossbar class.

Both the Message Queue object and Packet Stack object are composed of data items that traverse the network links during phases of communication. Because a Packet class and a Message class contain common instance variables and operations, an abstract class, Data, was designed to collect the common components of each class (see Fig. 14.3). The

goal of the abstract class definition is to reuse as much of the data and methods as possible. In this case, both the Message class and the Packet class inherit from the abstract Data class. In addition, a Header Route List class composes each Packet class. The Route List class contains one or more Route objects that possess the information necessary to route a packet through the network to its destination.

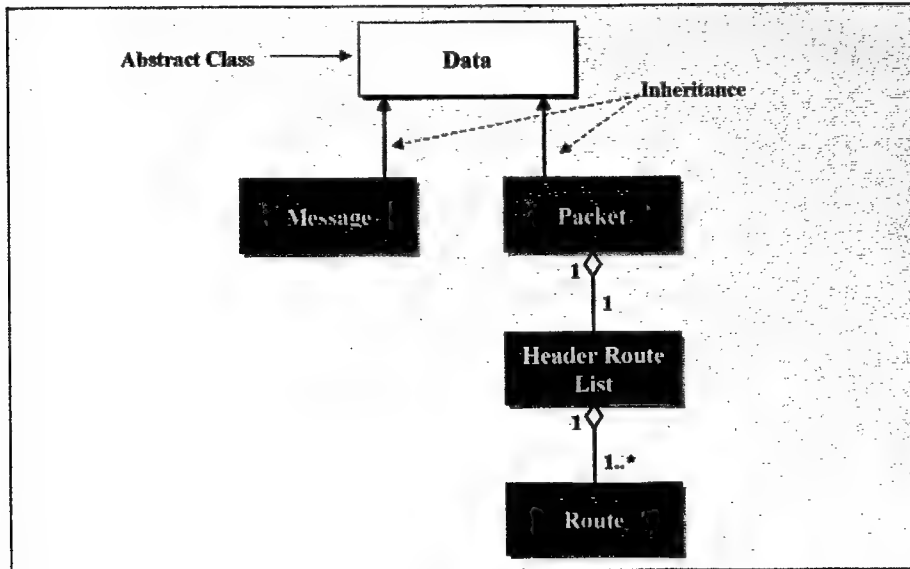


Fig. 14.3 A UML class diagram of the Data class.

## 14.2 Refining Class Operations

Once the classes in the solution space for the development of the simulator were defined, the next step involved formulating the operations for each class. In general, the operations defined for each class may be classified into three broad categories: (1) operations that manipulate the data; (2) operations that perform a computation; and (3) operations that monitor an object for the occurrence of an event [44].

The operations and class refinement of the Network class are shown in Fig. 14.4. Once instantiated, an instance of a Network class dynamically constructs the appropriately sized network based on the required number of CEs. After allocation of the crossbars and the generation of the connections between each level, the instantiated Network object

proceeds with the following two tasks. First, the object enables the correct number of CNs that equates to the number of required CEs. Second, a Routing Table object is dynamically constructed, based on the size of the network, that defines the routing between any two CNs in the network. This information is used to generate the source-to-destination packet header routing information for each packet prior to transmission.

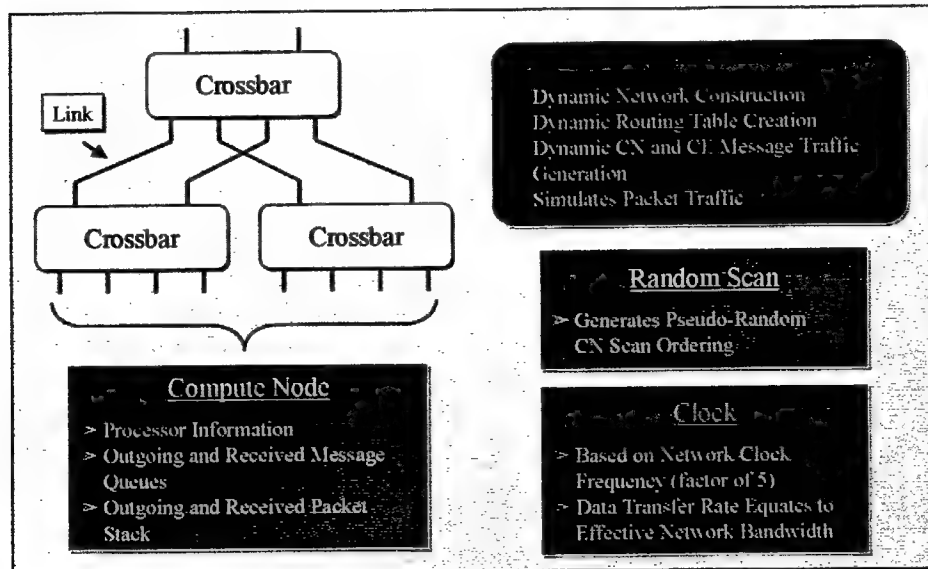


Fig. 14.4 Network class refinement and operations.

Before simulation, the outgoing messages queues of the Compute Node objects are loaded with the appropriate data messages for transmission. Recalling from Fig. 14.1, the Network object *gets data from* the Data Cube. The Data Cube object requests the configuration of the process set from the Process Set object. Using the process set configuration, the Data Cube object generates a CE message traffic matrix, which defines the required communications. The Network object requests the information in the traffic matrix. Based on the values in the matrix, the Network object generates the required message traffic for each CN or CE to accomplish either corner-turn communication pattern. To model (through simulation) the effects associated with how data is mapped onto the CNs of the Mercury system using a sub-cube partitioning approach, the messages in the

outgoing message queues at each CN are randomly ordered prior to message communication.

The complexity of simulating, in software, the message traffic of a real-time embedded parallel system requires significant management. During phases of communication in a real-time embedded system, possibly numerous data items are making connections and transmitting information simultaneously. Simulating the concurrency of such events in a single threaded software simulator is challenging. One approach to solving this problem would be to generate a separate thread of execution for each data packet that is currently transmitting data or attempting to establish a path to its respective destination in the network. Unfortunately, the overhead associated with managing the potentially high volume of currently executing threads at a given time would severely degrade the performance of the simulator. Furthermore, the crossbars and their associated connections would be a shared resource amid all the concurrently executing threads; as a result, critical sections, mutexes, or semaphores would be required to protect the shared resources by ensuring that only one thread can modify a shared resource at any given time. Implementing the necessary requirements to solve the data dependency problem would also require significant processing resources.

A second approach to simulating the real-time aspect of the network involves implementing a single thread of execution and scanning the compute nodes with current packets, during a given clock cycle, in a random order. Although this approach does not realistically simulate the exact execution of the real multicomputer, it does introduce some equality amongst the current packets. Additionally, this approach eliminates any shared resource problem that surfaced in the first approach.

To facilitate the necessity to scan the enabled Compute Node objects in random order, a Random Scan object was incorporated into the design. An instance of a Random Scan object generates a pseudo-random sequence of the enabled CNs. The simulator then proceeds, in the order designated by the Random Scan object, to evaluate and potentially alter the state of a packet at the specified CN. Prior to the execution of pass 1 of each simulation cycle, a new random scan ordering is generated by the instantiated Random Scan object. Details pertaining to the simulation cycle will be discussed later in the section.



The final object encompassing the Network Object is the Clock object. The clock object is based on the RACE multicomputer clock of 40 MHz (i.e., .025  $\mu$ s period); however, the simulation clock operates at 5 times the frequency of the actual clock (i.e., .125  $\mu$ s period). The reasons for selecting a multiple of the true clock cycle are three fold. First, the initial packet start-up cost is consumed in one simulation clock cycle. Second, the time required to arbitrate through a crossbar takes more than one actual clock cycle. Third, because a majority of the operations require more than one cycle to complete and implementing a simulation clock cycle of .025  $\mu$ s would increase the number of required simulation cycles while degrading overall performance, an appropriate multiple of the actual clock frequency was selected for the simulation clock. Obviously, certain side effects result from the multiple-cycled simulation clock. First, because the effective data transfer rate of the actual network is 157.5 MB/s, the simulator transfers approximately 20 data bytes per simulation clock cycle. Second, during one simulation clock cycle, a packet can arbitrate through two crossbars.

A major operation of the Crossbar object entails the implementation of the hardware priority arbitration algorithms. Clearly, the RACEway architecture supports a large number of simultaneous data transactions where each of these transactions can occur along independent paths that have no crossbar ports in common [45]. However, not all data transactions occur along independent paths. Whenever two or more transactions are contending for the same port at a given crossbar, arbitration is required. Recalling from Section 3.2, a user-programmable packet priority is provided to give the user some level of control over the given data transfer transaction's priority [45]. Unfortunately, user-programmable priorities do not eliminate the need for arbitration at the hardware level. For example, the hardware priority associated with a given path through a crossbar (defined by the entry and exit ports on that crossbar) comes into play when the two or more transactions having identical user-defined packet priorities are contending for the same exit port on a given crossbar [45].

Each Crossbar object is configured to implement both the Standard crossbar priority algorithm and the Top-Level crossbar arbitration algorithm (see Fig. 14.5). The selection of the appropriate algorithm depends on the location of the crossbar in the network.

Crossbars located at the top of a hierarchy of crossbars utilize the Top-Level algorithm, and all other crossbars employ the Standard algorithm. Both the Standard and Top-Level priority arbitration algorithms are defined as a function of the transaction entry and exit ports and transaction status. The assignment of the hardware priorities to crossbar transactions paths is far from trivial. Details of these two arbitration algorithms are provided in Section 11.2.

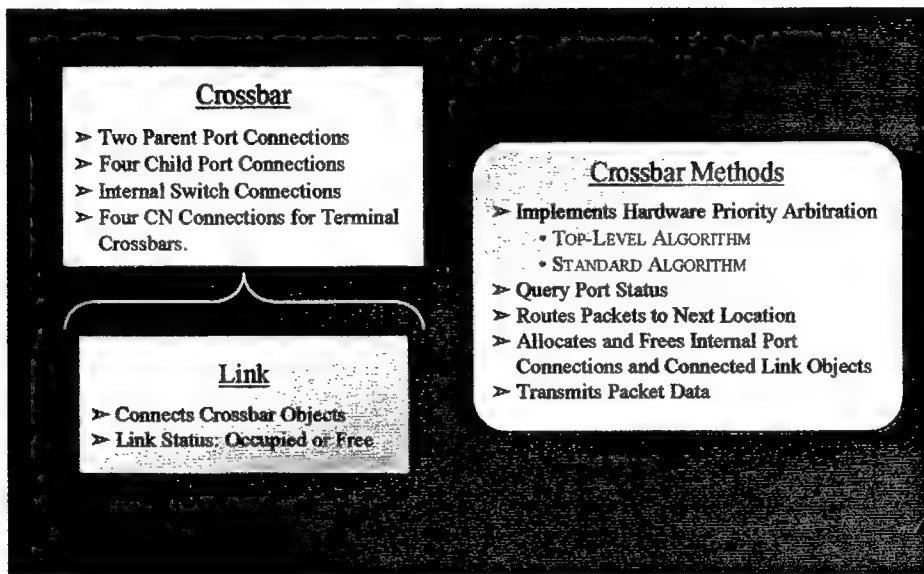


Fig. 14.5 Crossbar class refinement and operations.

In addition to the hardware arbitration, a Crossbar object exams the status of its internal and external ports and routes packets through the crossbar to the next location. A crossbar is also responsible for freeing its connections when a packet has completed or been suspended or killed. Finally, once the connection is established from the source to the destination CN, the crossbar transmits the data through the occupied connection.

The primary focus of the Compute Node class involves the management of the message queues and packet stacks (see Fig. 14.6). Because data is transferred from source to destination node across the RACEway network in packets of up to 2048 data bytes in length, each message in the outgoing message queue must be *exploded* into the appropriate number of corresponding packets. During simulation, the top message in the outgoing

message queue is *exploded* into packets. After each of the packets for that message has been transmitted to its respective destination node, the next message at the top of the queue is *exploded* into packets. This process repeats for each CN until all the outgoing messages queues are empty.

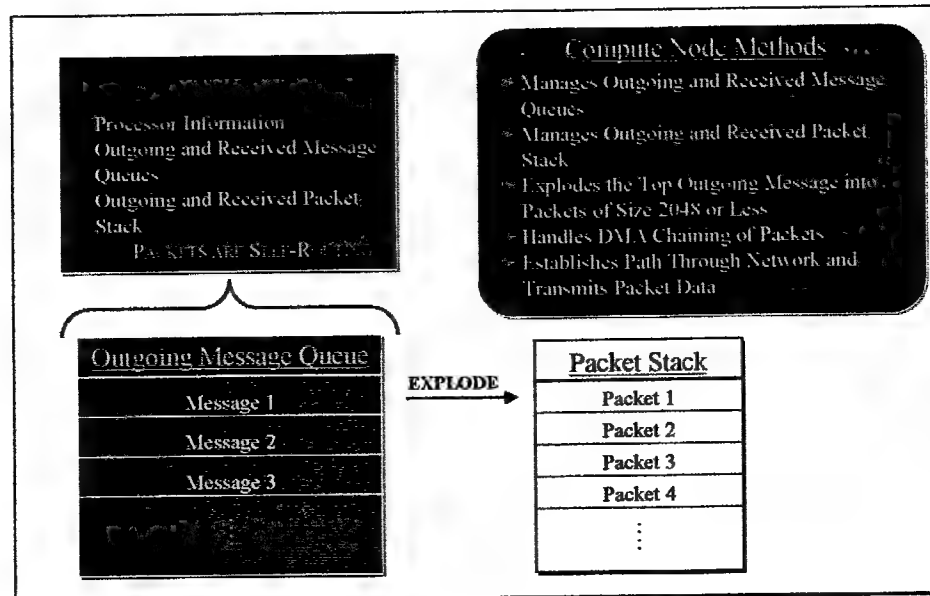


Fig. 14.6 Compute Node class refinement and operations.

During the generation of a packet, a packet header is constructed. The packet header (i.e., the Route List object) contains the information for routing a packet through the sequence of crossbars from the source CN to the destination CN. The routing information is retrieved from the Routing Table object within the given Network object. Via user selection, packets destined for the same location may be direct memory access (DMA) chained together. Essentially, DMA chaining provides a mechanism for transferring blocks of data to the same location without paying the startup cost for each packet. Furthermore, the Compute Node object is responsible for initiating the request for arbitration through the first terminal crossbar. Once access to the terminal crossbar is established, the crossbars are responsible for routing the packet through the network to the destination. Finally, when

an active, transmitting packet is suspended by another packet, the Compute Node object is responsible for generating a new packet composed of the unsent packet data.

### 14.3 UML Statecharts and Activity Diagrams of the Simulator

The UML statechart models are based on finite state machines using an extended Harel state chart notation with modifications to make them object-oriented [21]. A statechart diagram represents a state machine and illustrates the sequence of states that an object goes through during its life cycle. The states are represented by a rectangular box with rounded corners, and the transitions are represented by arrows connecting the states. The initial (pseudo) state is shown as a small solid filled dot representing any transition to the enclosing state [46]. A final (pseudo) state is shown as a small filled dot enclosed by a circle representing the activity in the enclosing state [46]. In a state diagram, the occurrence of an event may trigger a state transition.

A UML Activity model is a variation of a state machine in which the states are activities representing the performance of operations, and the transitions are triggered by the completion of an event [46]. The purpose of an activity diagram is to focus on the flows driven by internal processing. Statecharts and not Activity Diagrams should be used in situations where asynchronous events occur.

Fig. 14.7 shows a UML Activity model of the software simulator. The ovals represent action states, and the transitions, which are triggered by the end of the activity, are depicted as lines with directed arrows. A diamond represents a decision process. After the user enters information relating to the size of the network, the size of the STAP data cube, and the size of the process set, the simulator proceeds to build the network, the data cube, and the process set. Next, the simulator enables the appropriate setting for phase 1 or phase 2 communication traffic phase (described in the following paragraph), DMA chaining, and CN or CE message traffic pattern. Once the input parameters have been initialized, the simulator simulates the designated traffic pattern and displays the timing results.

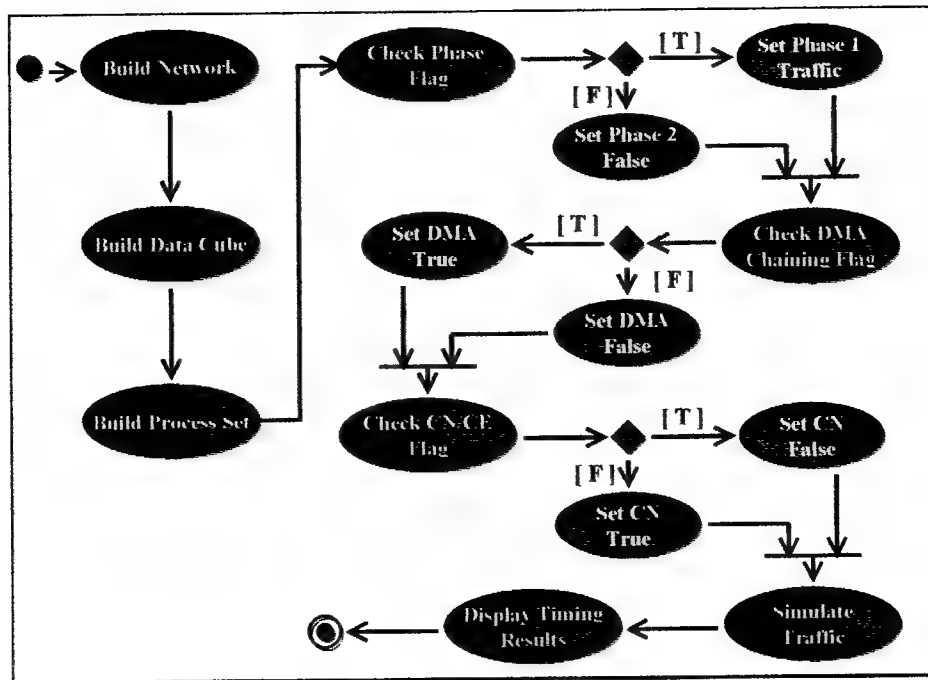


Fig. 14.7 A UML Activity Model of the Simulator.

The simulation of the network traffic is a complex and detailed process. Therefore, the low-level details of the operation of the simulator will not be discussed. Nevertheless, the important control-flow aspects of the simulation warrant explanation. The simulator's design incorporates a two-pass approach to simulating the packet traffic in the network. During one simulation clock cycle, both passes are executed. The primary objectives of the first pass are to build the packets, allocate link and crossbar resources for the packets, and transfers the packet data for each CN. The second pass performs necessary clean-up work that must be accomplished for suspended and completed packets prior to the next simulation clock cycle for each CN. The simulation process continues until all the messages have been transferred. As previously stated, the order in which the enabled CNs are scanned, during each simulation clock cycle, is random. Specifically, prior to the execution of pass 1, during a given simulation clock cycle, a new pseudo-random sequence of the CNs is generated, and CNs are scanned in that order. The CNs are also visited in

random order in pass 2, but the actual ordering of the visits in this pass has no affect on the network performance.

A combination of the Compute Node objects and the Crossbar objects are responsible for the transferring of the packets through the network. The CNs implement the two-pass simulation architecture that is required to deliver a packet from its source node to its destination node. The crossbars handle the arbitration of the connections at the switches as well as the allocation of the interconnected links. A UML statechart best illustrates the process performed by each CN object (see Fig. 14.8). First, an instantiated CN object determines if a current packet has already been removed from the packet stack. If so, the Compute Node object transitions to the Pass 1 state. In this view, the Pass 1 state is a superstate. (The states and transitions that occur during the simulation of Pass 1 will be elaborated on later in this section.) Otherwise, the Packet Stack is evaluated for the existence of an available packet. If the Packet Stack is not empty, the top packet is popped from the top of the stack and becomes the current packet. Afterwards, the CN object transitions to the Pass 1 state. An error code is generated if a failure occurs during the popping of the Packet Stack. In cases where the Packet Stack is empty, the Message Queue is evaluated for available messages. At this point, if the Message Queue is empty, the CN is tagged as completed, and control is passed back to the calling state. Otherwise, the top message is exploded into packets of size 2048 data bytes or less, and the CN transitions to the Pop the Top of Stack state. As illustrated in the figure, a CN is tagged as *done* only when the both the Message Queue and Packet Stack are empty and a current packet does not exist.

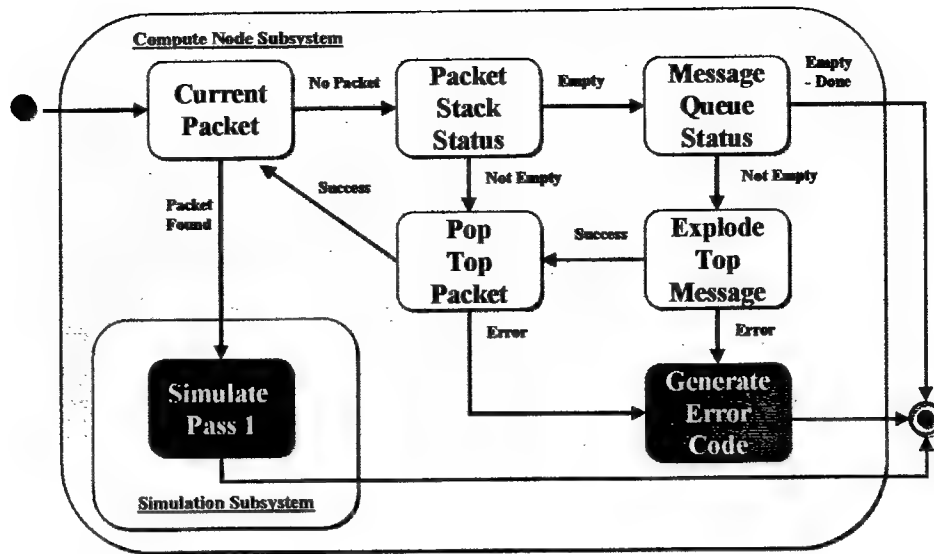


Fig. 14.8 A UML Statechart of the Compute Node class simulation Pass 1.

The Compute Node statechart diagram of the operations executed during Pass 2 of the simulation is significantly more simplistic than that of Pass 1 (see Fig. 14.9). If a current packet exist, a transition to the Pass 2 superstate takes place. On the other hand, if there is not a current packet at the current CN, a transition to the exit state occurs.

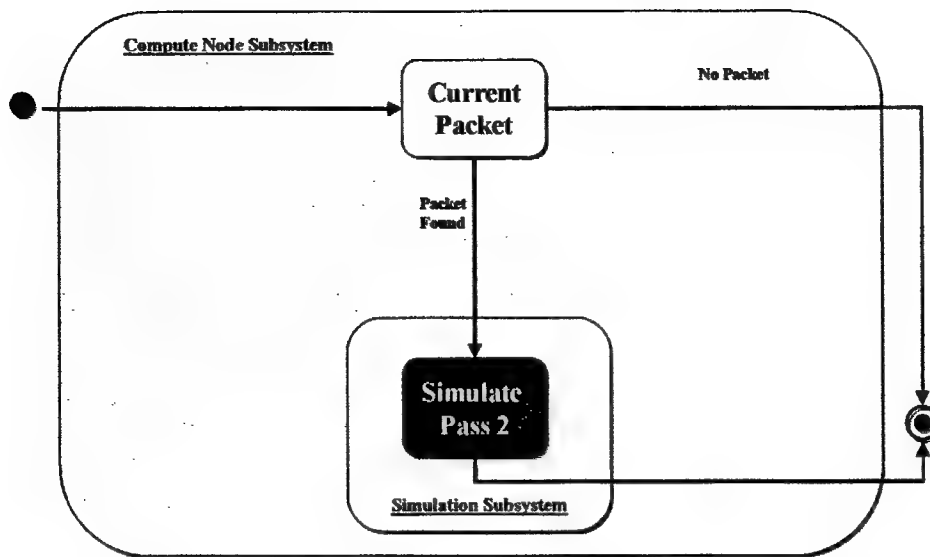


Fig. 14.9 A UML Statechart of the Compute Node class simulation Pass 2.

One of the main objectives of the software simulator is to transfer data (i.e., packets) through the network. The transitions of the Packet objects detail the underlying operation of the simulator. Fig. 14.10 illustrates the state diagram for a Packet object. The blue arrows represent transitions that can occur only during Pass 1 of the simulation, while transitions that take place during Pass 2 are indicated by red arrows. Initially, a given packet begins in either the Start Up state or Ready state. Normally, a packet begins in the Start Up state; however, for cases where DMA chaining of packets to the same destination CN is utilized, the packet's initial state is Ready. A packet in the Ready state is ready for route arbitration to the destination node. After the packet header is constructed, a packet in the Start Up state transitions to the Ready state. A Ready packet may transition to either an Active state, a Blocked state, or stay in the current state. A change to the Active state transpires only if the connection to the destination node is established. If the packet successfully acquires a partial path through the network but does not occupy a complete route to its destination, the packet transitions to the Blocked state. Finally, if the packet is unable to make any progression through the network, the packet remains in the Ready state.



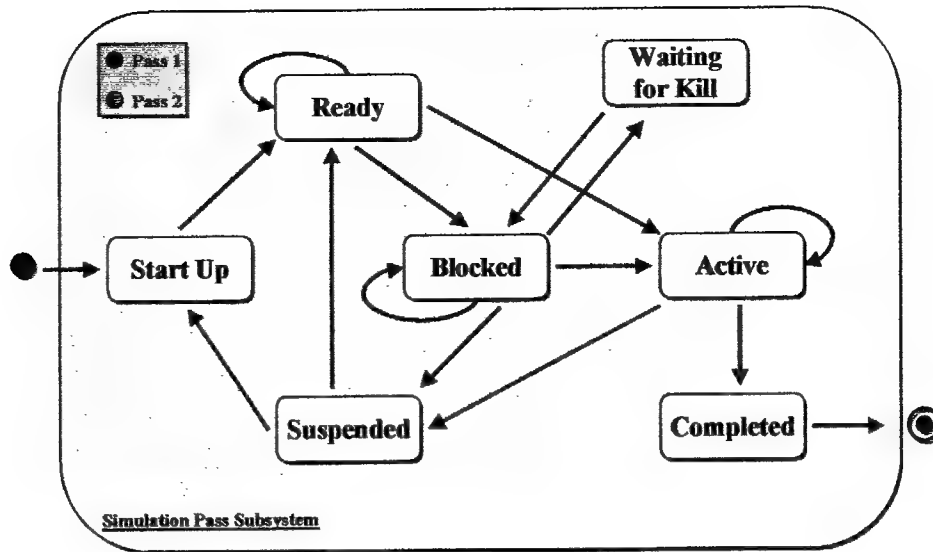


Fig. 14.10 A UML Statechart of the Packet class

A Blocked packet is categorized as a packet that occupies at least one connection in the network but has yet to make a complete connection to its final destination. A packet may be blocked for two reasons. First, the exit port that the packet requires at a particular crossbar is occupied by another packet, and the hardware arbitration algorithm does not allow for the suspension of that packet. Second, the simulation clock cycle completed before the arbitration to the destination was achieved. A Blocked packet may transition to any of four possible states. It changes to the Active state if a connection to the destination node is established. A Blocked packet may also transition to the Suspended state if it is terminated by a another packet. If the currently blocked packet suspends another packet, the current packet transitions to the Waiting for Kill state. Finally, a packet may remain in the Blocked state for the same two reasons that it first arrived in this state.

A packet that is transmitting its contents is in the Active state. Throughout the transfer of packet data, the packet remains in the Active state unless suspended. A packet in the Active state may be suspend by another packet based on the complicated hardware arbitration algorithms at the crossbar level. Once an Active packet transfers its data, a transition to the Completed state occurs.

Packets that are terminated prior to completion transition to the Suspended state. While in the Suspended state, packets that were previously Active require the construction of a new packet with the remaining data content. Additionally, the connections occupied by the packet are freed during the next simulation clock cycle, and the newly formed packet transitions to the Start Up state. Packets arriving at the Suspended state that were previously in the Blocked state are handled differently. Because none of the data was transferred, a new packet is not required; however, the packet header does require updating. After updating the packet header with new routing information, a transition to the Start Up state occurs.

Finally, packets in the Wait for Kill state are waiting for a suspended packet to release its occupied connections. During the pass 2, the waiting packet transitions to the Blocked state. Once in the Blocked state, the packet may be able to gain access to the newly freed connections in the next clock cycle, but because this is a real-time system, there is no guarantee that the connection will be available in the next clock cycle. For instance, another packet may allocate the connection before the waiting (now currently blocked) packet can occupy the connection.

#### 14.4 Implementation

The software simulator was written in Java, although the design is language independent. Java was selected for its portability and the need for Internet access to the simulator. The actual implementation, which is based on the design described in this section, was developed in Borland's JBuilder 1.0. Although the studies on network traffic described in the next chapter were conducted based on STAP algorithms, the simulator is designed to simulate any communication pattern requirement. That is, the simulator can take as input any CE traffic matrix. After implementation, the software simulator was extensively tested prior to the collection of data.

## CHAPTER XV

### PRELIMINARY NUMERICAL STUDIES

Recalling that the objective of this research is the design and implementation a network simulator to model the effect data mapping and communication scheduling has on the performance of a STAP algorithm on the Mercury RACE system. Determining the optimal communication schedule of queued messages during the two phases of data re-partitioning is beyond the scope of this research. In addition to scheduling, one could consider the complexity of determining the optimal routing of the queued messages (recall that there are multiple paths connecting pairs of CNs in the RACEway system). The goal of the research is not to *solve* these types of optimizations, but to *simulate* the effects different schedules and data mapping have on performance. The scope of the research involved the investigation of the following four areas: process set configuration, CN and CE message traffic, adaptive routing settings, and DMA chaining options.

#### 15.1 Process Set Configuration

In a sub-cube bar partitioning approach, the STAP data cube is distributed to the available CEs by partitioning the data cube into sub-cube bars by applying a two-dimensional process set to the data cube. Before processing can take place at the next phase, the data vectors must be re-distributed to form contiguous vectors of the next dimension. Five separate studies were conducted related to the size of a process set. Each simulation involved recording both the phase 1 and phase 2 completion times for fifty randomly selected schedules. After these fifty completion times for each phase were collected, the resulting data was placed in histogram format.

### 15.1.1 Performance Metric for a 3x12 and 4x12 Process Set

Fig. 15.1 shows the timing results collected from both a 3x12 and a 4x12 process set. For a 3x12 process set, which includes thirty-six CEs or 12 CNs, the horizontal dimension is 3, and the vertical dimension is 12. Intuitively, a 4x12 process set contains 16 CNs or 48 CEs, and the horizontal dimension is 4 while the 12 represents the vertical dimension. The notation above the illustrated graph, which is consistent throughout this chapter, defines the additional parameters of the simulation. The first label, CN, signifies that the message traffic pattern generated was CN traffic. The parameters of the STAP data cube are defined by the sizes of the range (R) dimension, the pulse (P) dimension, and the channel (C) dimension. For this simulation, an antenna array including sixteen channels obtained two hundred range samples from a CPI of twenty-two pulses. Additionally, adaptive routing is used to adaptively route the packets that enter the child ports to an exit parent port. In this instance, the F parent port is evaluated prior to the E parent port (i.e., adaptive routing, F first). The x-axis expresses the time line in milliseconds, and the y-axis denotes the tallied appearance of a particular time interval.

Notice that the communication time for the 3x12 process set is zero. Recall that each CN contains 3 CEs, so in this case, the data required for the second processing phase is located on the correct CN because the horizontal dimension contains exactly 3 CEs. For the CN 16 case, where the horizontal dimension contains 4 CEs, communication is required before processing of the next dimension can commence. In this instance, the 4x12 process set is outperformed by the 3x12 process set.

An examination of the communication times for the second corner-turn reveals a different outcome (see Fig. 15.2). In this simulation, the communication times are quite similar, although the CN 12 configuration again records the smallest time by approximately .25ms. Intuitively, the CN 12 configuration has fewer messages to communication because there are fewer processors and more data locate at each CN. However, in the CN 16 case, the STAP data is distributed to more processors, which results in a larger number of messages during the phase 2 corner-turn distribution of the data.

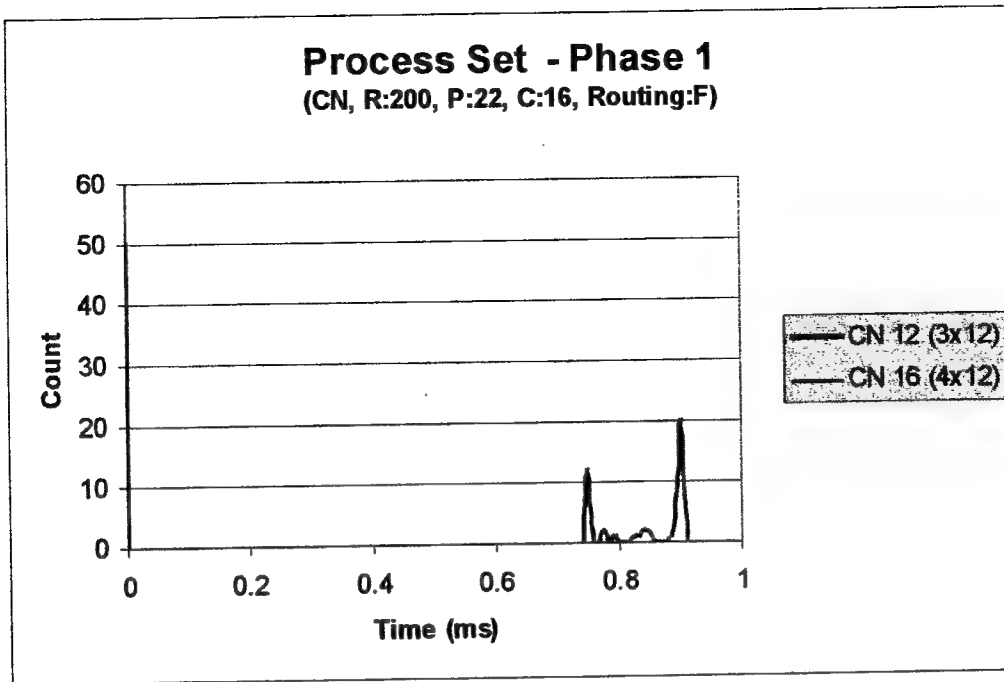


Fig. 15.1: Phase 1 performance metric for a 3x12 and 4x12 process set with range: 200, pulses: 22, channels: 16, and adaptive F routing.

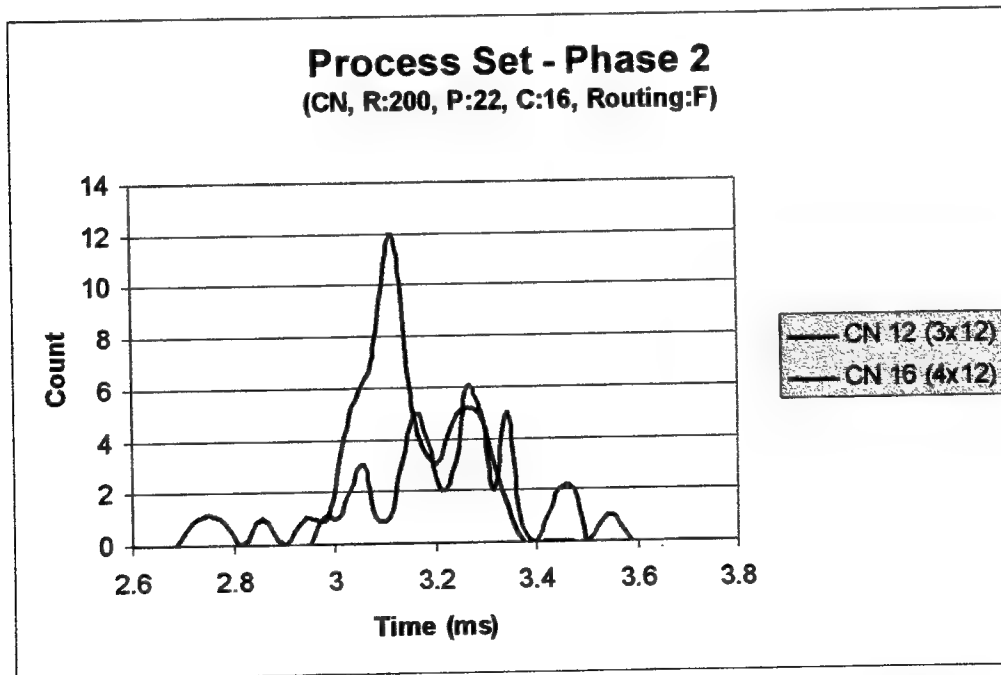


Fig. 15.2: Phase 2 performance metric for a 3x12 and 4x12 process set with range: 200, pulses: 22, channels: 16, and adaptive F routing.

### 15.1.2 Performance Metric for a 6x4 and 4x6 Process Set

Fig. 15.3 illustrates the phase 1 simulation timing results obtained for an 8 CN system configured with the STAP data cube partitioned by a 6x4 and a 4x6 process set. In this example, the communication pattern for the 6x4 process set records the same time for each iteration. Because the horizontal dimension is a factor of three, the communication pattern is a more predictable. In the first phase, CEs are sending messages to other CEs in the same row (i.e., the horizontal dimension). Additionally, because the data cube size is particularly large (i.e., 800 range samples, 32 pulses, and 22 channels), the messages are of significant size, which translates to a high number of packets sent from the same source node to the same destination node. Furthermore, there is only one message in the outgoing queue of each CN, so the number of possible orderings at each CN is one. Unfortunately, this is not the case with the 4x6 process set. In this instance, there is more than 1 message in the outgoing queues, and the messages are not uniform in size, resulting in a more diverse recording of completed simulation times.

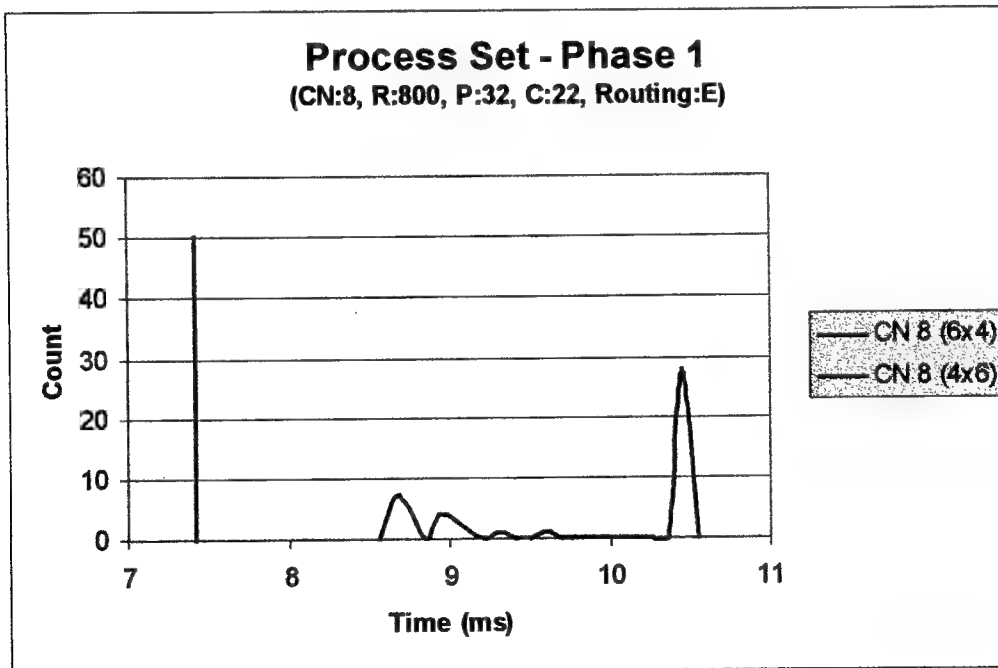


Fig. 15.3: Phase 1 performance metric for a 6x4 and 4x6 process set with range: 800, pulses: 32, channels: 22, and adaptive E routing.

In contrast to the different communication times in phase 1, the times in phase 2 have less variation between the two process sets (see Fig. 15.4). Furthermore, the completion times for phase 2 are a factor of 3 to 4 greater than phase 1 times because there is more data to distribute in this phase. In fact, the phase 2 communication dominates the total completion time for each case presented in this chapter. This simulation reveals that the 6x4 process set size would yield the shortest total completion time.

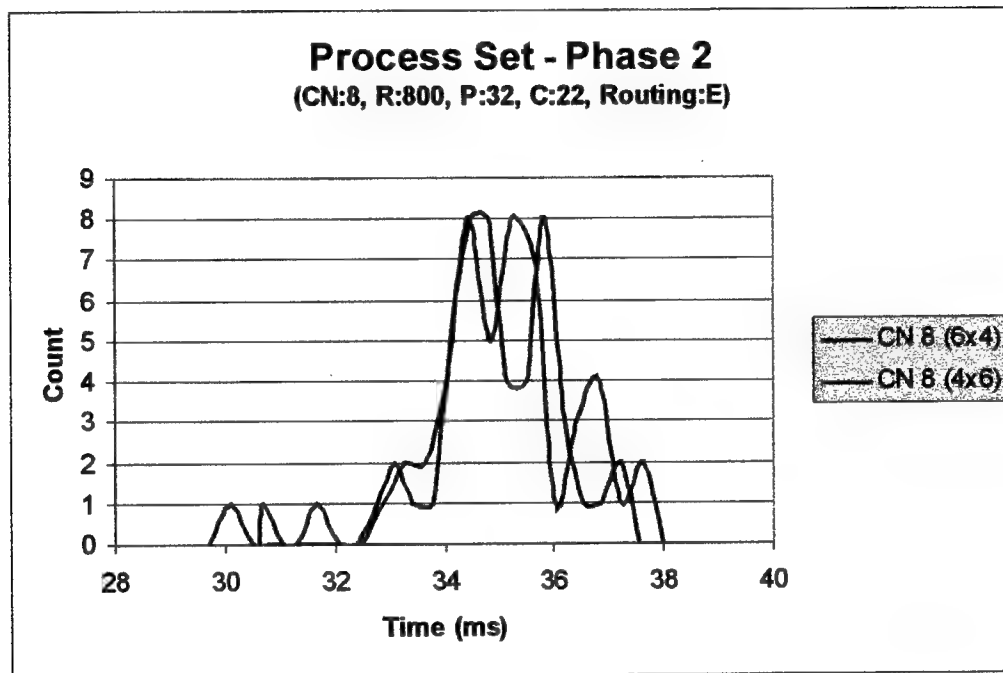


Fig. 15.4: Phase 2 performance metric for a 6x4 and 4x6 process set with range: 800, pulses: 32, channels: 22, and adaptive E routing.

### 15.1.3 Performance Metric for a 12x3, 9x4, 6x6, and 4x9 Process Set

The object of this simulation is to illustrate, for a given 12 CN system configuration, the effects the process set choice can have on performance. Figs. 15.5 and 15.6 display the simulation timing results for a 12x3, 9x4, 6x6, and 4x9 process set for communication phases 1 and 2, respectively. For a 6x6 process set, the communication pattern for phase 1

is very regular which results in a low degree of variation of the recorded completion time. In addition, the 4x9 process set performs better in phase 1 than both the 12x3 and 9x4 process set. Furthermore, this simulation unveiled that the phase 1 communication benefited from lower horizontal dimension process set value. However, it is important to note that this may not be true for all horizontal cases (i.e., data cube sizes, routing, options, etc.).

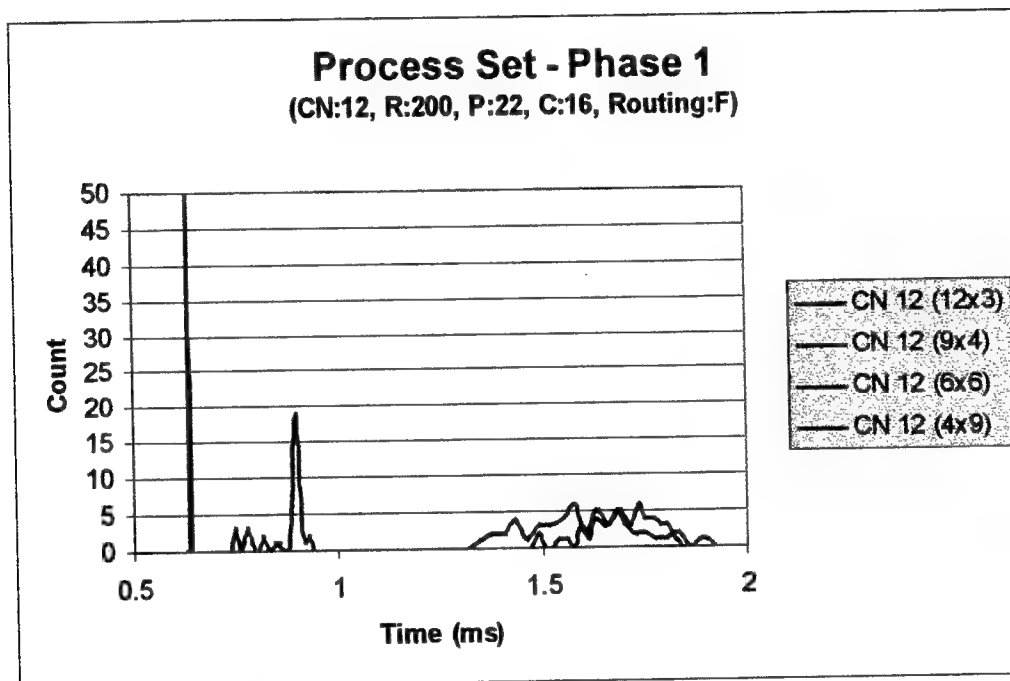


Fig. 15.5: Phase 1 performance metric for a 12x3, 9x4, 6x6, and 4x9 process set with range: 200, pulses: 22, channels: 16, and adaptive F routing.

In phase 2, the variation of completion times ranged from 2.9 to 3.75 milliseconds, with the 12x3 process set on average performing poorest. The 4x9 arrangement of compute elements registers the lowest communication time, while the remaining three process sets recorded slightly higher times.

Once a process set is selected, the dimension sizes of the process set may not change between phases 1 and 2. Recall that phase 2 communication depends on the resultant



communication of phase 1. So a change in the dimension of the phase 2 process set does not reflect the actual location of the source data. As a result, the process sets with the lowest times for corner-turn communication phases 1 and 2 must have the same process set. However, the ordering of the CEs within the structure of the process could be altered between phases.

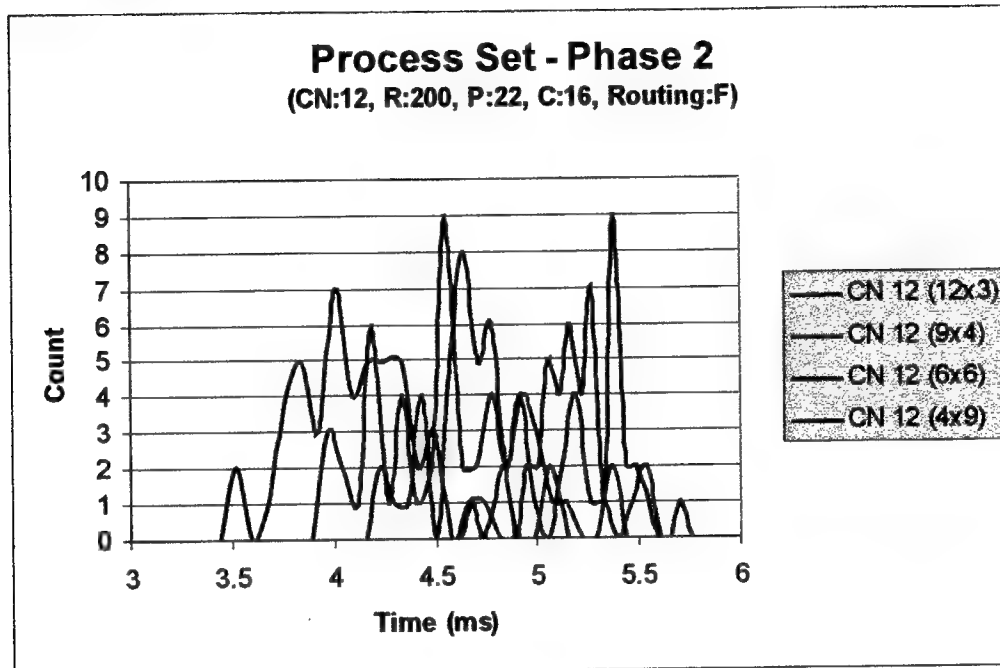


Fig. 15.6: Phase 2 performance metric for a 12x3, 9x4, 6x6, and 4x9 process set with range: 200, pulses: 22, channels: 16, and adaptive F routing.

#### 15.1.4 Performance Metric for a 3x12, 12x3, and 4x9 Process Set

One of the possible process sets neglected in Section 15.1.3 was the 3x12 set. The 3x12 process set was not included in the above section because of the limited graphing space. The 3x12 process set requires no communication during the phase 1 communication cycle due to the dimensions of the process set (see Fig. 15.7). The horizontal dimension of the process set corresponds to the number of available CEs on a

given CN; as a result, the data required for both range compression and Doppler filtering are currently available on the same CN. In this instance, there is no data transfer requirement for phase 1. The 12x3 and 4x9 process set, which are elaborated on in Section 15.1.3, are include here for comparison only.

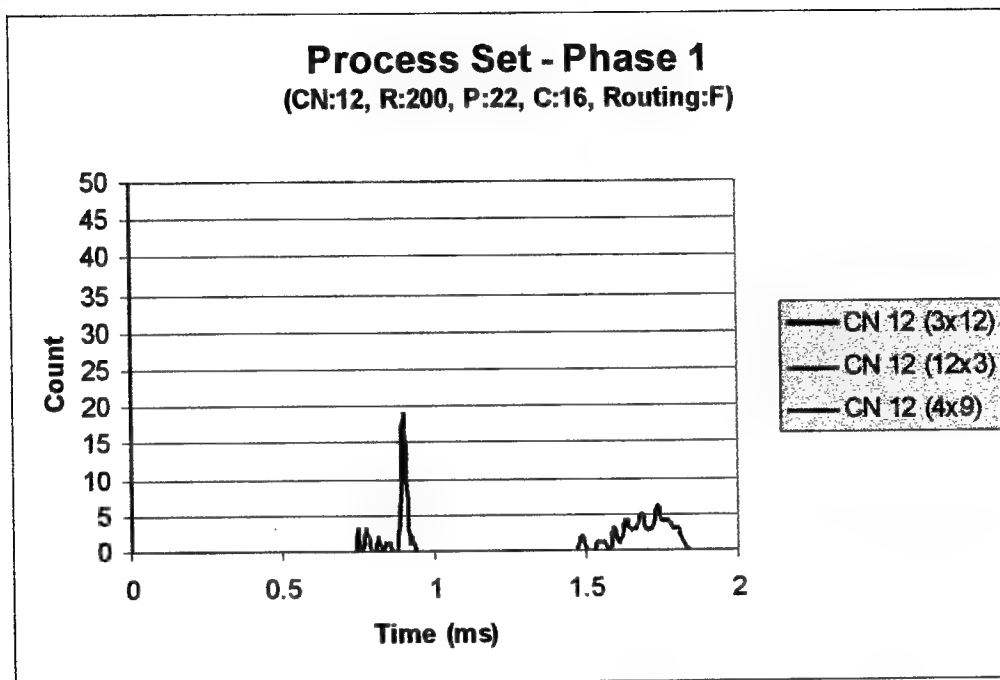


Fig. 15.7: Phase 1 performance metric for a 3x12, 12x3, and 4x9 process set with range: 200, pulses: 22, channels: 16, and adaptive F routing.

In the second corner-turn phase, the 3x12 partitioning of the data cube also performs the data redistribution in the shortest period (see Fig. 15.8). Recalling from Section 15.1.3, the 4x9 process set was the best overall performer; nevertheless, the 4x9 process set is, on average, roughly a millisecond slower compared to the 3x12 process set in phase 2. In addition, because the 3x12 process set does not require communication in the first phase, it is the best process set for the listed problem parameters.

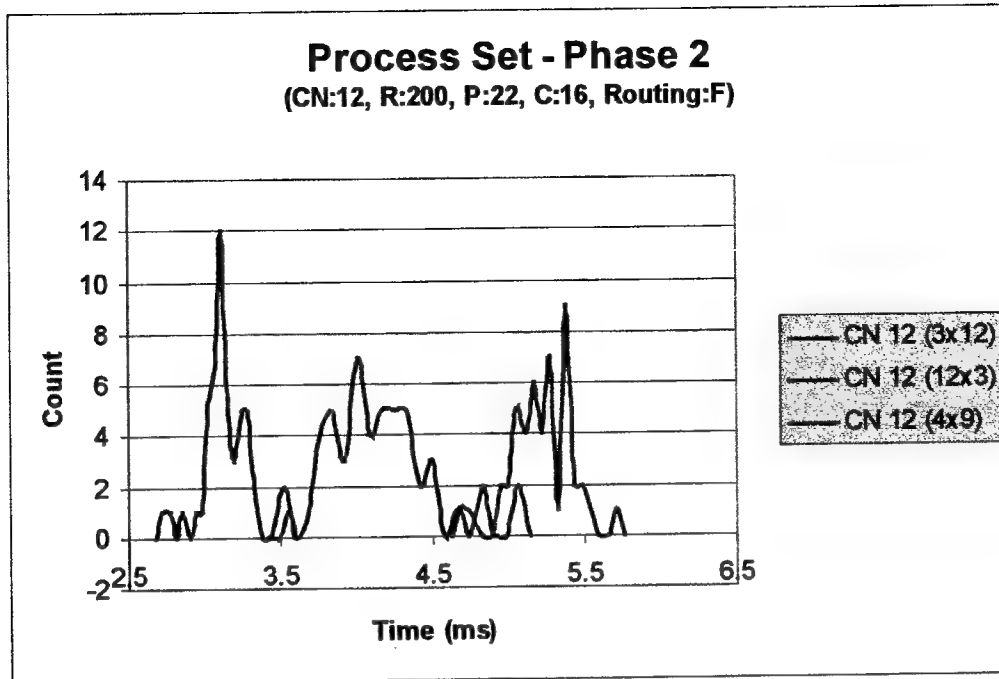


Fig. 15.8: Phase 2 performance metric for a 3x12, 12x3, and 4x9 process set with range: 200, pulses: 22, channels: 16, and adaptive F routing.

#### 15.1.5 Performance Metric for a 12x4, 8x6, and 4x12 Process Set

The final process set performance metric compares a subset of the possible process set combinations for a 16 CN system. If the minimum sized dimension of a process set is 2, the number of possible process set size combinations is 10. For illustration purposes, only a subset of the possible process sets is presented for a set of fixed problem parameters. Fig. 15.9 shows the phase 1 communication times recorded for a 12x4, 8x6, and 4x12 process set. In phase one, there is slightly less than a 50% difference in the separation between that shortest and longest time recorded. As illustrated in the graph, the 4x12 process set is, on average, approximately 20% faster than the 8x6 process set and 28% percent faster than the 12x4 process set. The 8x6 has a longer interval of recorded

completed times, which indicates that the ordering (i.e., the scheduling) of the messages impacts the performance of this process set more than the others.

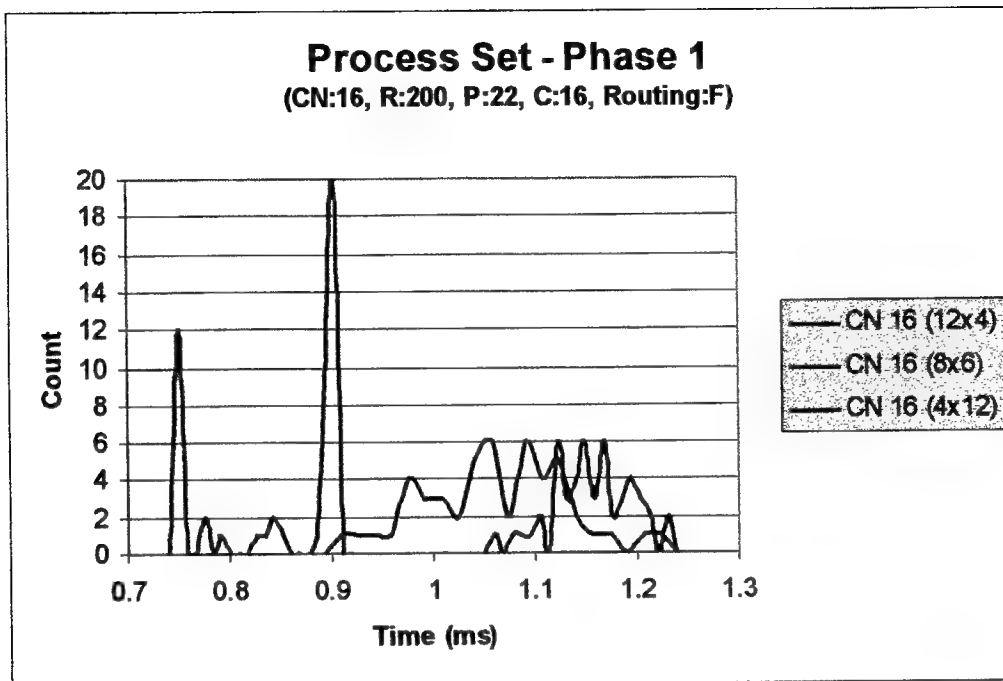


Fig. 15.9: Phase 1 performance metric for a 12x4, 8x6, and 4x12 process set with range: 200, pulses: 22, channels: 16, and adaptive F routing.

The 4x12 process set also produces the best completion time for the phase 2 communication pattern (see Fig. 15.10). In fact, the performance increase is almost thirty percent. The 12x4 and 8x6 process sets generate comparable results during phase 2, but each are roughly 1 to 1.25 ms slower in the best case. In addition, there is around a 1 ms variation in completion times which indicates message ordering affects performance.

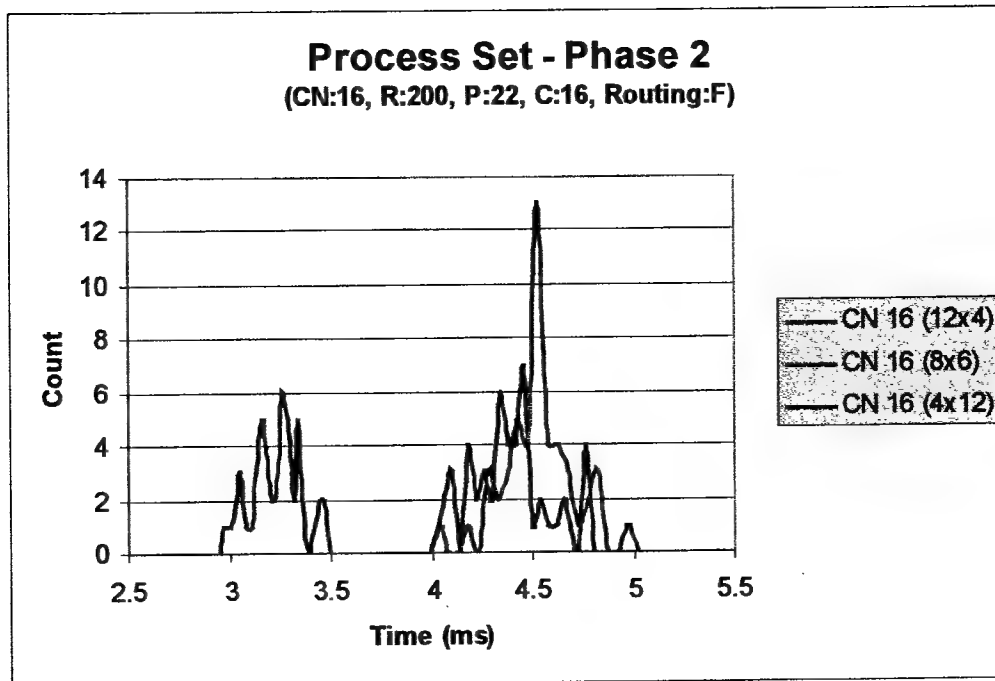


Fig. 15.10: Phase 2 performance metric for a 12x4, 8x6, and 4x12 process set with range: 200, pulses: 22, channels: 16, and adaptive F routing.

## 15.2 Compute Node and Compute Element Traffic Investigation

The simulator is capable of generating either compute node traffic or compute element traffic. Messages from the same CN to the same destination CN are combined to form one message in a CN traffic approach to message generation. In contrast to CN traffic, each CE generates its own message to the destination CE, and messages to the same "CN destination" are not combined together in a CE traffic approach. In general, CN traffic contains larger and fewer messages than CE traffic. Simulations involving CE traffic contain more messages but each message is smaller. Three distinct investigations were conducted related CN and CE traffic. As in Section 15.1, each simulation involved recording both the phase 1 and phase 2 completion times for fifty simulations. After the fifty completion times for each phase were collected, the resulting data was placed in a histogram format.

### 15.2.1 Message Traffic Performance Metric for 16 CN (12x4) Configuration

A message traffic investigation of a 16 CN system with a 12x4 process set configuration is provided in Fig. 15.11. From the graph, the CE traffic is approximately 10% faster than the CN traffic on average. In a CN traffic approach to message generation, the larger messages tend to allocate the same path through the network for longer back-to-back periods. However, when CE traffic is utilized, there is an increase in the number of messages, but the messages are smaller. By having more messages, the number of possible orderings in the outgoing queues increase. Additionally, because the messages are smaller and the orderings more diverse, the same CN is not necessary requesting a connection to the same destination node repeatedly. Furthermore, notice the variation in communication times for CE traffic. This indicates that the ordering of the messages in the queues affects the completion time of the communication pattern.

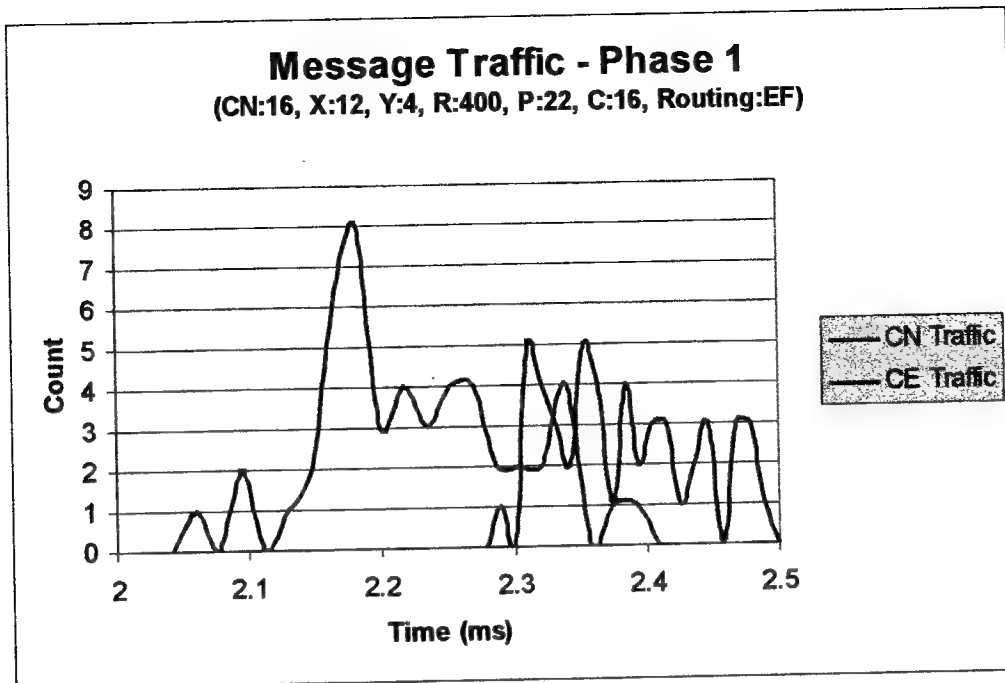


Fig. 15.11: Phase 1 message traffic performance metric for a 16 CN (12x4) configuration with range: 400, pulses: 22, channels: 16, and adaptive E/F routing.

The phase 2 message traffic results for the same set of parameters are opposite of phase 1 (see Fig. 15.12). In phase 2, the CN traffic appears to dominate the CE traffic. In fact, the CE traffic is approximately 25% slower than the CN traffic. For this scenario, it would be possible and advisable to employ a CE distribution of messages in phase 1, and a CN deployment of messages in phase 2. Also worth mentioning in the phase 2 CN message traffic results is the variation in completion time. This again indicates that the ordering of the outgoing messages is correlated to the completion time.

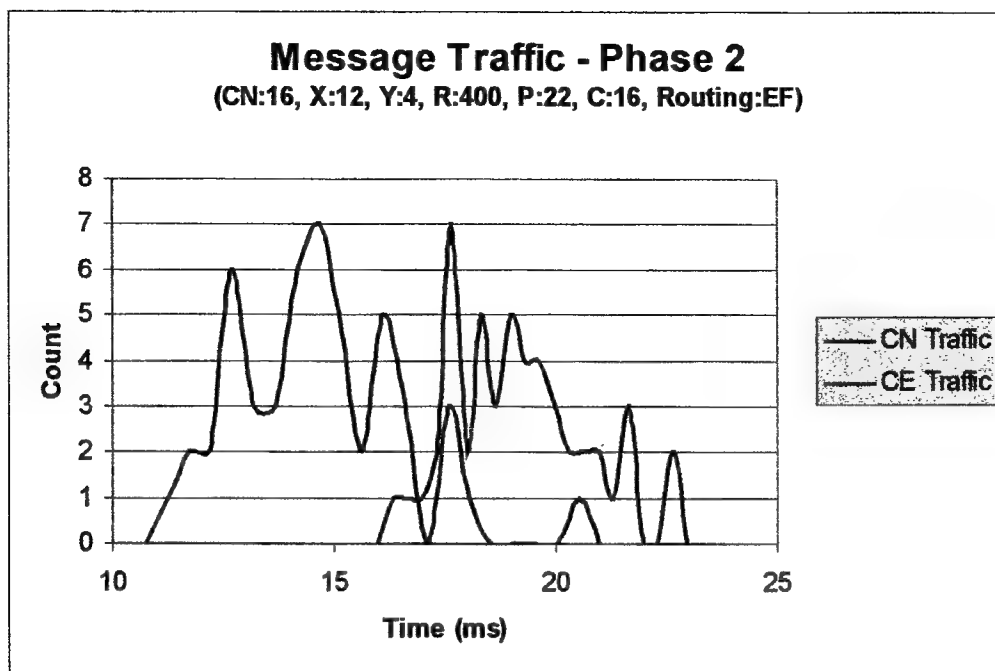


Fig. 15.12: Phase 2 message traffic performance metric for a 16 CN (12x4) configuration with range: 400, pulses: 22, channels: 16, and adaptive E/F routing.

### 15.2.2 Message Traffic Performance Metric for 16 CN (6x8) Configuration

In Section 15.2.1, the CN and CE traffic was examined for a 16 CN system configured with a 12x4 process set. In this section, a 16 CN system is studied, but the process set configuration is 6x8. Fig. 15.13 illustrates the results from the phase 1 corner-turn of the STAP data cube. In this scenario, the times for CN and CE message traffic in phase 1 are almost identical. In addition, the completion time for the 6x8 process set is approximately 60% faster than for the 12x4 process set in Section 15.2.1.



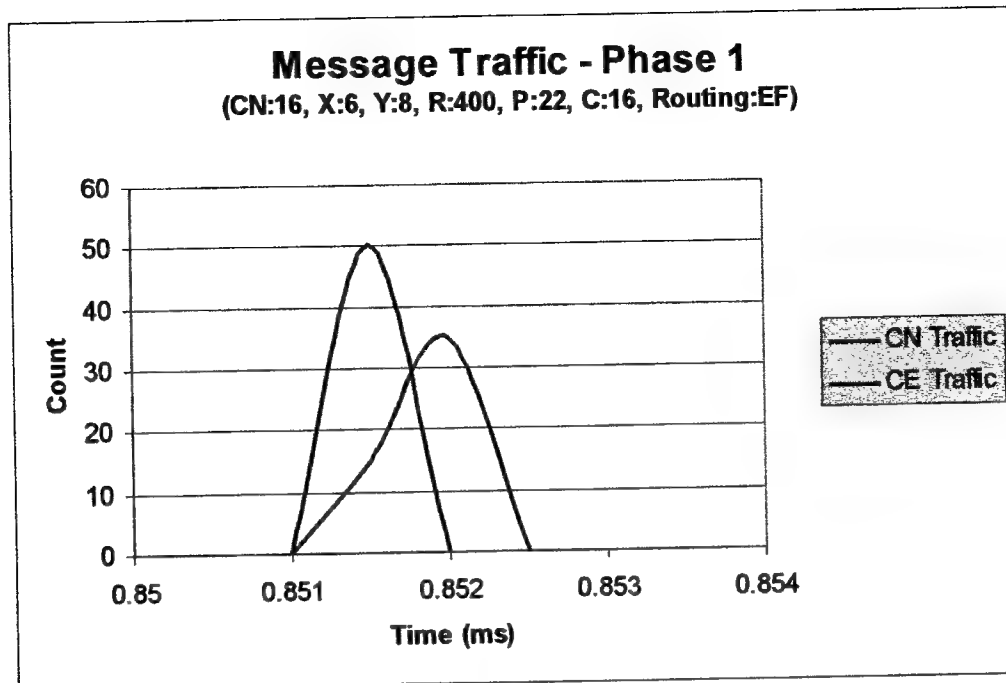


Fig. 15.13: Phase 1 message traffic performance metric for a 16 CN (6x8) configuration with range: 400, pulses: 22, channels: 16, and adaptive E/F routing.

As in Section 15.2.1, the CN traffic performs better than the CE traffic in phase 2 (see Fig. 15.14). On average, the CN traffic is approximately 30% quicker than the CE traffic. The variation in the CN traffic completion times also indicates that the ordering of the messages in the queues at each compute node is related to the performance of the communication pattern.

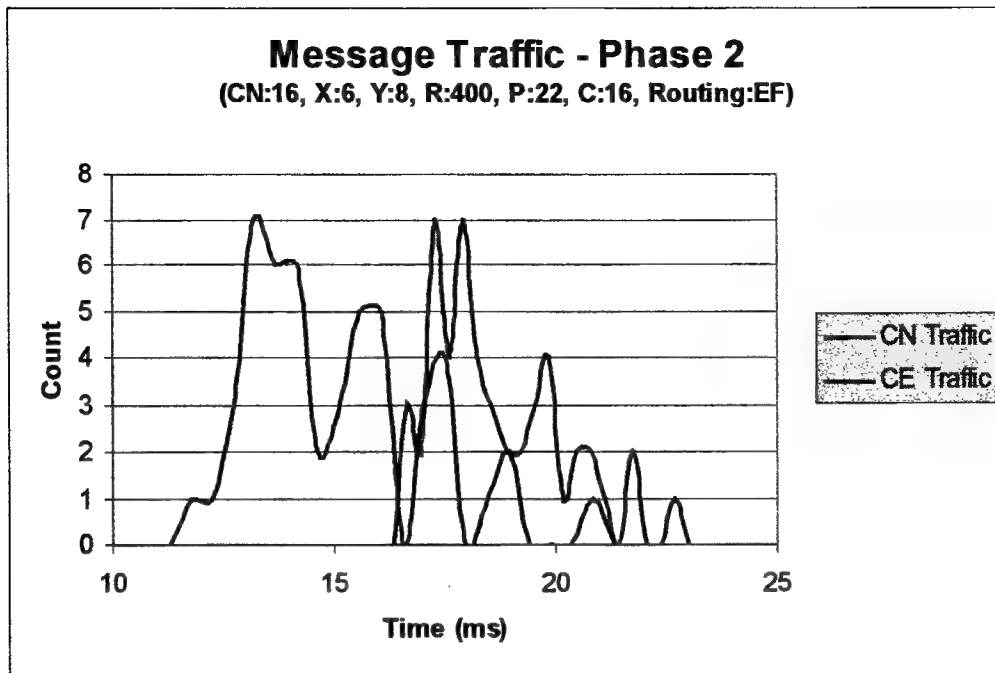


Fig. 15.14: Phase 2 message traffic performance metric for a 16 CN (6x8) configuration with range: 400, pulses: 22, channels: 16, and adaptive E/F routing.

### 15.2.3 Message Traffic Performance Metric for 12 CN (6x6) Configuration

In the previous two investigations, the communication phase prior to QR-Decomposition (i.e., phase 2), was best suited to CN traffic. However, this scenario will reveal that CE traffic could be best served for the phase 2 corner-turn. Fig. 15.15 illustrates the differences recorded in the completion times of both CN and CE traffic for a 12 CN system with a 6x6 process set configuration. In this example, the CE traffic is only slightly slower than the CN traffic in phase 1. In fact, the overall difference is less than 1%.

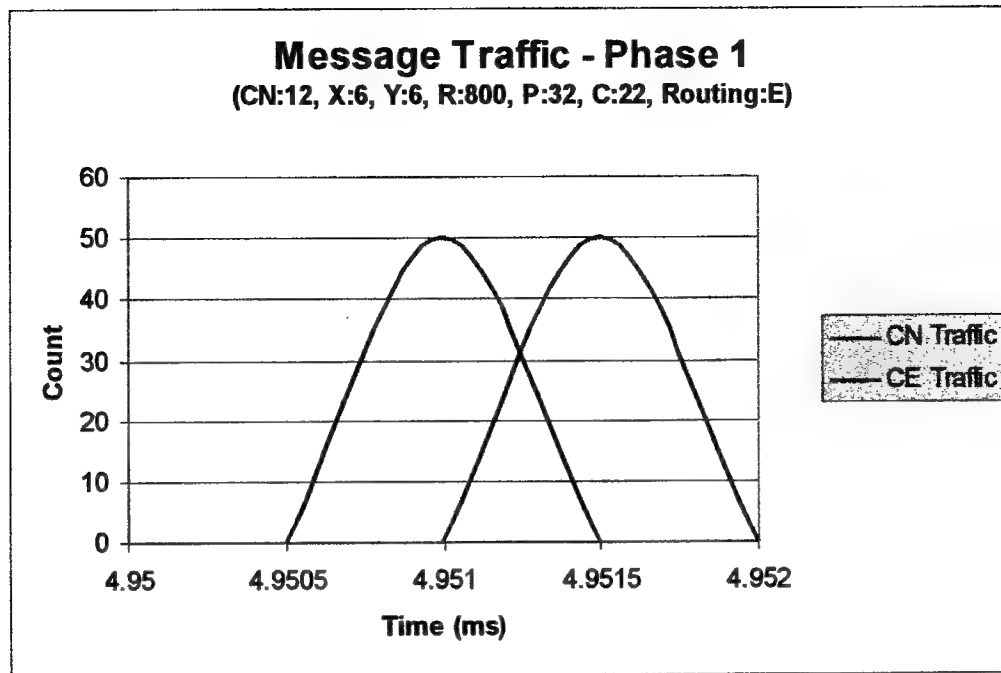


Fig. 15.15: Phase 1 message traffic performance metric for a 12 CN (6x6) configuration with range: 800, pulses: 32, channels: 22, and adaptive E routing.

In phase 2, the best completion times for both CN and CE traffic are approximately identically (see Fig. 15.16). In the above two examples, the CN traffic clearly outperformed the CE traffic. For this example, the number of CNs, the size of the data cube, and the arrangement of the process set were altered to demonstrate that CE traffic, under the right conditions, could prove valuable during the phase 2 communication. In this simulation, the results indicate that the number of CNs, size of the STAP data cube, and the layout of the process set significantly affects the message traffic performance of communication phases on the Mercury network. In addition, there is a 10% variation in the completion times for the CN traffic. Consequently, the ordering of the messages can both improve and degrade the communication performance.

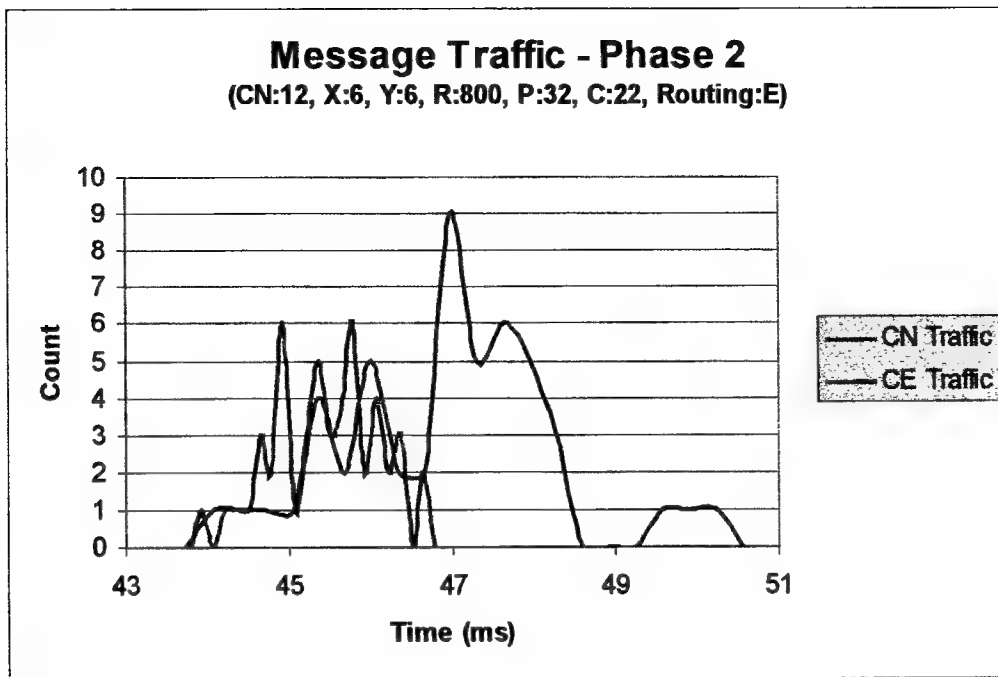


Fig. 15.16: Phase 2 message traffic performance metric for a 12 CN (6x6) configuration with range: 800, pulses: 32, channels: 22, and adaptive E routing.

### 15.3 Adaptive Routing Configurations

An adaptive routing technique may be used to route packets through the connections at each crossbar. Packets exiting one of the parent ports may be routed to the other parent port if the first port is not free and adaptive routing is used. Because each crossbar contains two parent ports, the adaptive routing option may be set to evaluate either E or F first. Additionally, a combination of both adaptive E and adaptive F could be used to arbitrate packets through the interconnection of crossbars. The following sections illustrate the effects of adaptive routing on the communication time. As before, each simulation involved recording both the phase 1 and phase 2 completion times for fifty simulations.

### 15.3.1 Adaptive Routing Performance Metric 1 for a 16 CN (8x6) Configuration

In the first simulation, a 16 CN system configured with an 8x6 process set was studied. The STAP data cube size for this simulation was eight hundred range bins, thirty-two pulses, and twenty-two channels. For this simulation the combination of adaptive E and adaptive F routing recorded the shortest communication times (see Fig. 15.17). Additionally, the adaptive E/F configuration accounted for the smallest completion time interval. When configured with adaptive E routing only, the simulation record the widest variation in completion times. Again, this indicates that the scheduling of the messages impacts the performance of the communication pattern.

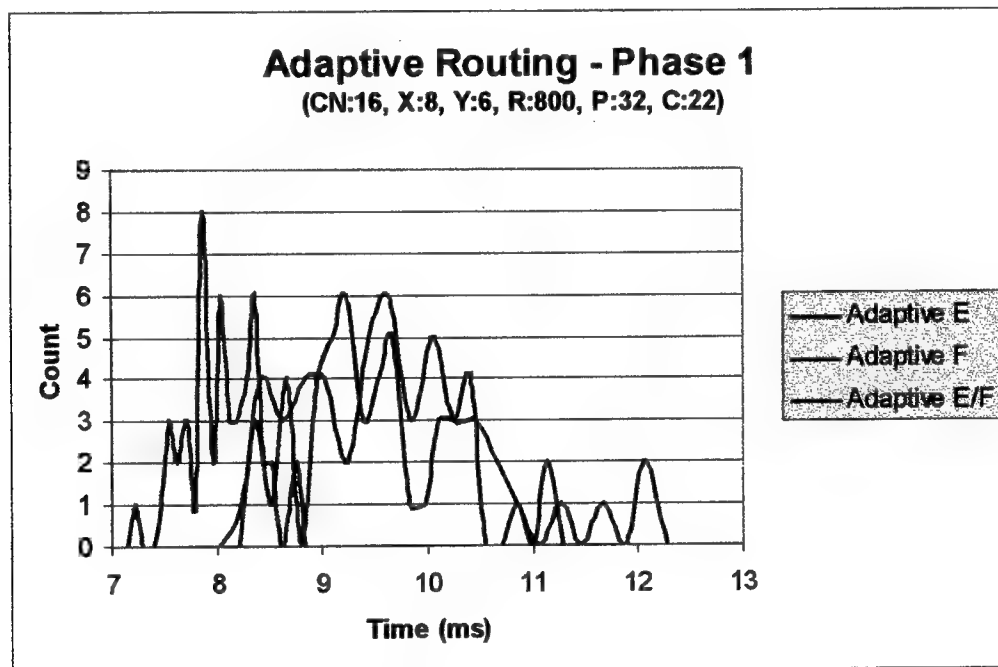


Fig. 15.17: Phase 1 adaptive routing performance metric for a 16 CN (8x6) configuration with range: 800, pulses: 32, and channels: 22.

In the second phase of communication, the adaptive E/F routing outperforms the other two adaptive routing options (see Fig. 15.18). The adaptive E/F routing completed, on average, 5 ms faster than adaptive F routing (i.e., approximately a 15% decrease) and 10

ms faster than adaptive E routing (i.e., approximately a 25% decrease). The adaptive E routing completed last. This is primarily due to the two hardware priority arbitration algorithms at the crossbars. Packets entering port F are given a higher hardware priority than those entering port E. For this simulation, a combination of adaptive E/F routing produces the smallest completion times for both phase 1 and 2.

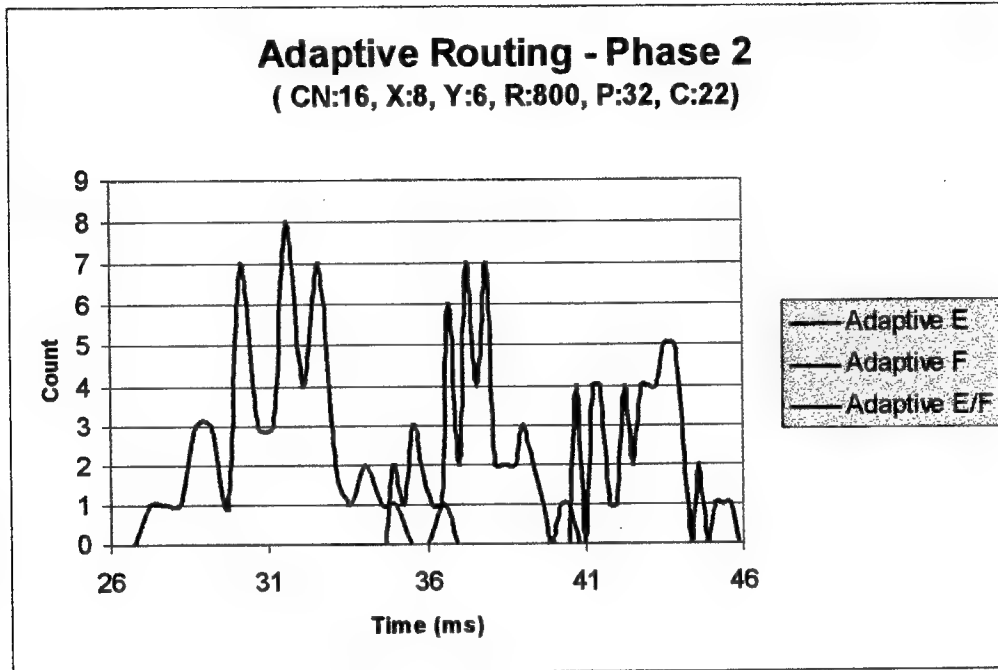


Fig. 15.18: Phase 2 adaptive routing performance metric for a 16 CN (8x6) configuration with range: 800, pulses: 32, and channels: 22.

### 15.3.2 Adaptive Routing Performance Metric 2 for a 16 CN (8x6) Configuration

By applying a slight modification to the simulation parameters in Section 15.3.1, different timing results were obtained. In this simulation, the input parameters of the STAP data cube were modified to produce a smaller data sample. In this case, the range samples were reduced to four hundred, the pulses to twenty-two, and the channels to sixteen. As a result of the changes, the adaptive E/F routing approaches the completion times of adaptive E and F routing in both phases (see Figs. 15.19 and 15.20), although the adaptive E/F

combination still records the shortest time for each phase. In addition, the adaptive E routing options continued to account for the longest completion times.

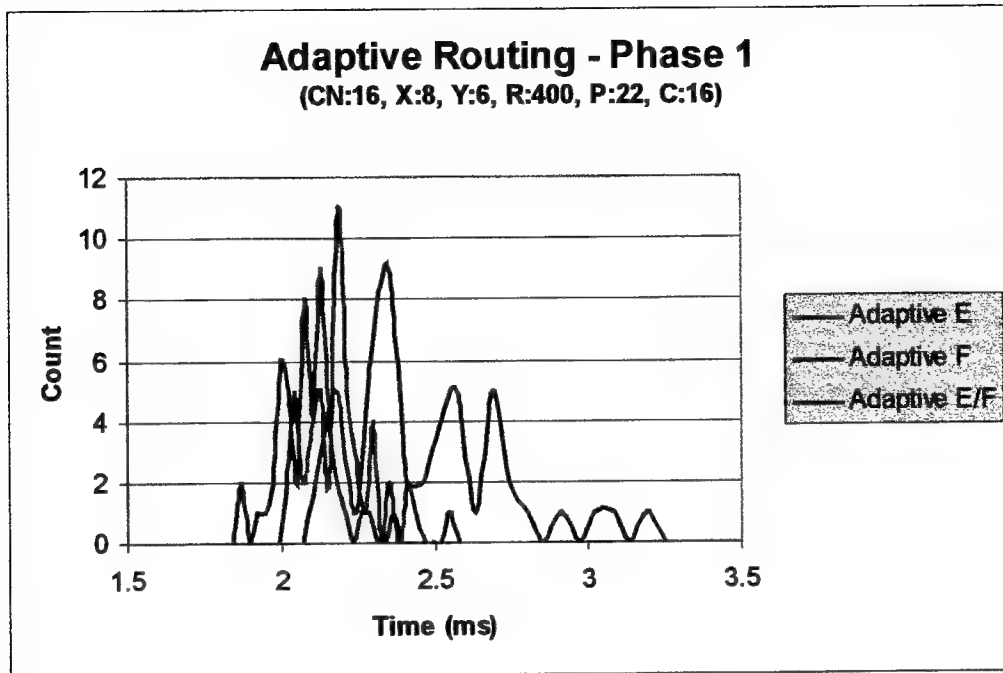


Fig. 15.19: Phase 1 adaptive routing performance metric for a 16 CN (8x6) configuration with range: 400, pulses: 22, and channels: 16.

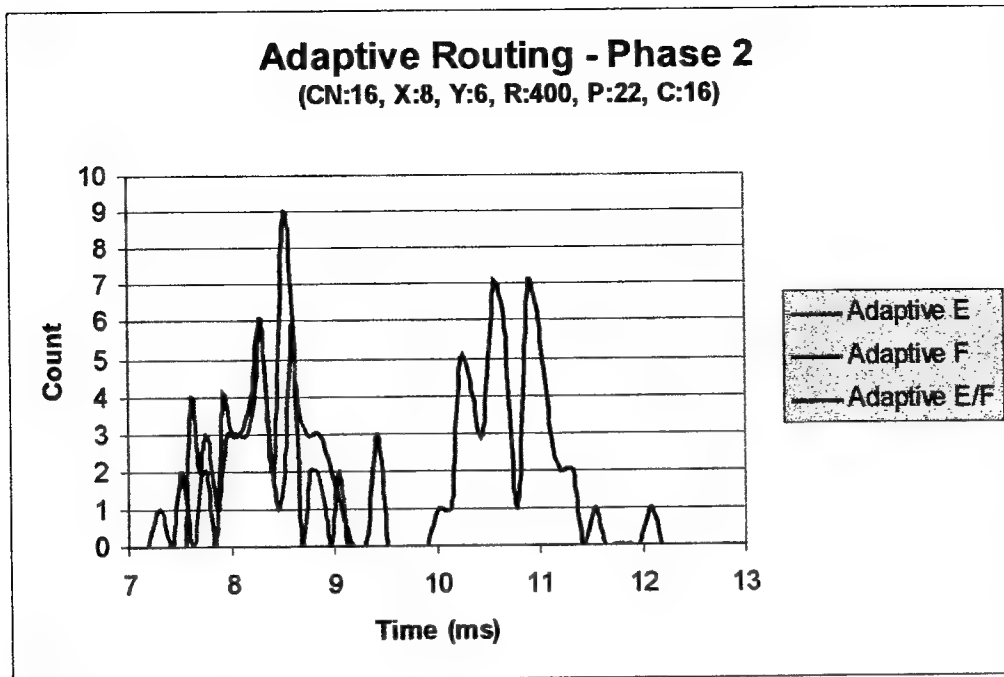


Fig. 15.20: Phase 2 adaptive routing performance metric for a 16 CN (8x6) configuration with range: 400, pulses: 22, and channels: 16.

#### 15.4 DMA Chaining Options

Direct Memory Access (DMA) block transfers may be utilized to send multiple packets to the same destination CN. When packets destined for the same location are DMA chained together, only the first packet is assessed the DMA start up time, which is required to start the DMA controller. The remaining packets do not incur a start up cost, and proceed directly to the route arbitration state. Three distinct investigations were conducted related to DMA chaining. Each of the three simulations generate CE traffic. CE traffic was selected because it tends to create more, but smaller messages than CN traffic. Each simulation involved recording both the phase 1 and phase 2 completion times for fifty simulations.



#### 15.4.1 DMA Chaining Performance Metric 1 for a 24 CE (8x3) Configuration

In the first DMA chaining investigation, the parameters of the system studied included twenty-four CEs, an 8x3 process set, two hundred range samples, twenty-two pulses, sixteen channels, and adaptive F first routing. Fig. 15.21 illustrates the timing results collected from the simulator with DMA chaining enabled and disabled. Under these conditions, the DMA chaining option has only a limited effect on the timing results. Disabling the DMA chaining for this scenario achieves the shortest completion time. Furthermore, each option contains a disparity of approximately .3 ms in recorded completion times. The variation is a product of message ordering prior to communication. This alone suggests that the ordering of the messages influences the performance of the communication pattern.

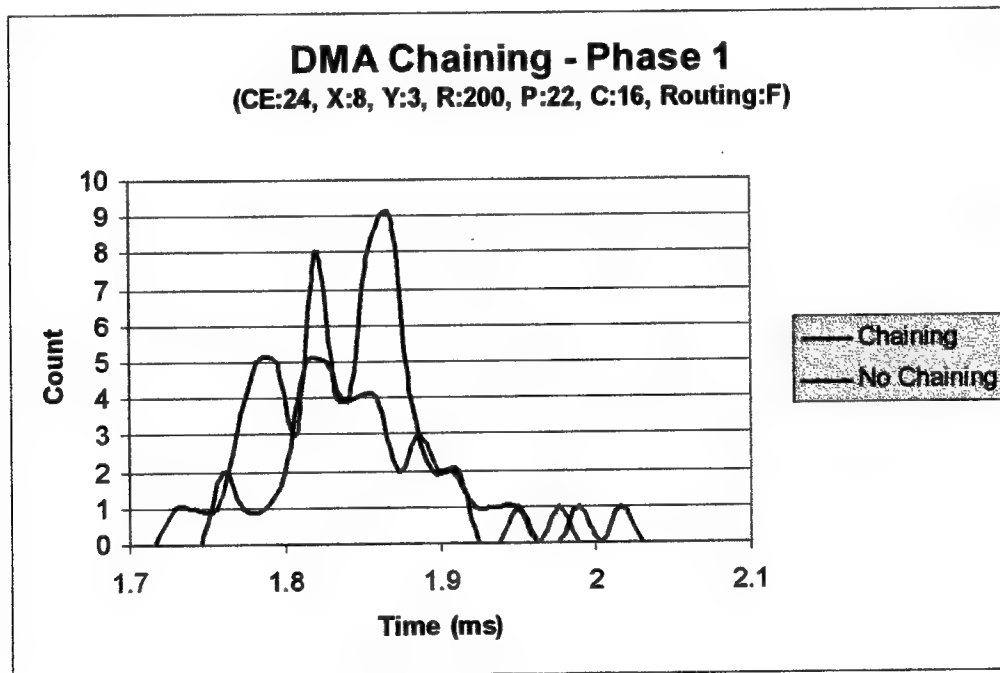


Fig. 15.21: Phase 1 DMA chaining performance metric for a 24 CE (8x3) configuration with range: 200, pulses: 22, channels: 16, and adaptive F routing.

The phase 2 communication details yield a similar results (see Fig. 15.22). The overall performance for both DMA chaining enabled and disabled are comparable. In this instance, the shortest possible completion time is recorded by both options. In addition, the average completion time for no chaining is approximately .2 ms better than with chaining enabled.

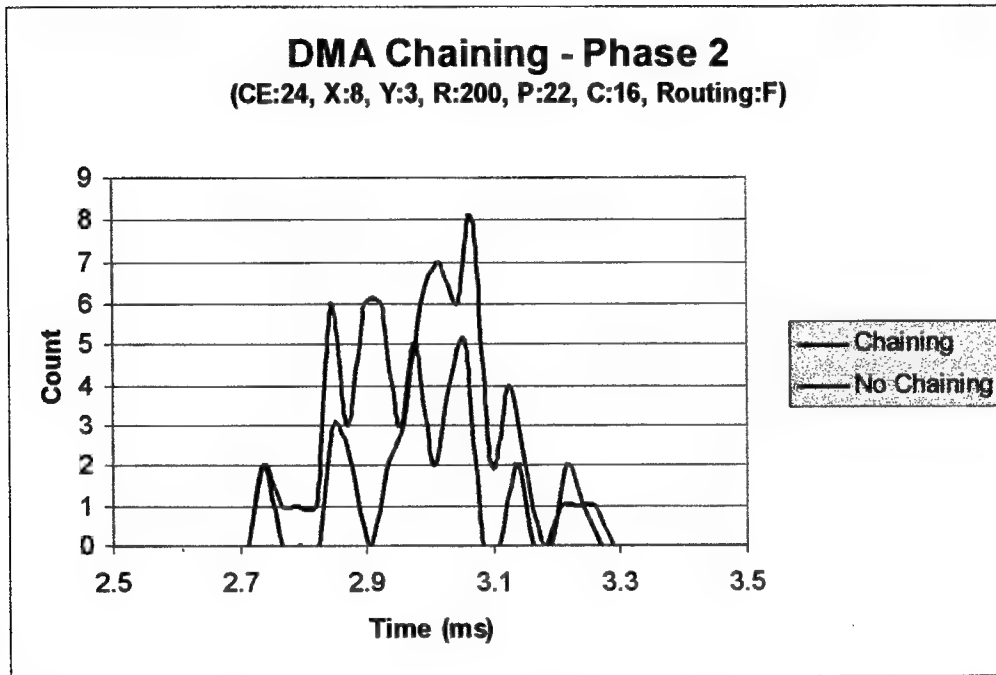


Fig. 15.22: Phase 2 DMA chaining performance metric for a 24 CE (8x3) configuration with range: 200, pulses: 22, channels: 16, and adaptive F routing.

#### 15.4.2 DMA Chaining Performance Metric 2 for a 24 CE (8x3) Configuration

In section 15.4.1, DMA chaining had only a small effect on the performance of the communication pattern. The second performance metric involved investigating the same hardware configuration (i.e., a twenty-four CE system, an 8x3 process set, and adaptive F first routing) but with a larger data cube. Increasing the size of the data cube equates to increasing the amount of packets transmitted during corner-turn phases. The results of increasing the data cube range parameter from two hundred to four hundred for both phases of communication are illustrated in Figs. 15.23 and 15.24. In each phase, disabling the DMA block transfers accounted for the shortest completion time. For this simulation configuration, packets chained together tended to occupy the same connection path repeatedly. Consequently, certain packets remained blocked until the entire DMA block transfer was completed. Disabling the DMA chaining yielded a greater diversity of packets successfully arbitrating through the network. Finally, the variation in recorded completion times for each phase signifies the importance of the outgoing order of messages.

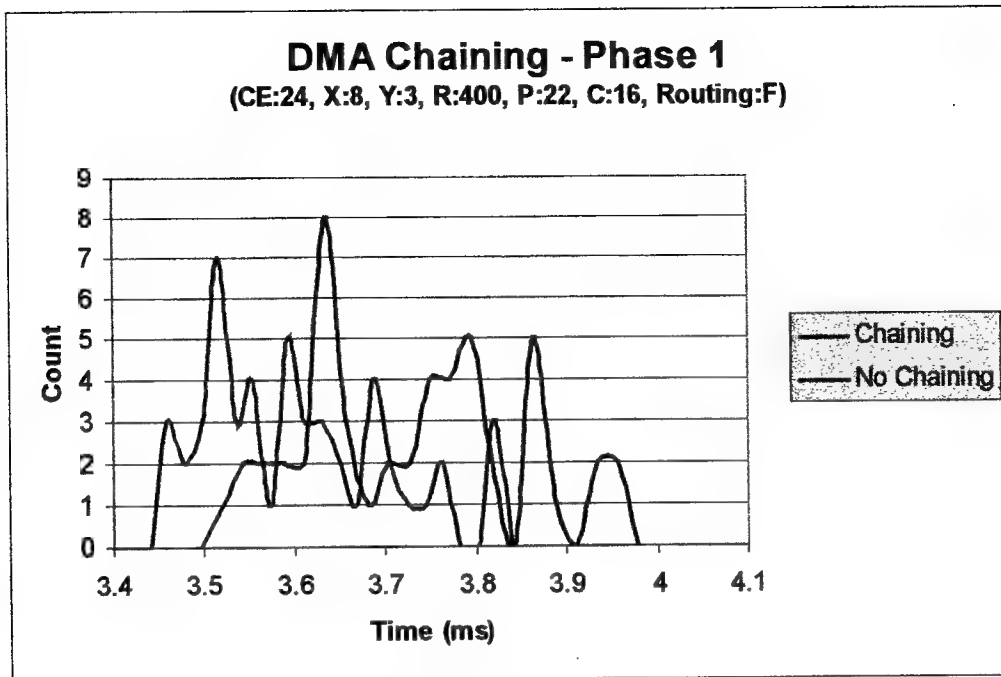


Fig. 15.23: Phase 1 DMA chaining performance metric for a 24 CE (8x3) configuration with range: 400, pulses: 22, channels: 16, and adaptive F routing.

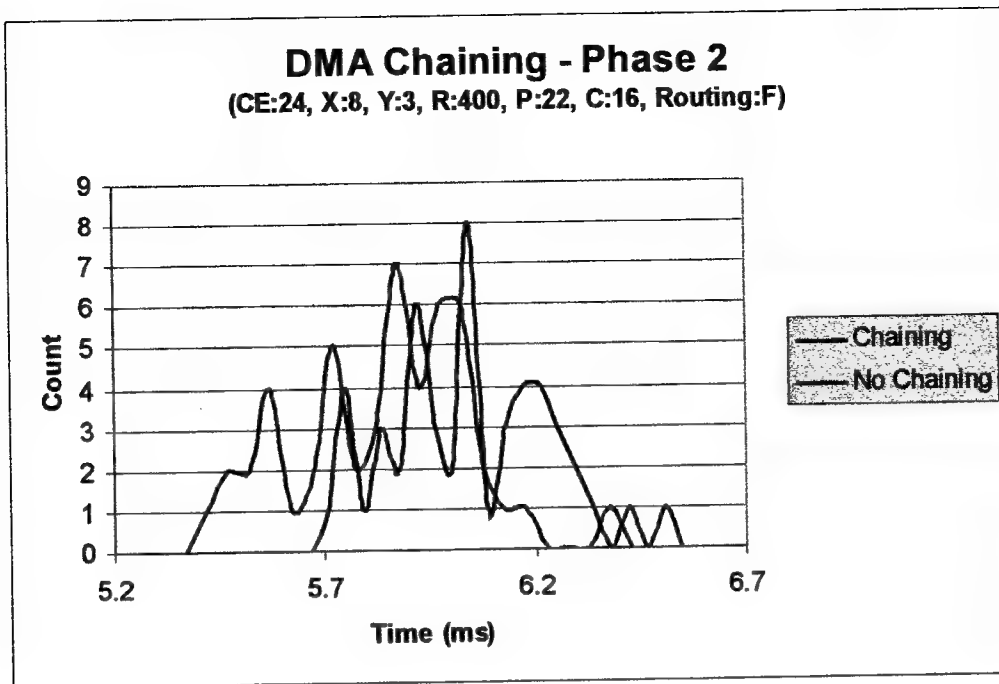


Fig. 15.24: Phase 2 DMA chaining performance metric for a 24 CE (8x3) configuration with range: 400, pulses: 22, channels: 16, and adaptive F routing.

#### 15.4.3 DMA Chaining Performance Metric 3 for a 24 CE (8x3) Configuration

The third and final performance metric involved investigating the same hardware configuration (i.e., a twenty-four CE system, an 8x3 process set, and adaptive F first routing) but with a larger data cube size than either of the first two investigations. Again, increasing the size of the data cube equates to increasing the amount of data communicated during corner-turn phases. The results of increasing all three of the data cube parameters for both phases of communication are illustrated in Figs. 15.25 and 15.26. In both cases, DMA block chaining significantly improved the performance of data transfers. The disparity between the completion times illustrates the fact that message order effects the communication performance whether DMA chaining is enabled or not.

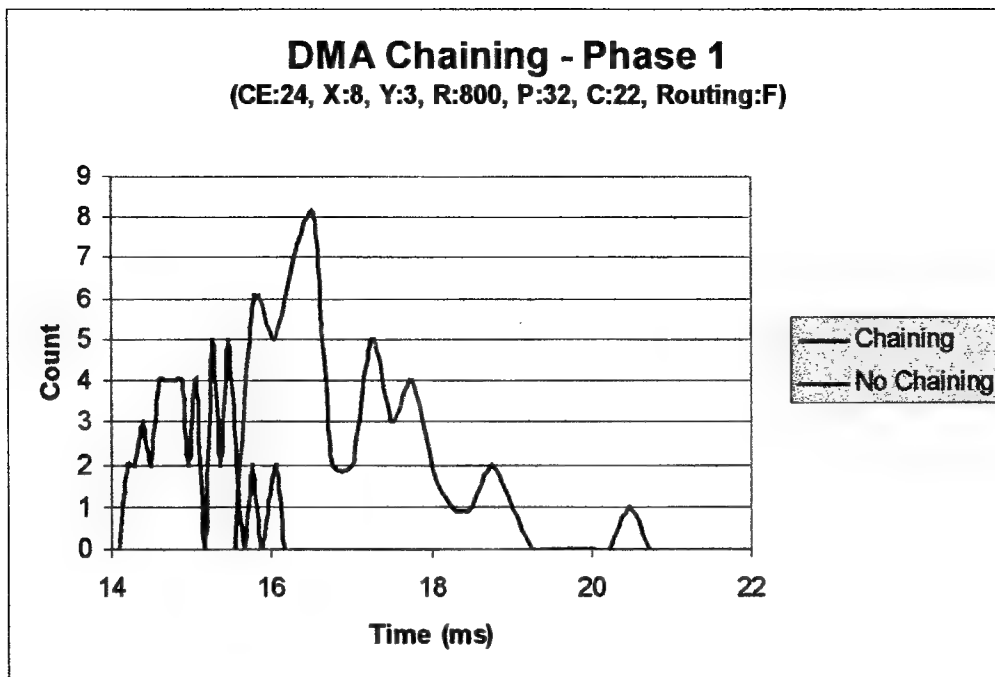


Fig. 15.25: Phase 1 DMA chaining performance metric for a 24 CE (8x3) configuration with range: 800, pulses: 32, channels: 22, and adaptive F routing.

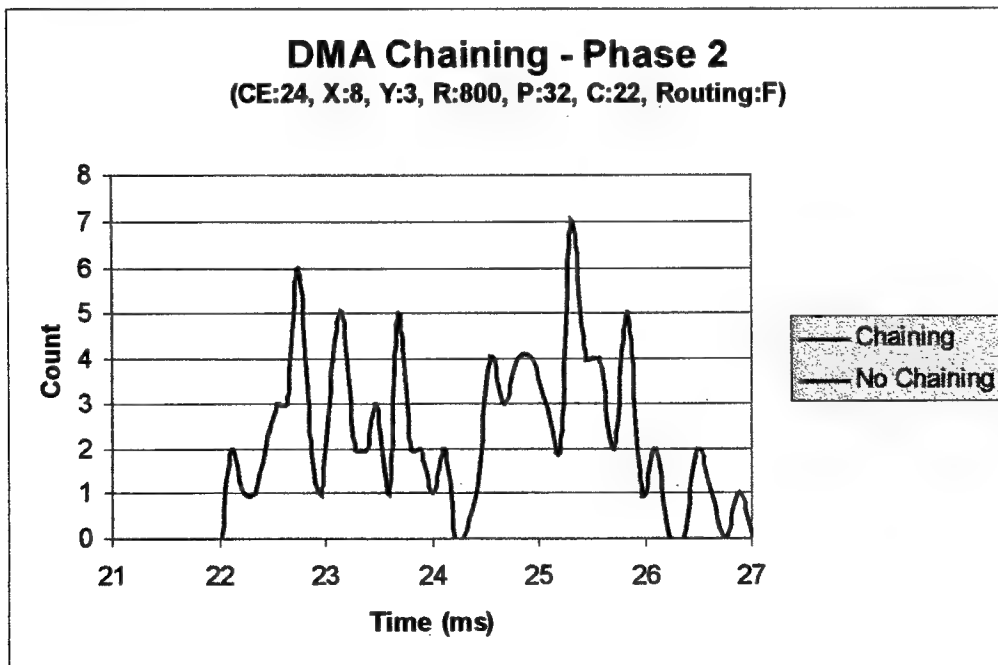


Fig. 15.26: Phase 2 DMA chaining performance metric for a 24 CE (8x3) configuration with range: 800, pulses: 32, channels: 22, and adaptive F routing.

## CHAPTER XVI

### CONCLUSIONS FOR PART 2

Achieving real-time performance for STAP algorithms on parallel embedded systems like the Mercury RACE multicomputer, largely depends two major issues. First is determining the best method for distributing the 3-D STAP data cube across CNs, composed of multiple processors, of the multiprocessor system (i.e., the mapping strategy). Second is determining the scheduling of communications prior to Doppler filtering and weight computation and beamforming. In general, STAP algorithms contain three phases of processing, one for each dimension of the data cube (i.e., range, pulse, and channel). During each phase of processing, the vectors along the dimension of interest are distributed as equally as possible among the CNs for parallel processing. In a sub-cube bar approach, before processing can take place at the next phase, the data vectors must be re-distributed to form contiguous vectors of the next dimension.

Determining the optimal communication schedule of queued messages during the two phases of data re-partitioning may be classified as an NP-hard problem. The goal of the research was to model (through simulation) the effects associated with how data is mapped onto the CNs of the Mercury system using a sub-cube partitioning approach and how the data transfers are scheduled.

Chapter XIV described the design and implementation of the network simulator for the RACE system. Chapter XV provided numerical studies of a subset of the data recorded from simulation scenarios investigated. In general, five parameters can be modified to produce different results from the simulator. The five parameters are: the data cube size, the process set size, the DMA chaining options, the adaptive routing options, and CN or CE message traffic. Investigating all possible combinations of the above simulation parameters is far beyond the scope of this work. However, the results obtained illustrate the importance a network simulator in investigating the effects of communication on performance.

Although used here to study the communication times for parallel STAP algorithms, the simulator is generic enough to be used to predict communication times for any communication pattern. The simulator is very complex and; its implementation required over 6500 lines of Java code. Future work will involve systematic approaches to parameter selection for optimizing performance.



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## APPENDIX

This appendix includes a representative portion of the MATLAB code that was significant in the generation and analysis of the data presented in this work. Note that the actual data was produced by many separate application files, often duplicating common portions of code. This approach was taken in favor of creating one monolithic application that required numerous input arguments or prompts and the associated internal conditional statements. Such an approach inevitably would have resulted in a more cumbersome program, in terms of both execution and maintainability. The code included in the following pages often represents only code fragments, not complete “.m” files.

### Parameter Initialization

%This complete file serves as a general template for  
%power minimization in both the ISMM and CNCM. In addition,  
%the capability to determine integer daughtercard numbers is  
%included. Execution of the complete file therefore produces  
%solutions to the optimal power, mixed card-type configuration  
%of the CNCM with integer card numbers. This configuration  
%can be treated as the most general case with the real-value  
%card number case taken as a specific instance of this program  
%with the integer card-number code removed. Furthermore, the  
%ISMM can similarly be treated as a specific case of the CNCM,  
%where the code concerning CNs is removed.

%This code also serves as the template for the other objective  
%functions, although modifications are necessary in the order  
%of variable calculations (for example, in resolution  
%minimization the azimuth kernel size cannot be calculated  
%until a resolution is known) and to the number of parameters  
%passed. This latter modification is necessary also when  
%switching between the ISMM and CNCM. (Note that in the case  
%of the other objective functions, changing of several  
%variable names would be in order.)

%Because of the occasional lockup of the optimization routine,  
%it is beneficial to allow other starting indices besides 1.  
%The user can obtain the index from the last output. In  
%addition, certain spots in the surface can be recalculated by  
%entering the appropriate index and then aborting the program  
%when the desired sequence is complete. When initiating a  
%fresh run, it is advised that RESULTS are set to [] before  
%invocation of the program. Otherwise they are never cleared  
%in order to allow for midway restarts. The indices range  
%from 1 to the total number of samples. For several reasons,  
%this method proved easier than dealing with both row and  
%column indices.

```
index = input('Enter starting index: ');
```

%Global variables are used to transfer data to the CONSTR  
%optimization routine. Although passing of non-optimization

%variables is provided for in CONSTR, the large number of  
 %variables necessary lent global variables to be a much more  
 %efficient option, without causing unnecessary problems in  
 %program flow or maintainability. Variables are defined as  
 %they are introduced below.

```
clear delta v C1 C2 CNMem CNPow Fa Ka Mr Pr Ma Pa
clear R Rs c T alpha beta gamma lambda Meg
```

```
global delta v C1 C2 CNMem CNPow Fa Ka Mr Pr Ma Pa
global R Rs c T alpha beta gamma lambda Meg
```

%Parameter initializations

```
Meg = 10^6;           %1 million
c = 3*10^8;          %speed of light (m/s)
T = 2048*2/c;        %pulse width (s)
R = 100000;          %range (m)
Rs = 20000;          %range swath (m)
lambda = .03;         %wavelength (m)
alpha = 127/360;      %range non-fast-convolution load (MFlops)
beta = 1061/1170;     %azimuth non-fast-convolution load (MFlops)
gamma = 94;           %fast-convolution load (MFlops)
```

```
CNs = [2 1];          %CNs per card type
CNPow = [6.1 9.6];    %power per CN by card type (w)
CNMem = [16 64];      %memory per CN by card type (MB)
CNCE = [3 2];         %processors per CN by card type
```

```
lb = [1,0,0];         %lower bound for optimization parameters
options = [];          %clear options for optimization functions
```

```
x = [1500,0,0];       %Initial Guess
```

```
%convert index to row and column loop counters
i = ceil(index/25);
j = rem(index-1,25)+1;
```

```
vIter1 = 1;           %inner loop flag
```

```
%resolution sample-space vector
dvector = linspace(.5,2,25);
%velocity sample-space vector
```

```

vvector = linspace(50,400,25);

%outer loop for resolution
for delta=dvector(i:25)
    %check flag for first iteration to use INDEX value
    if ~vIter1
        %set start point of inner loop at 1 after first iteration
        j = 1;
        %clear flag
        vIter = 0;
    end
    %inner loop for velocity
    for v=vvector(j:25)

        FirstIter=1;          %flag for setting first solution value
        Reduced = 0;          %statistics of rounding integer cards
        NonReduced = 0;

        %range values can be calculated statically for each
        %new resolution-velocity pair
        %range FFT size
        Fr = 2^ceil(log2(Rs/delta+c*T/(2*delta)));
        %number of range processors
        Pr = v*(alpha*Rs*gamma+10*Fr*delta*log2(Fr)...
            +6*Fr*delta)/delta^2/Meg/gamma;
        %megabytes of range memory
        Mr = 16*v*Rs*(alpha*Rs*gamma+10*Fr*delta*log2(Fr)...
            +6*Fr*delta)/delta^3/Meg^2/gamma;

        %Although not very flexible, the execution of the
        %following two loops enumerates all the feasible
        %combinations of CN configurations in the CNCM.
        %A more versatile and generalized approach could be
        %formulated by following the derivation of the chapter
        %on the CNCM, using recursive functions or hardcoding
        %the number of loops and employing several conditionals.
        %However, for the task at hand of generating initial
        %test data, the brute force method below was efficient
        %and convenient.
        %C1 and C2 represent X and Y (as in the CNCM chapter),
        %or the first and second card configurations. Note that

```



```

%these two loops would be removed in the case of the
%ISMM.
for C1=[1,1,0; 1,2,0; 1,3,0; 2,1,0; 2,2,0; ...
        2,1,1; 1,1,1; 1,2,1; 1,1,2]'
    for C2=[1,0,1; 1,0,2; 1,0,3; 2,0,1; 2,0,2; ...
            2,1,1; 1,1,1; 1,2,1; 1,1,2]'

        %Do not test for two simultaneous heterogeneous
        %configurations. Only one is necessary.
        if ~( C1(2) & C1(3) & C2(2) & C2(3) )

            %Azimuth kernel size
            Ka = ceil(R*lambda/(2*delta^2));
            %Power of two for azimuth FFT size (Fa)
            FFTk = ceil(log2(Ka+1));

            %flag for yet-active section size constraint
            ConActive=1;

            %while section size constraint still active
            while ConActive

                %compute (next) value of Fa
                Fa = 2^FFTk;

                %x: vector of optimization variables
                %options: optimization toolbox options
                %OptFun: user-defined .m file with
                %    objective function and constraints
                [x,options]=constr('OptFun',x,options,lb);

                %get objective function value and constraints
                %values to check validity
                [f,g] = OptFun(x);

                %Check if all constraints are less than the
                %tolerance (default). CONSTR will display a
                %warning if no feasible solution is found but
                %will still return a rational value for the
                %objective function even if infeasible
                if g <= options(4)

```

```

    %Feasible flag set
    Feasible = 1;
else
    %Feasible flag cleared
    Feasible = 0;
    %objective function value set to infinity
    %for comparison's sake in subsequent
    %iterations
    options(8) = inf;
end %end if-else

%if solution is not feasible, continue
%Note that this section is only when integer
%card numbers are desired. This long
%conditional statement can be skipped if
%real-valued solutions are sought.
if Feasible

    %integer card values, to be determined
    Real = [0 0 0 inf];
    %original values returned by optimization
    Ideal = [x options(8)];

    %The below code is hardcoded for the two
    %daughtercards researched. For a general
    %formulation, the variable CNS should
    %be used.
    %Use modulus to round up to nearest card,
    %dependent on number of CNs per card.
    if C1(1) == 1      %S2T16B
        ceilx(1) = ceil(x(2))...
            + mod(ceil(x(2)),2);
        floorx(1) = floor(x(2))...
            - mod(floor(x(2)),2);
    else                %S1D64B
        ceilx(1) = ceil(x(2));
        floorx(1) = floor(x(2));
    end %end if-else

    if C2(1) == 1      %S2T16B
        ceilx(2) = ceil(x(3))...

```

```

        + mod(ceil(x(3)),2);
    floorx(2) = floor(x(3))...
        - mod(floor(x(3)),2);
else
    %S1D64B
    ceilx(2) = ceil(x(3));
    floorx(2) = floor(x(3));
end %end if-else

```

```

%Check mixed card usage. Otherwise any
%rounding down is infeasible. If mixed,
%check for possible use of floor of one
%card type. Floor of both types is
%always infeasible.
if ceilx

```

```

    %Check for ceiling:floor of C1:C2 if
    %power of such a mix is >= to optimal
    %value. If not, configuration is
    %infeasible by definition of minimum
    %power.
    if( CNPow(C1(1))*ceilx(1) + ...
        CNPow(C2(1))*floorx(2) ...
        >= Ideal(4) )
        %restrict optimization to within
        %new ceiling:floor bounds by setting
        %upper bounds.
        ub=[inf,ceilx(1),floorx(2)];
        %reoptimize with new upper bounds
        [x,options]=constr('OptFun',...
            Ideal(1:3),options,lb,ub);
        %get constraints values
        [f,g] = CNHetFun(x);
        %check for validity of solution
        if all(g <= options(4))...
            & all(x <= (ub + options(4)))
            %set new card and power values
            Real = [round(x(1)) ceilx(1)...
                floorx(2)...
                CNPow(C1(1))*ceilx(1)...
                + CNPow(C2(1))*floorx(2)];
        end %if
    end %if

```

```

end %if

%Check as above but floor:ceiling for
%C1:C2 instead.
if (CNPow(C1(1))*floorx(1)...
    + CNPow(C2(1))*ceilx(2)...
    >= Ideal(4) )
ub=[inf,floorx(1),ceilx(2)];
[x,options]=constr('OptFun',...
    Ideal(1:3),options,lb,ub);
[f,g] = CNHetFun(x);
if all(g <= options(4))...
    & all(x <= (ub + options(4)))
    TmpPow = CNPow(C1(1))*floorx(1)...
        + CNPow(C2(1))*ceilx(2);
    %compare present value to value
    %(if any) of ceiling:floor
    %solution and take best
    if TmpPow < Real(4)
        Real = [round(x(1)) floorx(1)...
            ceilx(2) TmpPow];
    end %if
end %if
end %if
end %if

%check for any valid answer, else...
if Real(4) == inf
    Real = [round(x(1)) ceilx(1) ceilx(2) ...
        CNPow(C1(1))*ceilx(1)...
        + CNPow(C2(1))*ceilx(2)];
    %increment tally of ceiling:ceiling
    %solutions
    NonReduced = NonReduced + 1;
else
    %increment tally of either ceiling:floor
    %or floor:ceiling solutions
    Reduced = Reduced + 1;
end %if-else
end %if

```

```

        %check if answer is feasible and is best so far
        if Feasible & ( FirstIter...
            | ( Real(4) <= Best(8) ) )
            %new best solution
            Best = [delta v Fa Ka Real C1' C2'];
            %clear flag
            FirstIter = 0;
        end %if

        %Check for active section size constraint.
        %If active, increase FFT size and try again.
        %Otherwise, discontinue.
        if ceil(x(1)+Ka)<Fa
            ConActive=0;
        else
            FFTk=FFTk+1;
        end %if-else

    end %while

end %if

    end %for C2
end %for C1

%RESULTS holds the best for each velocity-resolution pair.
%If first iteration flag still set, then no valid solution
%was found during all iterations. Set flags of 0 or
%infinity in values to mark infeasibility.
if FirstIter
    results(index,:) = [delta v Fa Ka 0 0 0 inf...
        0 0 0 0 0 0 0 0];
else
    %REDUCED and NONREDUCED hold tallies for one entire
    %resolution-velocity pair
    results(index,:) = [Best Reduced NonReduced];
end %if-else

%Print to screen results. Note that the results could
%also be printed to a file instead by including a file

```

```

%handle instead of the '1', opened at the start of the
%program.
fprintf(1,'%d\t%.2f\t%.1f\t%d\t%d\t%4d\t%.1f\t%.1f...
\t%.1f\t%d%d%d\t%d%d%d\t%d/%d\n',...
index,Best,Reduced,NonReduced);

%Increment RESULTS array index by one.
index = index + 1;

end %for v
end %for delta

%clear global variables
clear global

```

### Function File Preliminary Code

%The code fragments below are representative of the function  
%files associated with each optimization objective and  
%configuration. The preliminary common to each function  
%is given below, with the unique blocks of code displayed  
%below. Although in reality each block would be a uniquely  
%name file, all the files are grouped here under the name  
%OptFun. This file is the one referenced in the previous  
%example of the main program. Note that modification is  
%necessary in the main program to accomodate the variously  
%sized optimization variables vector and removal of range  
%variable calculations where appropriate.

%Common preliminary code

%X: vector of optimization variables  
%f: objective function value  
%g: vector of constraints values  
function [f,g] = OptFun(x)

%Global parameters

global delta v C1 C2 Fa Ka Mr Pr Ma Pa  
global R Rs c T alpha beta gamma lambda Meg

### ISMM Optimal Single-Card Power

%Single card-type function

%Note that there are only two optimization variables

Sa = x(1);  
C1 = x(2);

Pa = v\*Rs/delta^2/Meg\*(beta+(10\*Fa\*log2(Fa)+6\*Fa)/Sa/gamma);  
Ma = 12\*Rs/delta\*(Sa+1/2\*R\*lambda/delta^2)/Meg;

%Objective Function

f = 12.2\*C1;

%Constraints

g(1) = (Pr + Pa) - 6\*C1;

g(2) = (Mr + Ma) - 32\*C1;

g(3) = Sa - (Fa-Ka);

### ISMM Nominal Mixed Power

Sa = Ka;

C1 = x(1);

C2 = x(2);

%Objective Function

f = 12.2\*C1 + 9.6\*C2;

%Constraints

g(1) = (Pr + Pa) - (6\*C1 + 2\*C2);

g(2) = (Mr + Ma) - (32\*C1 + 64\*C2);

g(3) = Sa - (Fa-Ka);

### ISMM Optimal Mixed Velocity

v = x(1);

Sa = x(2);

C1 = x(3);

C2 = x(2);

Fr = 2<sup>ceil(log2(Rs/delta+c\*T/(2\*delta)))</sup>;

Pr = v\*(alpha\*Rs\*gamma+10\*Fr\*delta\*log2(Fr)+6\*Fr\*delta)...  
/delta^2/Meg/gamma;



```
Mr = 16*v*Rs*(alpha*Rs*gamma+10*Fr*delta*log2(Fr)+6*Fr*delta)...
    /delta^3/Meg^2/gamma;
```

```
Pa = v*Rs/delta^2/Meg*(beta+(10*Fa*log2(Fa)+6*Fa)/Sa/gamma);
Ma = 12*Rs/delta*(Sa+1/2*R*lambda/delta^2)/Meg;
```

```
%Objective Function
f = -v;
```

```
%Constraints
g(1) = 12.2*C1 + 9.6*C2 - P;
g(2) = Sa - (Fa-Ka);
g(3) = (Pr + Pa) - (6*C1 + 2*C2);
g(4) = (Mr + Ma) - (32*C1 + 64*C2);
```

### ISMMS Optimal Mixed Velocity with Set Hardware

```
v = x(1);
Sa = x(2);
```

```
Fr = 2^ceil(log2(Rs/delta+c*T/(2*delta)));
Pr = v*(alpha*Rs*gamma+10*Fr*delta*log2(Fr)+6*Fr*delta)...
    /delta^2/Meg/gamma;
Mr = 16*v*Rs*(alpha*Rs*gamma+10*Fr*delta*log2(Fr)+6*Fr*delta)...
    /delta^3/Meg^2/gamma;
```

```
Pa = v*Rs/delta^2/Meg*(beta+(10*Fa*log2(Fa)+6*Fa)/Sa/gamma);
Ma = 12*Rs/delta*(Sa+1/2*R*lambda/delta^2)/Meg;
```

```
%Objective Function
f = -v;
```

```
%Constraints
g(1) = Sa - (Fa-Ka);
g(2) = (Pr + Pa) - (6*C1 + 2*C2);
```

$$g(3) = (Mr + Ma) - (32*C1 + 64*C2);$$

### ISMM Optimal Mixed Resolution

```

delta = x(1);
Sa = x(2);
C1 = x(3);
C2 = x(4);

Ka=R*lambda/(2*delta^2);

Fr = 2^ceil(log2(Rs/delta+c*T/(2*delta)));
Pr = v*(alpha*Rs*gamma+12*Fr*delta*log2(Fr)+6*Fr*delta)...
    /delta^2/Meg/gamma;
Mr = 16*v*Rs*(alpha*Rs*gamma+12*Fr*delta*log2(Fr)+6*Fr*delta)...
    /delta^3/Meg^2/gamma;

Pa = v*Rs*(beta*Sa*gamma+12*Fa*log2(Fa)+6*Fa)/delta^2/Meg/Sa/gamma;
Ma = 6*Rs*(2*Sa*delta^2+R*lambda)/delta^3/Meg;

%Objective Function
f = delta;

%Constraints
g(1) = 12.2*C1 + 9.6*C2 - P;
g(2) = Sa - (Fa-Ka);
g(3) = (Pr + Pa) - (6*C1 + 2*C2);
g(4) = (Mr + Ma) - (32*C1 + 64*C2);

```

### CNCM Optimal Mixed Power

```

Sa = x(1);

```

```

NumCN1 = x(2);
NumCN2 = x(3);
PaReal = x(4);

Pa = v*Rs/delta^2/Meg*(beta+(10*Fa*log2(Fa)+6*Fa)/Sa/gamma);
Ma = 6*Rs*(2*Sa*delta^2+R*lambda)/delta^3/Meg;

%Objective Function
f = CNPow(C1(1))*NumCN1 + CNPow(C2(1))*NumCN2;

%Constraints
g(1) = Sa - (Fa-Ka);
g(2) = Pr - NumCN1*C1(2) - NumCN2*C2(2);
g(3) = PaReal - NumCN1*C1(3) - NumCN2*C2(3);
g(4) = C1(2)*Mr/Pr + C1(3)*Ma/PaReal - CNMem(C1(1));
g(5) = C2(2)*Mr/Pr + C2(3)*Ma/PaReal - CNMem(C2(1));
g(6) = Pa - PaReal;

```

### CNCM Naive Nominal Mixed Power

```

NumCN1 = x(1);
NumCN2 = x(2);

%Objective Function
f = CNPow(C1(1))*NumCN1 + CNPow(C2(1))*NumCN2;

%Constraints
g(1) = Pr - NumCN1*C1(2) - NumCN2*C2(2);
g(2) = Pa - NumCN1*C1(3) - NumCN2*C2(3);
g(3) = C1(2)*Mr/Pr + C1(3)*Ma/Pa - CNMem(C1(1));
g(4) = C2(2)*Mr/Pr + C2(3)*Ma/Pa - CNMem(C2(1));

```

### CNCM Sophisticated Nominal Power

NumCN1 = x(1);

NumCN2 = x(2);

Pa = x(3);

%Objective Function

f = CNPow(C1(1))\*NumCN1 + CNPow(C2(1))\*NumCN2;

%Constraints

g(1) = Pr - NumCN1\*C1(2) - NumCN2\*C2(2);

g(2) = Pa - NumCN1\*C1(3) - NumCN2\*C2(3);

g(3) = C1(2)\*Mr/Pr + C1(3)\*Ma/Pa - CNMem(C1(1));

g(4) = C2(2)\*Mr/Pr + C2(3)\*Ma/Pa - CNMem(C2(1));